Integrated Review 4: Preparing for Probability

Elementary Statistics Chapter 4: Probability

Objectives:

1. Apply operations to fractions.
2. Apply operations to decimals.
3. Evaluate factorials.
4. Evaluate $_nP_r$ and $_nC_r$.
5. Determine the intersection, union, and complement of two sets.

In this chapter of the Triola text, you will learn the basics of probability. The probability, or likelihood, that some event will occur can be written either as a decimal or a fraction. For this reason, when you are evaluating probability formulas you will be dealing with operations on decimals and fractions. We will begin this integrated review chapter by reviewing operations on fractions and decimals. Then we will review some basic calculations that you will use for calculating other probability formulas. Finally, we will review the common operations of intersection, union, and complement of sets, as these concepts tie in with some other probability calculations. Once again, we will not explain the probability concepts in this integrated review. We will review only the basic calculation techniques here, and you will see the probability applications in the textbook itself.

Objective 1: Apply operations to fractions.

We will review the basic operations of addition, subtraction, multiplication, division, and exponentiation of fractions. Although you may typically calculate these using a calculator within a statistics course, we will briefly review how to do them without technology. However, you may also want to try each operation using a calculator to make sure you know how to work with fractions on your calculator.

Adding Fractions

Example 1 Perform the indicated operation:

$$\frac{3}{16}+\frac{9}{16}$$

In order to add fractions, you need a common denominator. The two given fractions already have the common denominator of 16. So, all you need to do is add the two numerators together and keep the common denominator. Always write your answers in reduced form.

$$\frac{3}{16}+\frac{9}{16}=\frac{(3+9)}{16}=\frac{12}{16}=\frac{3}{4}\cdot\frac{4}{4}=\frac{3}{4}\cdot 1=\frac{3}{4}$$

Answer $\frac{3}{16}+\frac{9}{16}=\frac{3}{4}$

My Turn!

Perform the indicated operation:

$$\frac{5}{8}+\frac{1}{8}$$

Example 2 Perform the indicated operation:

$$\frac{1}{9}+\frac{2}{15}$$

Since we need a common denominator in order to add or subtract fractions, our first step will be to rewrite each fraction over the lowest common denominator. The lowest common denominator is 45. Then, you follow the same procedure as in Example 1 and add the two numerators together and keep the common denominator.

$$\begin{aligned}\frac{1}{9}+\frac{2}{15}&=\frac{1}{9}\cdot\frac{5}{5}+\frac{2}{15}\cdot\frac{3}{3}\\&=\frac{5}{45}+\frac{6}{45}\\&=\frac{(5+6)}{45}\\&=\frac{11}{45}\end{aligned}$$

Answer $\frac{1}{9}+\frac{2}{15}=\frac{11}{45}$

My Turn!

Perform the indicated operation:

$$\frac{3}{10}+\frac{5}{12}$$

Subtracting Fractions

Example 3 Perform the indicated operation:

$$\frac{5}{6}-\frac{1}{3}$$

You also need a common denominator when subtracting fractions. So, our first step will be to rewrite each fraction over the lowest common denominator. The lowest common denominator is 6. Once you have rewritten each fraction over the lowest common denominator, subtract the two numerators and keep the common denominator. Be sure that your final answer is reduced!

$$\begin{aligned}\frac{5}{6}-\frac{1}{3}&=\frac{5}{6}-\frac{1}{3}\cdot\frac{2}{2}\\&=\frac{5}{6}-\frac{2}{6}\\&=\frac{(5-2)}{6}\\&=\frac{3}{6}\\&=\frac{1}{2}\end{aligned}$$

Answer $\frac{5}{6}-\frac{1}{3}=\frac{1}{2}$

My Turn!

Perform the indicated operation:

$$\frac{11}{15}-\frac{5}{12}$$

Since there are several topics in statistics that require subtracting fractions from 1, we will cover that type of subtraction problem here.

Example 4 Perform the indicated operation:

$$1-\frac{3}{8}$$

The number 1 can always be rewritten as $\frac{a}{a}, a \neq 0$. We will select a as the value of the common denominator. So, we set $a = 8$ for this problem. Then

$$1-\frac{3}{8}=\frac{8}{8}-\frac{3}{8}=\frac{5}{8}$$

Answer $1-\frac{3}{8}=\frac{5}{8}$

My Turn!

