Integrated Review 5: Evaluating Formulas for Probability Distributions

Elementary Statistics Chapter 5: Discrete Probability Distributions

Objectives:

1. Evaluate formulas for probability distributions.
2. Evaluate the binomial probability formula.
3. Evaluate expressions with e.
4. Evaluate the Poisson formula.

Chapter 5 of the Triola text will continue to discuss probability, and several related formulas will be introduced. In this integrated review chapter, we will go over how to evaluate some of those formulas. You will learn the meaning of the formulas in the corresponding chapter of the textbook.

Objective 1: Evaluate formulas for probability distributions.

We reviewed summation notation in *Integrated Review* Chapter 3. As you may recall, the summation symbol, Σ , tells you to add up values.

In the related chapter from the text, we will often check to see if the sum of a table of values equals 1.

Example 1 Determine whether $\Sigma P(x)=1$.

P(x)
0.2
0.15
0.05
0.3
0.2
0.01

So, we will add up the six values that were given in the table and check to see if the sum is equal to 1.

$$\Sigma P(x)=0.2+0.15+0.05+0.3+0.2+0.01=0.91$$

$$0.91 \neq 1$$

Answer $\Sigma P(x)$ does not equal 1.

My Turn!

Determine whether $\Sigma P(x)=1$.

P(*x*)
0.03
0.22
0.15
0.2
0.25
0.15

Sometimes, the summation may require that you use values from two columns of a table.

Example 2 Find $\Sigma\left[x \cdot P(x)\right]$.

x	*P*(*x*)
0	0.2
1	0.15
2	0.05
3	0.3
4	0.2
5	0.1

This notation is telling us to add up the products of the first column of x values with the second column, $P(x)$. You can elect to find the sum in a horizontal fashion, or you can elect to insert a third column to the right with the products and then total that column.

$$\begin{aligned}\Sigma\left[x \cdot P(x)\right] &= (0 \cdot 0.2)+(1 \cdot 0.15)+(2 \cdot 0.05)+(3 \cdot 0.3)+(4 \cdot 0.2)+(5 \cdot 0.1)\\ &= 0+0.15+0.1+0.9+0.8+0.5\\ &= 2.45\end{aligned}$$

Now, we will perform the same calculations in a table format.

x	$P(x)$	$x \cdot P(x)$
0	0.2	0
1	0.15	0.15
2	0.05	0.1
3	0.3	0.9
4	0.2	0.8
5	0.1	0.5
$\Sigma[x \cdot P(x)] =$		2.45

Answer $\Sigma[x \cdot P(x)] = 2.45$

My Turn!

Find $\Sigma[x \cdot P(x)]$.

x	$P(x)$
0	0.3
1	0.35
2	0.15
3	0.2

Example 3 Find $\Sigma\left[x^2 \cdot P(x)\right]$ using the same given table as in Example 2.

x	$P(x)$
0	0.2
1	0.15
2	0.05
3	0.3
4	0.2
5	0.1

This notation is telling us to add up the products of the squares of the first column of x values with the second column, $P(x)$. Again, this could be done horizontally or by using extra columns in a table. Note that we are following the order of operations and will be squaring before multiplying and then finally adding.

$$\begin{aligned}\Sigma\left[x^2 \cdot P(x)\right] &= (0^2 \cdot 0.2) + (1^2 \cdot 0.15) + (2^2 \cdot 0.05) + (3^2 \cdot 0.3) + (4^2 \cdot 0.2) + (5^2 \cdot 0.1)\\ &= (0 \cdot 0.2) + (1 \cdot 0.15) + (4 \cdot 0.05) + (9 \cdot 0.3) + (16 \cdot 0.2) + (25 \cdot 0.1)\\ &= 0 + 0.15 + 0.2 + 2.7 + 3.2 + 2.5\\ &= 8.75\end{aligned}$$

Or, by expanding on the given table, we get the following:

x	x^2	$P(x)$	$x^2 \cdot P(x)$
0	0	0.2	0
1	1	0.15	0.15
2	4	0.05	0.2
3	9	0.3	2.7
4	16	0.2	3.2
5	25	0.1	0.25
	$\Sigma\left[x^2 \cdot P(x)\right] =$		8.75

Answer $\Sigma\left[x^2 \cdot P(x)\right] = 6.5$

My Turn!

Find $\Sigma\left[x^2 \cdot P(x)\right]$.

x	$P(x)$
0	0.3
1	0.35
2	0.15
3	0.2

Example 4 Evaluate $\mu + 2\sigma$ when $\mu = 25.2$ and $\sigma = 2.3$.

We need to substitute 25.2 for μ (the lowercase Greek letter mu) in the beginning of the expression and 1.3 for σ (the lowercase Greek letter sigma) at the end of the expression.

$\mu + 2\sigma = 25.2 + 2(2.3)$

Following the order of operations, we need to multiply before we add.

$25.2 + 2(2.3) = 25.2 + 4.6 = 29.8$

Answer $\mu + 2\sigma = 29.8$ when $\mu = 25.2$ and $\sigma = 2.3$.

My Turn!

Evaluate $\mu + 2\sigma$ when $\mu = 9.9$ and $\sigma = 0.4$.

Example 5 Evaluate $\mu = n \cdot p$ given $n = 150$ and $p = 0.35$.

We are evaluating a formula given the values for all but one variable, μ.

$$\mu = n \cdot p = 150 \cdot 0.35 = 52.5$$

Answer $\mu = 52.5$

My Turn!

