**Section 4.2**

A **quantitative random variable** x is a random variable if the value that x takes on in a given experiment or observations is a chance or random outcome.

A **discrete random variable** can take on only a finite number of values or a countable number of values. A **continuous random variable** can take on any of the countless numbers of values in a line interval.

* The **expected value** is often referred to as the ”long-term” average or mean. This means that over the long term of doing an experiment over and over, you would expect this average.
* The mean of a random variable X is µ.
* If we do an experiment many times and record the value of X each time, the average is likely to get closer and closer to µ as we keep repeating the experiment. This is known as another version of the Law of Large Numbers.

**Example** Which of the following are continuous variables, and which are discrete?

1. *x* = the number of siblings a person has.

1. *x* = a person’s height.

1. *x* = the amount of time it takes a statistics student to finish an exam.

1. *x* = the number of units a student takes in a quarter.





**Example** Consider the following probability distribution of a discrete random variable, *x*. Complete the table so that it is a valid probability distribution.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *x* | 0 | 1 | 2 | 3 |
| *P* (*x*) | 0.4 | 0.2 | 0.1 |  |

**Example** A dust mite allergen level that exceeds 2 micrograms per gram (*µg/g*) of dust has been associated with the development of allergies. Consider a random sample of 4 homes. Let *x* = number of homes with a dust mite allergen level that exceeds 2 *µg/g*. The probability distribution for *x* is given below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *x* | 0 | 1 | 2 | 3 | 4 |
| *P* (*x*) | 0.09 | 0.30 | 0.37 | 0.20 | 0.04 |

1. Find the probability that at least three of the homes have a dust mite level that exceeds 2 *µg/g*.

1. Find the probability that fewer that two of the homes have a dust mite level that exceeds 2 *µg/g*.

1. Compute and interpret *µ*.

**Example** Consider the game consisting of rolling a pair of fair dice and recording the sum. It will cost you $1*.*00 to play this game. If the sum is at least 10, then you will win $3*.*00. If the sum is 2, you will win $2*.*00. Otherwise, you will lose the game.

Probability Distribution: Winnings

|  |  |  |  |
| --- | --- | --- | --- |
| *x* | $3*.*00 | $2*.*00 | $0*.*00 |
| *P* (*x*) |  |  |  |

1. What are your expected earnings per dollar wagered? Show your work and interpret your result.

1. Are your expected earnings equal to the cost to play?

1. If you played this game one million times, how much money would you expect to earn/lose?

**Example** Suppose that Nigel wants to take out a $100*,* 000 term insurance policy. Nigel is 60 years old and according to the actuarial tables, there is a 1*.*191% chance Nigel will die this year. If the insurance company wants to make $700 profit per policy of this type, how much should the insurance company charge Nigel? Show your work and give your answer in the form of a sentence.

1. Determine the probability distribution table. There are only two possibilities, you either pay out on the policy this year or you don’t:

|  |  |  |
| --- | --- | --- |
| *x* | $100*,* 000*.*00 | $0*.*00 |
| *P* (*x*) |  |  |

1. Now calculate the expected pay out per person for this year.

1. If they want to make $700 per person (on average) how much should they charge?

**Example** To raise money, the ASFC is holding a raffle. The prize is a Walt Disney World vacation for two valued at $3000. The ASFC sold 1382 raffle tickets at $5 each.

1. Antonia bought 10 tickets. What is the probability that she will win?

1. What are Antonia’s expected earnings?

1. Are Antonia’s expected earnings equal to the cost to play?

**Example Revisited** Consider the game consisting of rolling a pair of fair dice and recording the sum. It will cost you $1*.*00 to play this game. If the sum is at least 10, then you will win $3*.*00. If the sum is 2, you will win $2*.*00. Otherwise, you will lose the game.

Probability Distribution: Winnings

|  |  |  |  |
| --- | --- | --- | --- |
| *x* | $3*.*00 | $2*.*00 | $0*.*00 |
| *P* (*x*) | 1 |  1  | 29 |
| 6 | 36 | 36 |

Find the Standard deviation of this Probability Distribution Table.