Perform the indicated operation:

$$1-\frac{16}{19}$$

Multiplying Fractions

Example 5 Perform the indicated operation:

$$\frac{5}{6}\cdot\frac{1}{3}$$

You do *not* need a common denominator in order to multiply fractions.

One way to multiply fractions is to multiply all numerators together and make that product the numerator, multiply all denominators together and make that product the denominator, and then reduce the fraction. Another way to do it is to "cancel out" any common factors that are in any numerator with any denominator (recall that you can do this because any number divided by itself is equal to 1) and then multiply across numerators and denominators. However, you do not have to decide on your preferred method for this particular example as there are no common factors in the numerators and denominators.

$$\frac{5}{6}\cdot\frac{1}{3}=\frac{5\cdot 1}{6\cdot 3}=\frac{5}{18}$$

Answer $\frac{5}{6}\cdot\frac{1}{3}=\frac{5}{18}$

My Turn!

Perform the indicated operation:

$$\frac{4}{15}\cdot\frac{5}{12}$$

Dividing Fractions

Example 6 Perform the indicated operation:

$$\frac{5}{6}\div\frac{1}{3}$$

You do *not* need a common denominator in order to divide fractions.

To divide, multiply by the reciprocal of the second fraction.

(Perhaps, you have heard the expression, “Keep. Change. Flip.” This pneumonic device helps some individuals remember to keep the first fraction the same, to change the division to multiplication, and to flip the divisor.)

Then, you follow the steps for multiplication. That is, multiply across the numerators, multiply across the denominators, and simplify.

$$\frac{5}{6}\div\frac{1}{3}=\frac{5}{6}\cdot\frac{3}{1}=\frac{5\cdot 3}{6\cdot 1}=\frac{15}{6}=\frac{5}{2}$$

You do not have to rewrite as a mixed number. Mixed numbers are generally not used in statistics.

Also, notice that you could have canceled out the common factor of 3 from the 3 in the second numerator and from the 6 in the first denominator before multiplying.

$$\frac{5}{6}\cdot\frac{3}{1}=\frac{5\cdot 1}{2\cdot 1}=\frac{5}{2}$$

Answer $\frac{5}{6}\div\frac{1}{3}=\frac{5}{2}$

My Turn!

Perform the indicated operation:

$$\frac{4}{9} \div \frac{7}{12}$$

Example 7 Perform the indicated operation:

$$\left(\frac{5}{6}\right)^3$$

Recall that an exponent is simply shorthand for repeated multiplication.

$$\left(\frac{5}{6}\right)^3 = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{125}{216}$$

Answer $\left(\frac{5}{6}\right)^3 = \frac{125}{216}$

My Turn!

Perform the indicated operation:

$$\left(\frac{2}{5}\right)^4$$

Objective 2: Apply operations to decimals.

We will review the basic operations of addition, subtraction, multiplication, division, and exponentiation of decimals. Although you may typically compute these using a calculator within a statistics course, we will briefly review how to do them without technology. Then, you may want to try these examples again using your calculator.

Adding Decimals

Example 8 Perform the indicated operation:

$$1.26 + 0.8$$

In order to add decimals, you need to line up the decimal points vertically. Then, add as you would normally and vertically line up the decimal point in the sum with those in the addends.

$$\begin{array}{r} 1.26 \\ \underline{+0.80} \\ 2.06 \end{array}$$

Answer $1.26 + 0.8 = 2.06$

My Turn!

Perform the indicated operation: $6.005 + 1.34$

Subtracting Decimals

Example 9 Perform the indicated operation:

$$1.26 - 0.8$$

You also need to line up the decimals points vertically when you are subtracting decimals.

$$\begin{array}{r} 1.26 \\ \underline{-0.80} \\ 0.46 \end{array}$$

Borrowing was involved in the subtraction. Since you will be using technology in this course, I encourage you to familiarize yourself with the procedure for performing basic operations on your preferred technology.

Answer $1.26 - 0.8 = 0.46$

My Turn!

