Integrated Review 10: Linear Equations

Elementary Statistics Chapter 10: Correlation and Regression

Objectives:

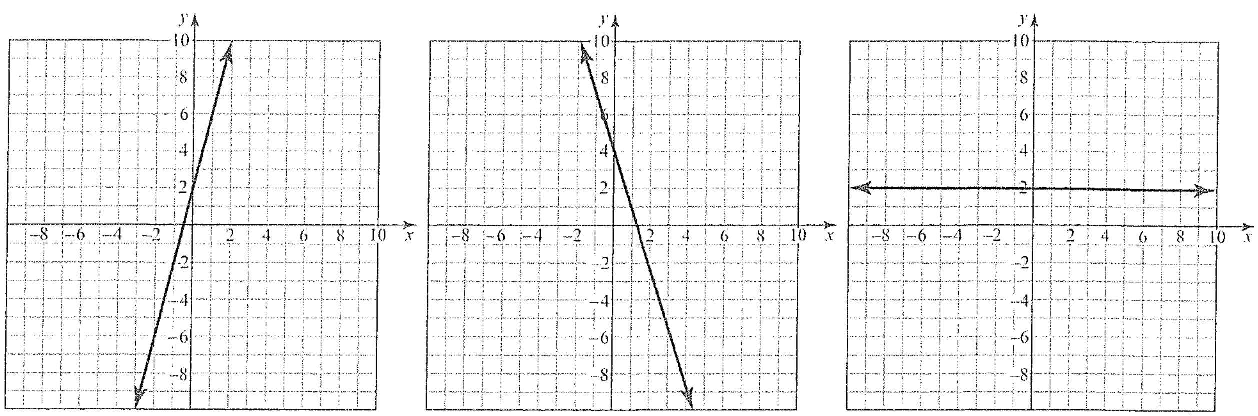
1. Find and interpret slope.
2. Find and interpret the y-intercept of a line.
3. Find values from a linear equation or graph.
4. Graph a linear equation.
5. Find and interpret a linear model ($y = mx + b$).

In this chapter of the Triola text, you will be modeling data with linear equations. In order to understand the statistical concepts, it is imperative that you are comfortable with lines in the coordinate plane. We will review slope, y-intercept, linear equations and their graphs, and linear modeling.

Objective 1: Find and interpret slope.

The slope of the line is its tilt or steepness.

Line with positive slope | Line with negative slope | Line with slope of 0



Here are some phrases related to slope that you may have heard:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y}}{\text{change in x}}$$

The slope of the line m between the points (x_1, y_1) and (x_2, y_2) can be given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1 Find the slope of the line that contains the points (3, 5) and (6, 9).

We can label the points as follows $(x_1, y_1) = (3,5)$ and $(x_2, y_2) = (6,9)$.

Now, we will substitute the given points into the formula for slope:

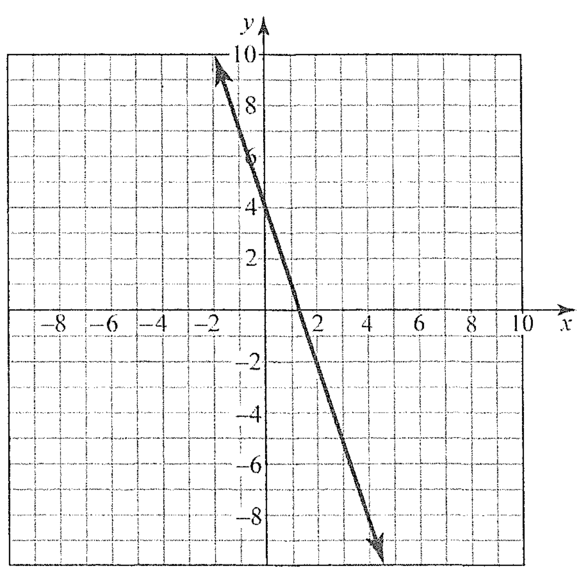
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-5}{6-3} = \frac{4}{3}$$

Answer The slope of the line that passes through the points (3, 5) and (6, 9) is $\frac{4}{3}$.

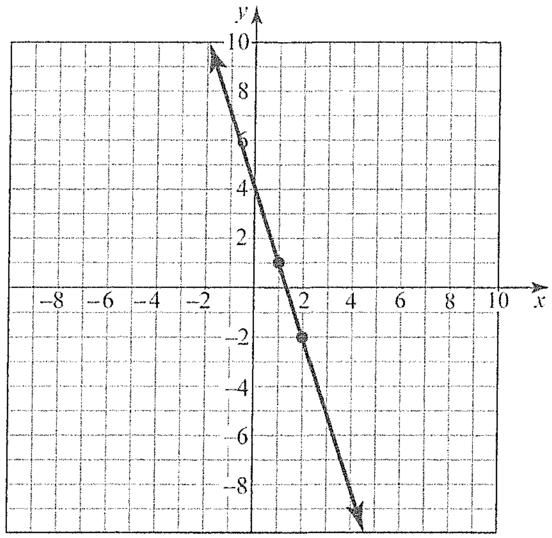
My Turn!

Find the slope of the line that contains the points (0.5, 2) and (0.7, 5). Round the answer to the nearest hundredth.

Example 2 Find the slope of the line given in the following graph.