Evaluate $\mu = n \cdot p$ when $n = 2000$ and $p = 0.04$.

Example 6 Evaluate $\sigma = \sqrt{np(1-p)}$ $n = 50$ and $n = 0.2$. Round the answer to the nearest tenth.

Begin by replacing n and p with the given values. You will be substituting 0.2 for p in two places. After you have done this, follow the order of operations.

$$\sigma = \sqrt{np(1-p)} = \sqrt{50 \cdot 0.2 \cdot (1-0.2)} = \sqrt{50 \cdot 0.2 \cdot (0.8)} = \sqrt{10 \cdot 0.8} = \sqrt{8} \approx 2.82 \approx 2.8$$

Answer $\sigma \approx 2.8$

My Turn!

Evaluate $\sigma = \sqrt{np(1-p)}$ for $n = 100$ and $p = 0.38$. Round the answer to the nearest tenth.

Objective 2: Evaluate the binomial probability formula.

Example 7 Evaluate ${}_nC_x \cdot p^x \cdot (1-p)^{n-x}$ for $n = 10$, $x = 2$, and $p = 0.3$. Round to three significant digits.

First, we must substitute the given values in for the given variables.

$${}_nC_x \cdot p^x \cdot (1-p)^{n-x} = {}_{10}C_2 \cdot 0.3^2 \cdot (1-0.3)^{10-2}$$

Next, we will clean up the third factor.

$$= {}_{10}C_2 \cdot 0.3^2 \cdot (0.7)^8$$

Then, we will evaluate the combination. Even though we are showing the detailed calculations here, you may want to practice evaluating combinations using technology.

$${}_{10}C_2 = \frac{10!}{2! \cdot (10-2)!} = \frac{10!}{2! \cdot 8!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{10 \cdot 9}{2 \cdot 1} = \frac{90}{2} = 45$$

We now have ${}_{10}C_2 \cdot 0.3^2 \cdot (0.7)^8 = 45 \cdot 0.3^2 \cdot (0.7)^8$. Per the order of operations, we will evaluate exponents next.

$$= 45 \cdot 0.09 \cdot 0.05764801$$

Next, we multiply.

$$= 0.2334744405$$

Finally, we round to 3 significant figures.

$$\approx 0.233$$

Answer ${}_nC_x \cdot p^x \cdot (1-p)^{n-x} \approx 0.233$ for $n = 10$, $x = 2$, and $p = 0.3$.

My Turn!

Evaluate ${}_nC_x \cdot p^x \cdot (1-p)^{n-x}$ for $n = 20$, $x = 5$, and $p = 0.4$. Round to 3 significant digits.

Objective 3: Evaluate expressions with *e*.

The mathematical constant *e* is the number $e \approx 2.71828$.

Example 8 Evaluate e^5. Round the answer to the nearest tenth.

You perform calculations with base *e* as you would with any other base. You will want to find powers of *e* using technology. (Most calculators have an e^x button.)

$$e^5 \approx 148.413 \approx 148.4$$

Answer $e^5 \approx 148.4$

My Turn!

Evaluate $3e^5$. Round the answer to the nearest tenth.

Objective 4: Evaluate the Poisson formula.

Example 9 Let $P(x)=\dfrac{\mu^x \cdot e^{-\mu}}{x!}$ and let $\mu = 2$. Find $P(3)$. Round your answer to three significant figures.

You may notice that this formula is written in function notation. We are being asked to evaluate the function when $x=3$. So, we will substitute 2 in for μ and 3 in for x. Once we have done that, we will follow the order of operations. We will finish by rounding.

$$P(3)=\frac{2^3 \cdot e^{-2}}{3!}\approx\frac{8\cdot 0.13533528}{3!}=\frac{1.08268224}{6}=0.18044704\approx 0.180$$

Answer $P(3)\approx 0.180$

My Turn!

Let $P(x)=\dfrac{\mu^x \cdot e^{-\mu}}{x!}$ and let $\mu = 4$. Find $P(3)$. Round your answer to 3 significant figures.

Answers to My Turn!

1. Yes
2. 1.25
3. 2.75
4. 10.7
5. 80
6. 4.9
7. 0.0746
8. 445.2
9. 0.195

Practice Problems

1. Determine whether $\Sigma P(x)=1$.

$P(x)$
0.18
0.02
0.35
0.2
0.05
0.15

2. Find $\Sigma[x \cdot P(x)]$.

x	$P(x)$
0	0.2
1	0.45
2	0.15
3	0.2

3. Find $\Sigma[x^2 \cdot P(x)]$.

x	$P(x)$
0	0.1
1	0.25
2	0.45
3	0.2

4. Evaluate $\mu - 2\sigma$ when $\mu = 265.9$ and $\sigma = 3.8$.
5. Evaluate $\mu = n \cdot p$ given $n = 1125$ and $p = 0.86$.
6. Evaluate $\sigma = \sqrt{np(1-p)}$ for $n = 200$ and $p = 0.48$. Round the answer to the nearest tenth.
7. Evaluate ${}_nC_x \cdot p^x \cdot (1-p)^{n-x}$ for $n = 15$, $x = 5$, and $p = 0.7$. Round to 3 significant digits.

8. Evaluate $6e^{-3}$. Round the answer to the nearest tenth.

9. Let $P(x)=\dfrac{\mu^{x}\cdot e^{-\mu}}{x!}$ and let $\mu=5$. Find $P(4)$. Round to 3 significant digits.