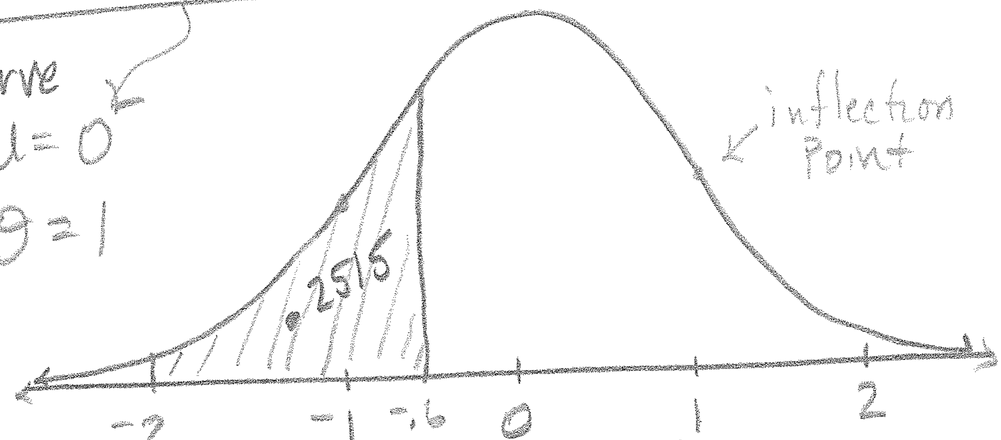
§6.1 The Standard Normal Distribution

Bell Shaped Curve

$$y = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

$\mu = 0$

$\sigma = 1$



DISTR

Area under Curve between t=-2 and t=-.6 $= P(-2 < Z < -.6) =$ normalcdf(-2, -.6, 0, 1)

$= .2515$

→ Total Area under Curve is 1 = 100%

Areas of Regions area - Proportion of Total Population

- % or Percent %

- Probability that a radomly Selected element (person) has z-Scores between those two Values

Find Area

Formula card

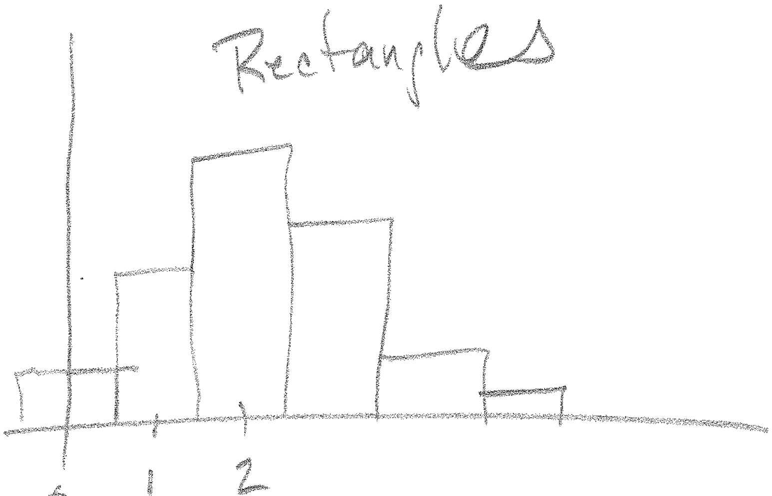
$$\boxed{\text{Area} = \text{Normalcdf}(LB, UB, \mu, \sigma) = P(LB < Z < UB)}$$

Ch. 5 | Ch 6, 7, 8, 9, ~~10~~
Discrete vs C+S

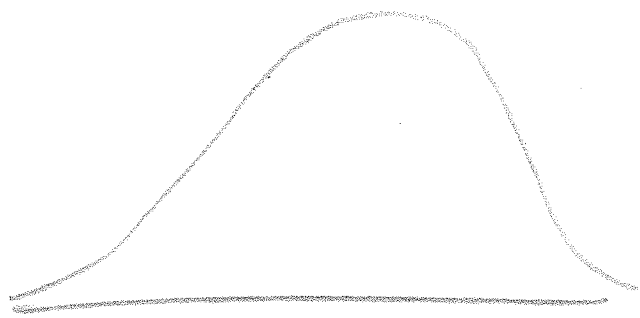
Counted
- # of Red M&Ms
- # hits
- # of hurricanes
- # of girls