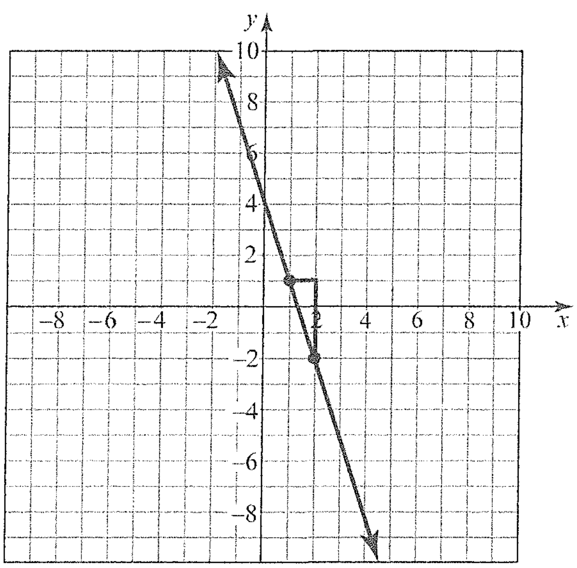
Begin by locating two points on the line. We will use the points $(1, 1)$ and $(2,-2)$, although you could use any two points on the line.



You could then either use the formula or map out the rise over run from the graph.

Formula Method: $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-1}{2-1} = \frac{-3}{1} = -3$$

Rise-Over-Run Method: Let's look at this problem as if we are travelling from $(2,-2)$ to $(1, 1)$. The rise from -2 to 1 is 3 units. The run from 2 to 1 is -1 . The run is negative because we went to the left, which is the direction of the negatives on the number line. (You could have also traveled from $(1, 1)$ to $(2,-2)$, which has a rise of -3 and a run of 1, resulting in the same slope.)

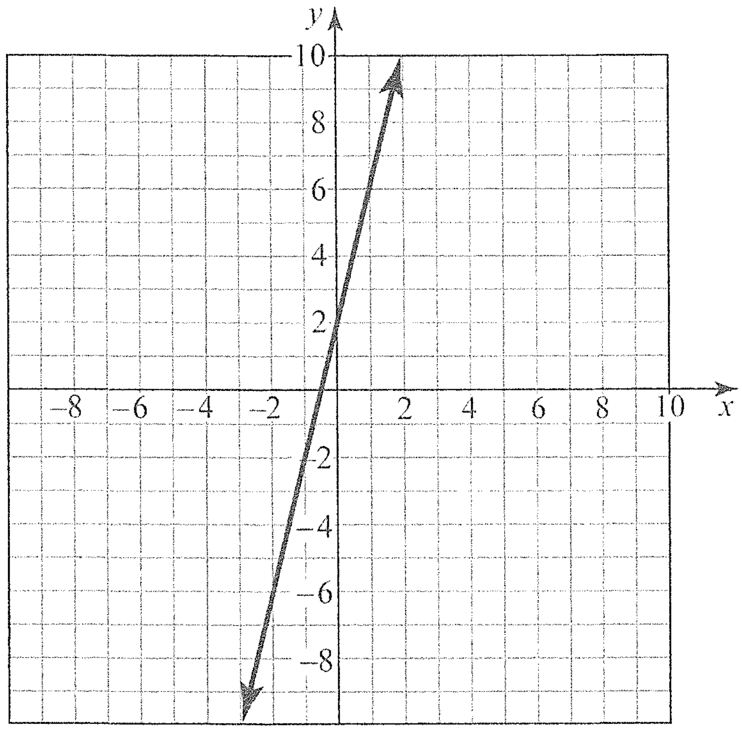


$$m = \frac{\text{rise}}{\text{run}} = \frac{3}{-1} = -3$$

Answer The slope is -3.

My Turn!

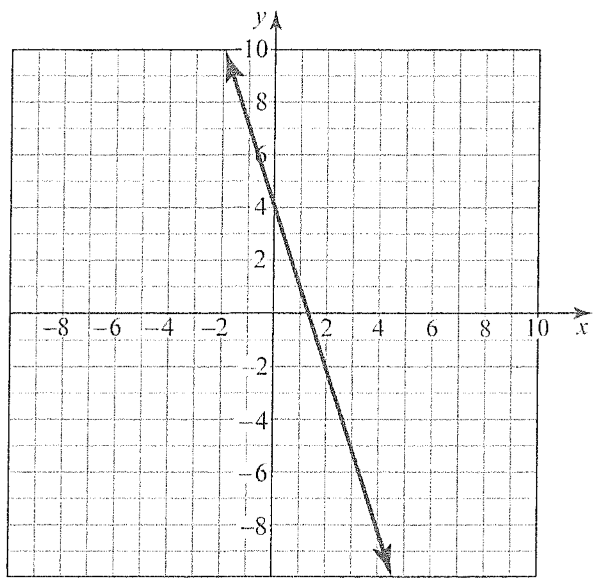
Find the slope of the line given in the following graph.