Perform the indicated operation:

$$9 - 1.004$$

Multiplying Decimals

Example 10 Perform the indicated operation:

$$3.628 \cdot 0.2$$

You do *not* line up decimal points for multiplication.

Count the number of places to the right of each decimal point in each factor. After completing standard multiplication, you will have to insert the decimal point in the answer. The product will have the same number of digits to the right of the decimal point as the sum of the number of places to the right of the decimal point from the factors.

3.628 has three digits to the right of the decimal point.

0.2 has one digit to the right of the decimal point.

The product will have $3+1=4$ digits to the right of the decimal point.

$$\begin{array}{r} 3.628 \\ \times\ 0.2 \\ \hline 0.7256 \end{array}$$

Answer $3.628 \cdot 0.2 = 0.7256$

My Turn!

Perform the indicated operation:

$$5.2 \cdot 31.57$$

Dividing Decimals

Example 11 Perform the indicated operation:

$$3.628 \div 0.2$$

First, we will set up the values for long division.

$$0.2\overline{)3.628}$$

Then we must move the decimal point in the divisor so that it is at the right-hand side of the divisor. In this case, we would need to move it one place value to the right (equivalent to multiplying by 10). We also need to do this same movement to the decimal point in the dividend. (You can think about why we are allowed to move the decimal point the same number of places in the divisor and dividend.)

So, we now have

$$2\overline{)36.28}$$

Perform long division.

$$\begin{array}{r} 18.14 \\ 2\overline{)36.28} \\ \underline{-2} \\ 16 \\ \underline{-16} \\ 02 \\ \underline{-2} \\ 08 \\ \underline{-8} \\ 0 \end{array}$$

Make sure you line up the decimal point in your quotient directly above the one in the dividend.

Even though we have briefly reviewed the procedure here, you are strongly encouraged to calculate these operations with technology.

Answer $3.628 \div 0.2 = 18.14$

My Turn!

Perform the indicated operation:

$$34.6953 \div 5.1$$

Objective 3: Evaluate factorials.

Now that we have reviewed operations on fractions and decimals, let's go over some concepts that we use in counting and probability. The first notation that we will look at is called factorial notation. The symbol $n!$ is read as "*n* factorial" and is telling us to multiply the factors beginning with *n* all the way down to 1.

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$$

Example 12 Evaluate $7!$

The $7!$ (7 factorial) is telling us to multiply the factors beginning with 7 all the way down to 1.

$$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$$

Answer $7! = 5040$

My Turn!

Perform the indicated operation:

$$6!$$

Objective 4: Evaluate $_nP_r$ and $_nC_r$.

You will learn the contextual meaning of $_nP_r$ (permutation) within the Triola text. Here you will just practice the calculation.

$$_nP_r = \frac{n!}{(n-r)!}$$

Example 13 Evaluate $_{10}P_3$.

We begin by substituting 10 and 3 into the formula. Then, we will perform the subtraction within the parentheses in the denominator. If you think about it, this makes sense, since simplifying parentheses come first in the order of operations. We will then expand the factorials. Once this is written out, you will quickly see that many factors cancel out. Our answer is ultimately determined by multiplying together the three decreasing factors beginning with 10. Note that you are always able to compute $_nP_r$ by multiplying the r decreasing factors starting with n.

$$_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 10 \cdot 9 \cdot 8 = 720$$

Answer $_{10}P_3 = 720$

My Turn!

Evaluate ${}_8P_2$.

Now we will practice the calculating of combinations, which are denoted by ${}_nC_r$.

$$ {}_nC_r = \frac{n!}{r!\cdot(n-r)!} $$

Example 14 Evaluate ${}_{10}C_3$.

We begin by substituting 10 and 3 into the formula. Then, we will perform the subtraction within the parentheses in the denominator. Once this is written out, you will quickly see that many factors cancel out. Our answer is ultimately determined by multiplying together the three decreasing factors beginning with 10 and dividing by the three decreasing factors beginning with 3. Note that you are always able to compute ${}_nC_r$ by dividing the product of the r decreasing factors starting with n by $r!$.

$$ {}_{10}C_3 = \frac{10!}{3!\cdot(10-3)!} = \frac{10!}{3!\cdot 7!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{3\cdot 2\cdot 1\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1} = \frac{720}{6} = 120 $$

Answer ${}_{10}C_3 = 120$

My Turn!