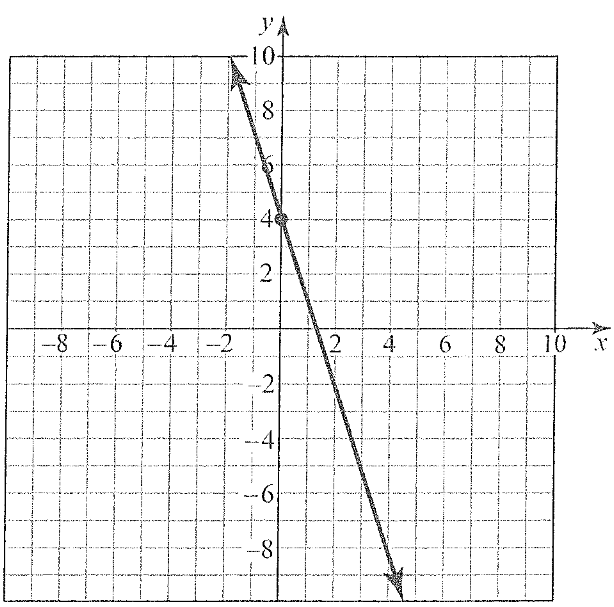
Objective 2: Find and interpret the y-intercept of a line.

The y-intercept(s) of a graph is (are) the point(s) where a graph crosses the y-axis. For a line, there is at most one y-intercept. Any y-intercept should be written as an ordered pair in the format $(0, b)$, where b is the y-coordinate of the y-intercept.

Example 3 Find the y-intercept of the following graph.



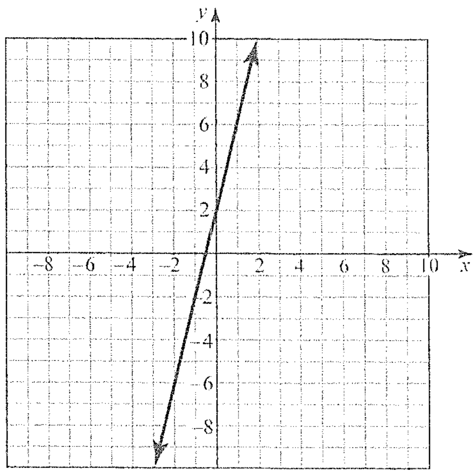
Looking at the graph, we can see that the line clearly crosses through the y-axis at the point (0,4).



Answer The y-intercept of the line is (0, 4).

My Turn!

Find the *y*-intercept of the following graph.



For any linear equation written in slope-intercept form, $y = mx + b$. The *y*-intercept can be written as $(0, b)$. That is, the constant value b is the *y*-coordinate of the *y*-intercept.

Example 4 Find the *y*-intercept of the line $y = 7x - 3$.

The equation is written in the form $y = mx + b$. The value of b is -3. Since the *y*-intercept should be written as an ordered pair $(0, b)$, the *y*-intercept for this line is $(0, -3)$.

Answer The *y*-intercept is $(0, -3)$.

My Turn!

Find the *y*-intercept of the line.

$$\hat{y} = \frac{2}{3}x + \frac{1}{2}$$

Objective 3: Find values from a linear equation or graph.

Example 5 Find three pairs of data values that would be on the line $\hat{y} = 5x + 7$.

There are actually an infinite number of points on any line. So, answers can vary.

If you plug in any x value, you will get a corresponding $\hat{y}$ (y-hat) value.

Let's plug in 0, 1, and 2 for x.

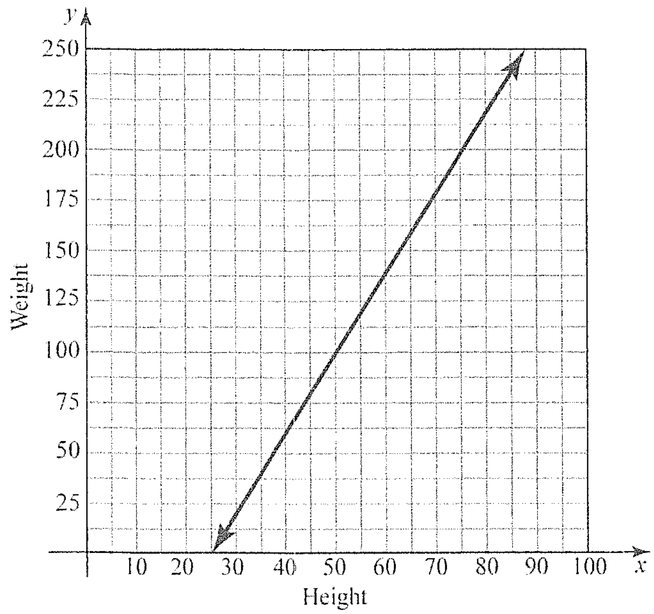
x	$\hat{y} = 5x + 7$	**Paired data values**
0	$\hat{y}=5(0)+7=0+7=7$	(0, 7)
1	$\hat{y}=5(1)+7=5+7=12$	(1, 12)
2	$\hat{y}=5(2)+7=10+7=17$	(2, 17)

Answer The points (0, 7), (1, 12), and (2, 17) are on the line $\hat{y} = 5x + 7$.

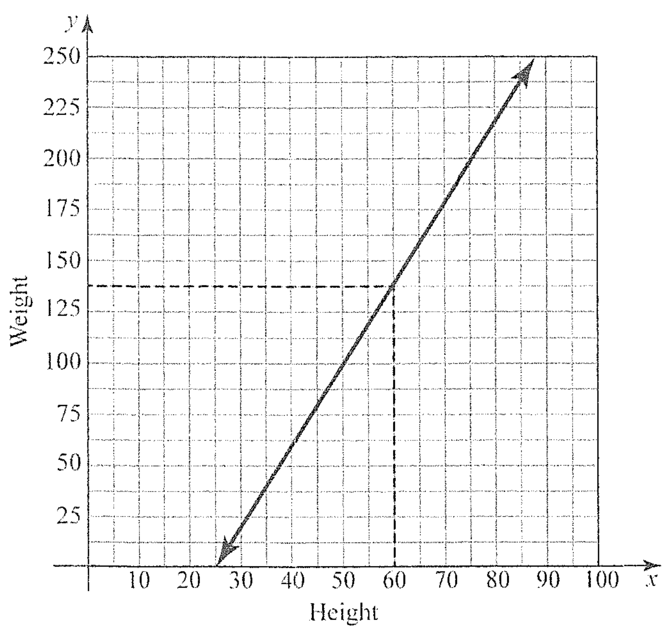
My Turn!

Find three pairs of data values that would be on the line $\hat{y} = -2x + 3$.

Example 6 In the following graph, the horizontal axis represents the height of human beings and the vertical axis represents the weight of human beings. Find the weight (in pounds) for somebody who is 60 inches tall.



The horizontal axis represents height and the vertical axis represents weight. So, we begin by locating the given value of height = 60 on the horizontal axis and then move up to locate the point on the given line. Then, we look over to the vertical axis and find the corresponding y value.

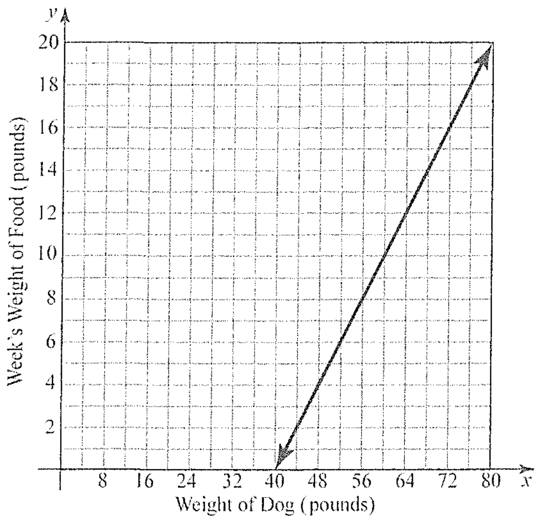


The corresponding weight for a height of 60 inches looks to be about 137.5 pounds, since it halfway between 125 and 150 pounds.

Answer 137.5 pounds

My Turn!

In the following graph, the horizontal axis represents the weight (in pounds) of dogs and the vertical axis represents the weight (in pounds) that the dog eats in a week. How many pounds of dog food does a dog who weighs 48 pounds eat in a week?

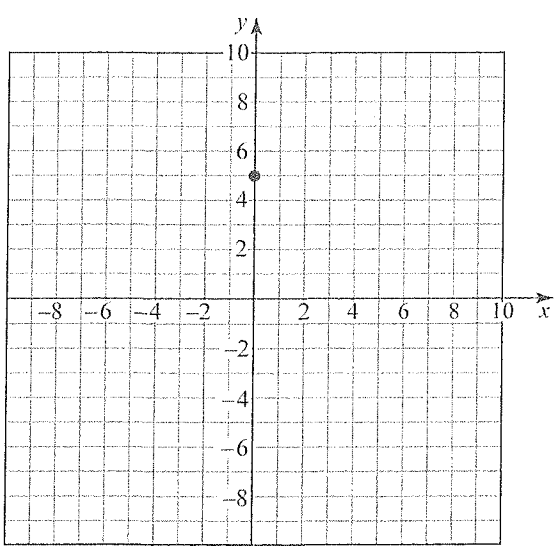


Objective 4: Graph a linear equation.

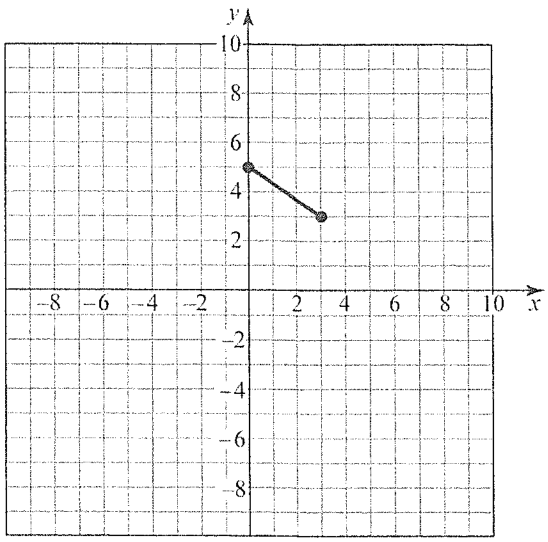
Example 7 Graph $y = -\frac{2}{3}x + 5$.

There are quite a few strategies for graphing lines from an equation. For this example, we will use the slope and the y-intercept to graph the line.