Evaluate ${}_8C_2$.

Objective 5: Determine the intersection, union, and complement of two sets.

A set is simply a collection of items. We will review three operations on sets.

The **intersection** of two sets consists of all the elements that are simultaneously in both sets. The symbol that we use to indicate intersection is $\cap$.

Let A be a set and let B be a set; then $A \cap B$ consists of the elements that are in A **and** in B.

The **union** of two sets consists of all the elements that are in at least one of the sets. The symbol that we use to indicate union is $\cup$.

Let A be a set and let B be a set; then $A \cup B$ consists of the elements that are in A **or** in B.

The **complement** of a set A consists of the items that are in the universal set, U (the set that contains everything or all elements under consideration), that are **not** in A. Several symbols are commonly used to denote the complement; however, we will use $\overline{A}$ (read "A-bar").

Example 15 Let $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{2,4,6,8\}$, and $B = \{1,2,3,4,5\}$.

a) Find $A \cap B$.
To find the intersection, we must look for the elements in A and in B.
$A = \{\boxed{2},\boxed{4},6,8\}$, and $B = \{1,\boxed{2},3,\boxed{4},5\}$ Those elements are 2 and 4.

$$A \cap B = \{2,4\}$$

b) Find $A \cup B$.
To find the union, we must look for the elements that are in A or in B (That is, join them all together). We only need to write each item once, though, in our solution set.

$$A \cup B = \{1,2,3,4,5,6,8\}$$

c) Find $\overline{A}$.

To find the complement of *A,* we must identify the elements in the universal set that are not in *A*. The elements in *A* are $A = \{2,4,6,8\}$. The elements that are in the universal set that are not in *A* are boxed: $U = \{\boxed{1},2,\boxed{3},4,\boxed{5},6,\boxed{7},8,\boxed{9}\}$. So, the complement of *A* consists of the elements 1, 3, 5, 7, and 9.

$$\overline{A} = \{1,3,5,7,9\}$$

Answer $A \cap B = \{2,4\}$, $A \cup B = \{1,2,3,4,5,6,8\}$, and $\overline{A} = \{1,3,5,7,9\}$

My Turn!

Let $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{1,2,3,4,5,6\}$, and $B = \{3,6\}$.

Find $A \cap B$, $A \cup B$, and $\overline{A}$.

Answers to My Turn!

1. $\frac{3}{4}$
2. $\frac{43}{60}$
3. $\frac{19}{60}$
4. $\frac{3}{19}$

5. $\frac{1}{9}$
6. $\frac{16}{21}$
7. $\frac{16}{625}$
8. 7.345
9. 7.996
10. 164.164
11. 6.803
12. 720
13. 56
14. 28
15. $A \cap B = \{3,6\}$, $A \cup B = \{1,2,3,4,5,6\}$, and $\overline{A} = \{7,8,9\}$

Practice Problems

1. Perform the indicated operation: $\frac{5}{18} + \frac{1}{18}$.
2. Perform the indicated operation: $\frac{3}{7} + \frac{7}{10}$.
3. Perform the indicated operation: $\frac{8}{15} - \frac{2}{5}$.
4. Perform the indicated operation: $1 - \frac{9}{10}$.
5. Perform the indicated operation: $\frac{3}{16} \cdot \frac{2}{9}$.
6. Perform the indicated operation: $\frac{4}{9} \div \frac{9}{16}$.
7. Perform the indicated operation: $\left(\frac{1}{4}\right)^3$.
8. Perform the indicated operation: $3.576 + 2.3$.

9. Perform the indicated operation: $5.6 - 2.354$.
10. Perform the indicated operation: $1.05 \cdot 2.5$.
11. Perform the indicated operation: $7.05 \div 0.5$.
12. Evaluate 9!.
13. Evaluate ${}_9P_4$.
14. Evaluate ${}_9C_4$.
15. Let $U =$ {red, blue, orange, yellow, green, purple, pink, white, black}, $A =$ {red, blue, yellow}, and $B =$ {red, pink, purple}.
    Find $A \cap B$, $A \cup B$, and $\overline{A}$.