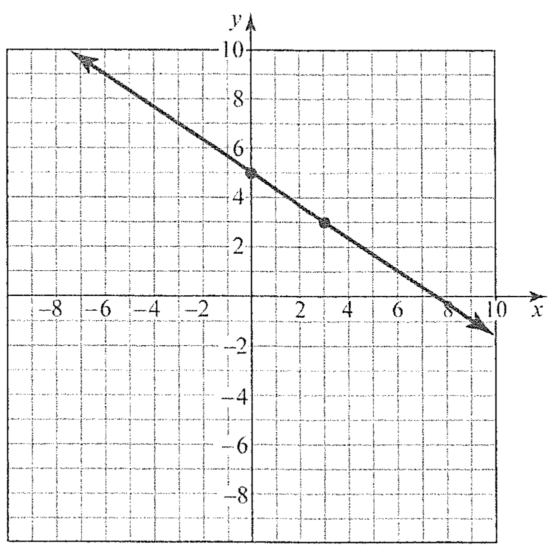
First, extract the y-intercept from the linear equation. Since $b = 5$, the y-intercept for this line is (0, 5). Plot the y-intercept.



Once you have the y-intercept, you can use the slope to locate another point. The slope in this problem is $-\frac{2}{3}$. So, another point will be down 2 units and to the right 3 units. (Remember rise/run.)

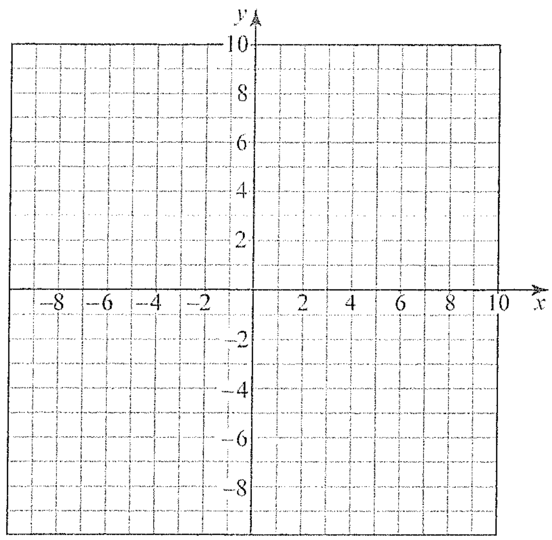


You can now extend this line using a straightedge.



My Turn!

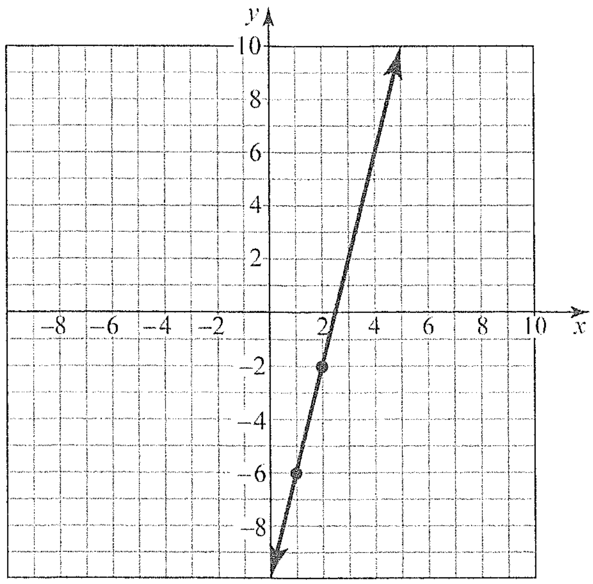
Graph $y = \frac{1}{4}x + 3$.



Example 8 Graph $y = 4x - 10$.

You could graph this equation using the method of Example 6 by using the y-intercept of $(0, -10)$ and the slope of 4. However, we will review another method in this example. You can find two points on the line and then connect them with a straightedge. Finding a third point is recommended as a check (or you could use the y-intercept as a check).

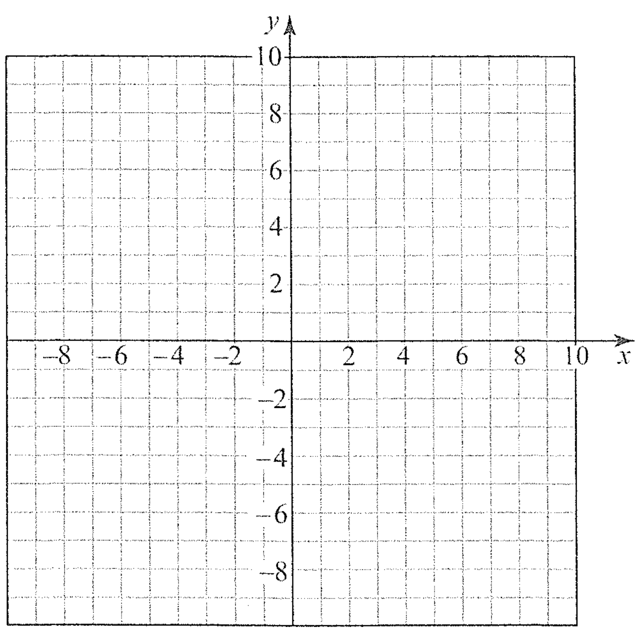
x	$y = 4x - 10$	**Paired data values**
1	$y = 4(1) - 10 = 4 - 10 = -6$	$(1, -6)$
2	$y = 4(2) - 10 = 8 - 10 = -2$	$(2, -2)$



We can see that the line also passes through the y-intercept $(0,-10)$, which serves as our check.

My Turn!

Graph $y=\frac{1}{2}x-5$. (Hint: You may want to substitute values for x that are multiples of the denominator.)



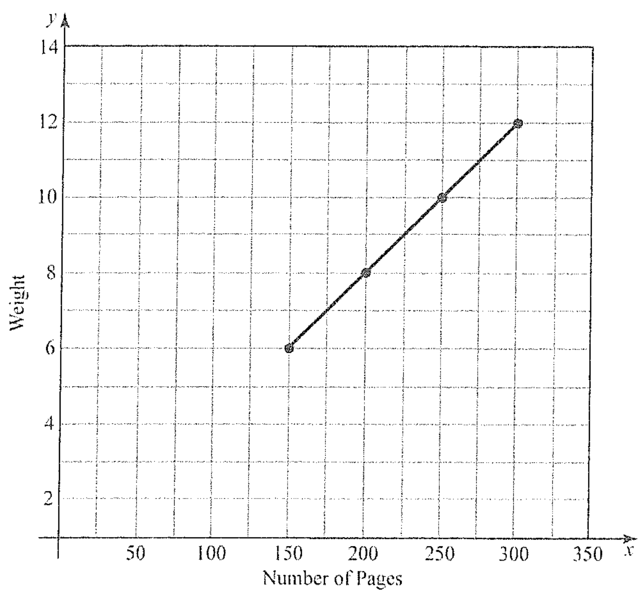
Objective 5: Find and interpret a linear model ($\hat{y} = mx + b$).

You will be studying linear models in great detail in the textbook. For those problems, you will be looking at real data that can sometimes be messy and may best be handled with a calculator. In this workbook, we will use a simplistic model to help prepare you.

Example 9 Find a linear model for the following data for the number of pages in a book and its weight in ounces.

Number of Pages	Weight (oz.)
150	6
200	8
250	10
300	12

The first thing you may want to do is plot the points to determine whether they fall on a line.



It does appear that the data fall on a line. So, we will use two points to find the slope.

$$m = \frac{8-6}{200-150} = \frac{2}{50} = \frac{1}{25}$$

We now need to find the y- intercept. Since it was not one of the given points, we can substitute any given point for x and $\hat{y}$ into $\hat{y} = \frac{1}{25}x + b$ and solve for b.

$$\hat{y} = \frac{1}{25}x + b$$
$$6 = \frac{1}{25}(150) + b$$
$$6 = 6 + b$$
$$0 = b$$

So, the linear model that fits the data is $\hat{y} = \frac{1}{25}x + 0$.

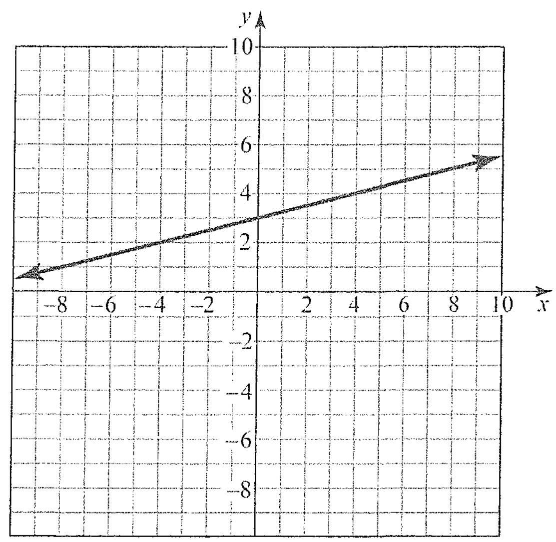
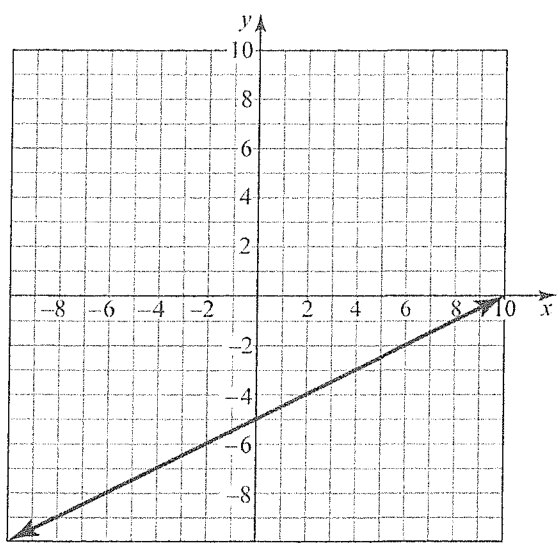
Answer $\hat{y} = \frac{1}{25}x$

My Turn!

Find a linear model for the following data for the number of hours spent studying and the final exam score.

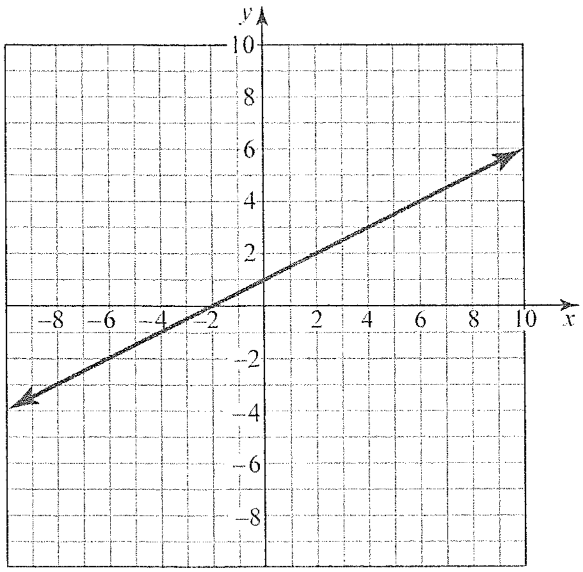
Number of Hours Spent Studying	**Final Exam Score**
4	40
6	50
12	80
18	100

Answers to My Turn!

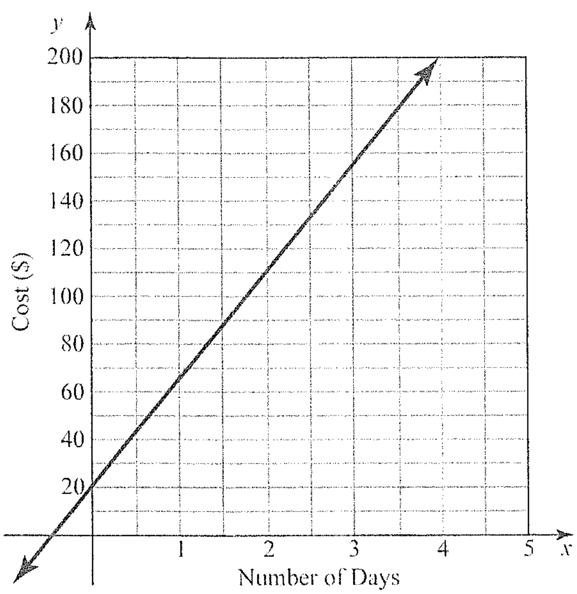
1. 15
2. 4
3. (0, 2)
4. (0, ½)
5. Answers will vary. (0,3), (1,1), (2, −1)
6. 4 pounds
7. 
8. 
9. $\hat{y} = 5x + 20$

Practice Problems

1. Find the slope of the line that contains the points (3.5, 2.4) and (4.5, 1.6).
2. Find the slope of the line in the following graph.



3. Find the y-intercept of the graph shown for Practice Problem 2.
4. Find the y-intercept of the line $\hat{y} = -0.28x + 1.5$.
5. Find three pairs of data values that would be on the line $\hat{y} = -x + 0.2$.
6. The horizontal axis in the following graph represents the number of days that a car is rented, and the vertical axis represents the cost in dollars. How much does it cost to rent a car for 2 days?



7. Graph $y = -x + 1$ using the slope and y-intercept.
8. Graph $y = \frac{4}{5}x - 3$ by plotting points.
9. Find a linear model for the following data for the number of courses that a student is enrolled in and the number of hours spent watching TV on a Wednesday night.

Number of Courses	**Number of Hours Watching TV**
2	4
3	3
4	2
5	1

Appendix A: Statistics Symbols

Symbol	Read as
α	Alpha ('ælfə)
β	Beta ('bi:tə)
μ	Mu ('mju:)
$\overline{x}$	x-bar
σ	Sigma ('sɪgmə)
ρ	Rho ('rəʊ)
χ	Chi ('kaɪ)
$\hat{p}$	p-hat
$\hat{q}$	q-hat
$\hat{y}$	y-hat
$\overline{d}$	d-bar
μ_d	Mu-sub-d
s_d	s-sub-d
$\overline{A}$	A-bar, the complement of event A (used for probability)
$\approx$	Is approximately equal to
$<$	Is less than
$>$	Is greater than
$\geq$	Is greater than or equal to
$\leq$	Is less than or equal to
df	Degrees of freedom

Appendix B: Units of Measurement

Metric		
Unit of Measurement	**Abbreviation**	**Measures**
Gram	g	Mass (weight)
Meter	m	Length
Liter	l	Volume
Degree Celsius	° C	Temperature

English		
Unit of Measurement	**Abbreviation**	**Measures**
Ounce	oz.	Mass (weight)
Pound	lb.	Mass (weight)
Ton	tn.	Mass (weight)
Inch	in.	Length
Foot (plural feet)	ft.	Length
Yard	yd.	Length
Mile	mi.	Length
Pint	pt.	Volume
Quart	qt.	Volume
Gallon	gal.	Volume
Degree Fahrenheit	° F	Temperature

Appendix C: Conversions between Units of Measurement

Within English System	
16 ounces	1 pound
2240 pounds	1 ton
12 inches	1 foot
3 feet	1 yard
5280 feet	1 mile
2 pints	1 quart
4 quarts	1 gallon

Within Metric System	
Kilo-	1000 basic units (meters, liters, or grams)
Hecto-	100 basic units (meters, liters, or grams)
Deka- (or deca-)	10 basic units (meters, liters, or grams)
Deci-	0.1 basic units (meters, liters, or grams)
Centi-	0.01 basic units (meters, liters, or grams)
Milli-	0.001 basic units (meters, liters, or grams)

Between English and Metric Systems	
1 pound	0.454 kilograms
1 inch	2.54 centimeters
1 mile	1.61 kilometers
1 pint	0.473 liters
1 fluid ounce	0.0296 liters

The conversion from degrees Fahrenheit (F) to degrees Celsius (C) requires the use of the following formula:

$$C = \frac{5}{9}(F - 32)$$

The conversion from degrees Celsius (C) to degrees Fahrenheit (F) requires the use of the following formula:

$$F = \frac{9}{5}C + 32$$

Answers to Practice Problems

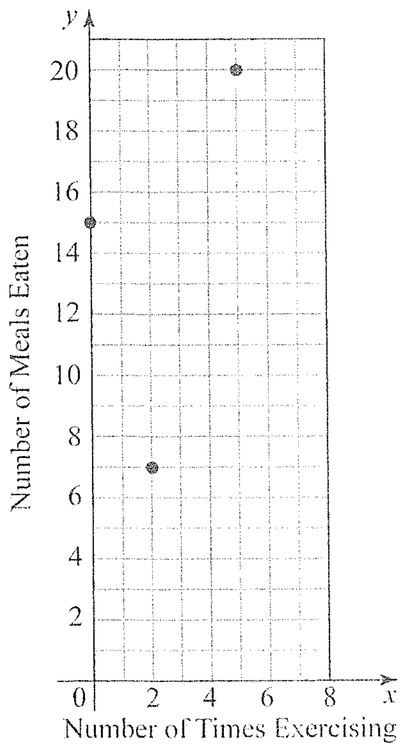
Integrated Review 1

1. $d = 12$
2. In the United States, we have presidential elections every four years. So, the possible values for t are 0 (election year), 1, 2, 3, or 4.
3. 16.4 feet
4. 35,000 meters
5. Integers, real numbers
6. 729
7. 7.06×10^{-4}
8. 2.7×10^{5}
9. 97,100,000
10. a) 7 significant digits
    b) 3 significant digits
    c) 1 significant digit
11. 8.67×10^{-19}

Integrated Review 2

1. 24,967.1 kilograms
2. 1.05 pounds
3. $\frac{3}{7}$
4. 0.025
5. 0.7%
6. 8%
7. $\frac{9}{500}$
8. 5%
9. 224
10. 66 subjects

11.



12. 14%
13. 22%

Integrated Review 3

1. 5
2. 14
3. $\frac{16}{85} \approx 0.19$
4. 16
5. 10 and 11
6. 15.97
7. 1.77
8. $\sigma \approx 3.4$
9. 6.5
10. 36.7

Integrated Review 4

1. $\frac{1}{3}$
2. $\frac{79}{70}$

3. $\frac{2}{15}$
4. $\frac{1}{10}$
5. $\frac{1}{24}$
6. $\frac{64}{81}$
7. $\frac{1}{64}$
8. 5.876
9. 3.246
10. 2.625
11. 14.1
12. 362,880
13. 3024
14. 126
15. $A \cap B = \{\text{red}\}$, $A \cup B = \{\text{red, blue, yellow, pink, purple}\}$ and $\overline{A} = \{\text{orange, green, purple, pink, white, black}\}$

Integrated Review 5

1. No
2. 1.35
3. 3.85
4. 258.3
5. 967.5
6. 7.1
7. 0.00298
8. 0.3
9. 0.175

Integrated Review 6

1. 0.3 in.^{2}
2. 10, 11
3. $l > 18, v < 60$
4. $z \approx -1.04$
5. $x \approx 98.2$
6. $x \approx 106.6$
7. $z = 4$

Integrated Review 7

1. 0.65
2. 10.105
3. 2.75
4. (3.5,11.9)
5. $0.63 < \mu < 0.92$
6. 25.2 ± 1.1
7. $E \approx 0.057$
8. $E \approx 0.24$
9. $n \approx 140$

Integrated Review 8

1. $z \approx -7.01$
2. $t \approx 3.46$
3. $\chi^2 \approx 161.8$

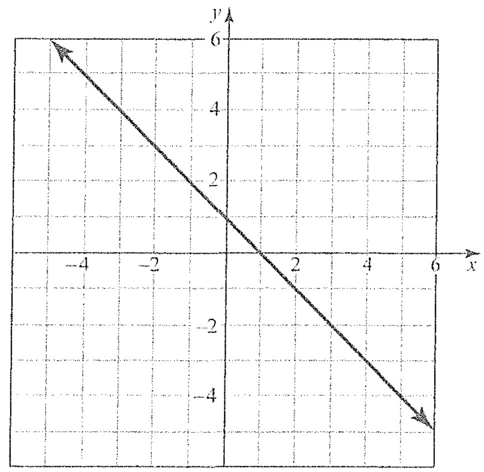
Integrated Review 9

1. $\overline{p} \approx 0.5533$
2. $z \approx 3.00$
3. $E \approx 0.0959$
4. $t \approx -13.6$
5. $E \approx 0.2$
6. $t = 60$

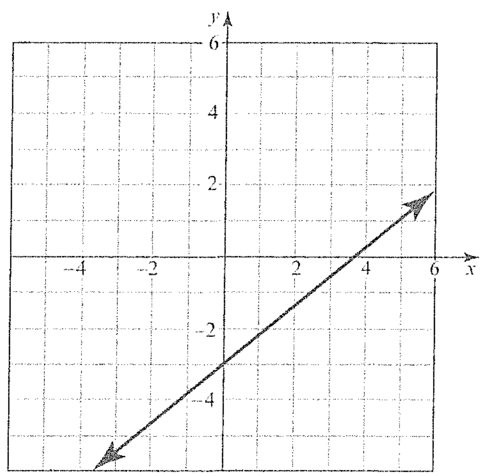
Integrated Review 10

1. –0.8
2. $\frac{1}{2}$
3. (0, 1)
4. (0, 1.5)
5. Answers will vary. (0, 0.2), $(1, -0.8)$, $(2, -1.8)$
6. $110

7.



8.



9. $\hat{y} = -x + 6$