Measured
- heights
- Pulse rates
- IQ
- large n

Rectangles



Smooth



Chapter 5
- Binomial
- Poisson
- Games of chance
- Insurence
- Geometric

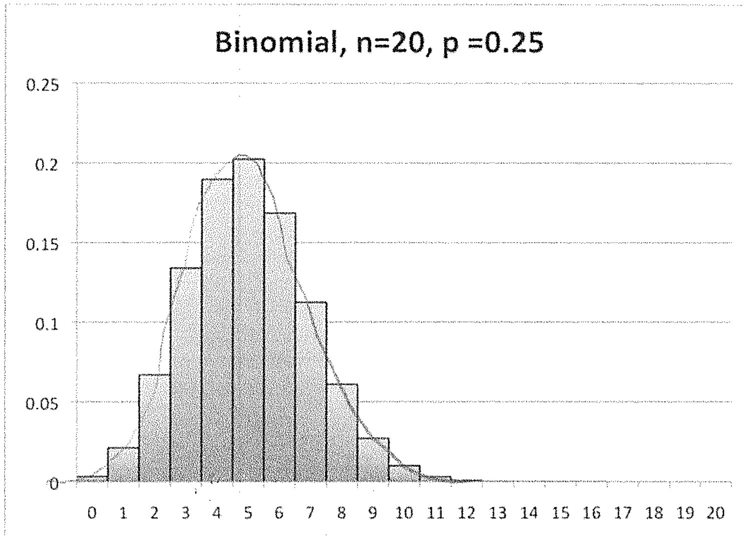
$p =$
n

X	0	1
P(x)		

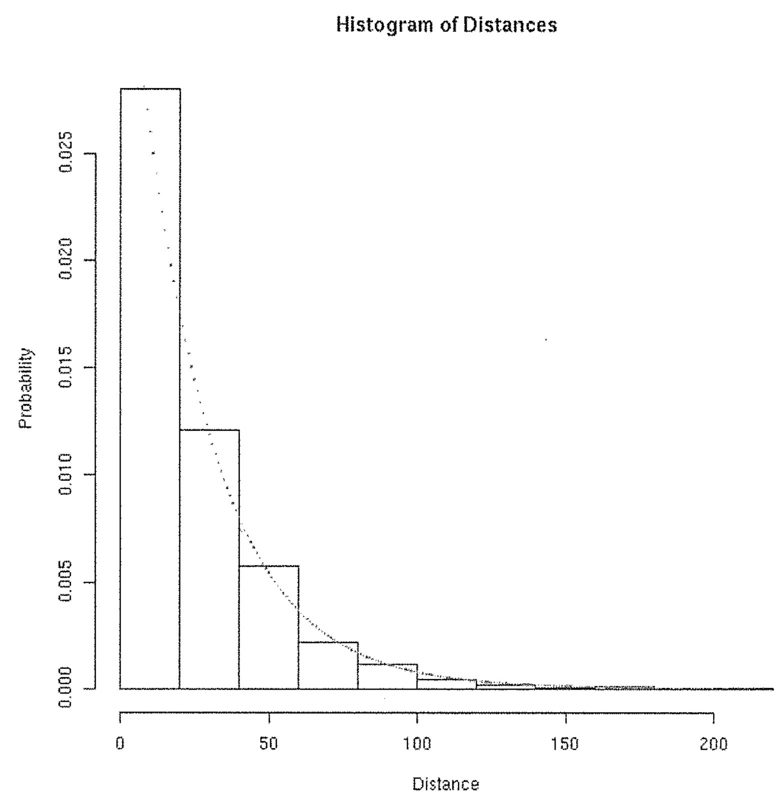
Chapter 6
- Normal
- T-dist
- χ^2-dist
- F-dist

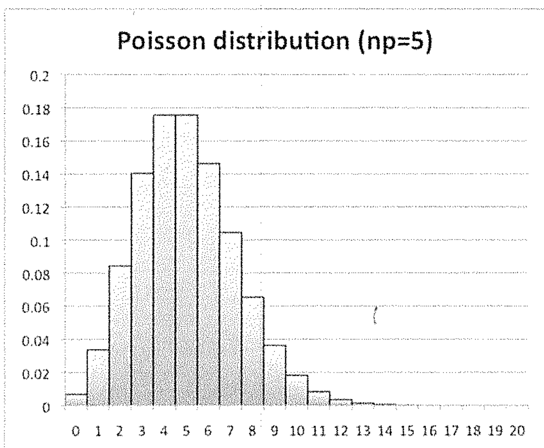
$\mu =$
σ

Discrete Probability Distributions



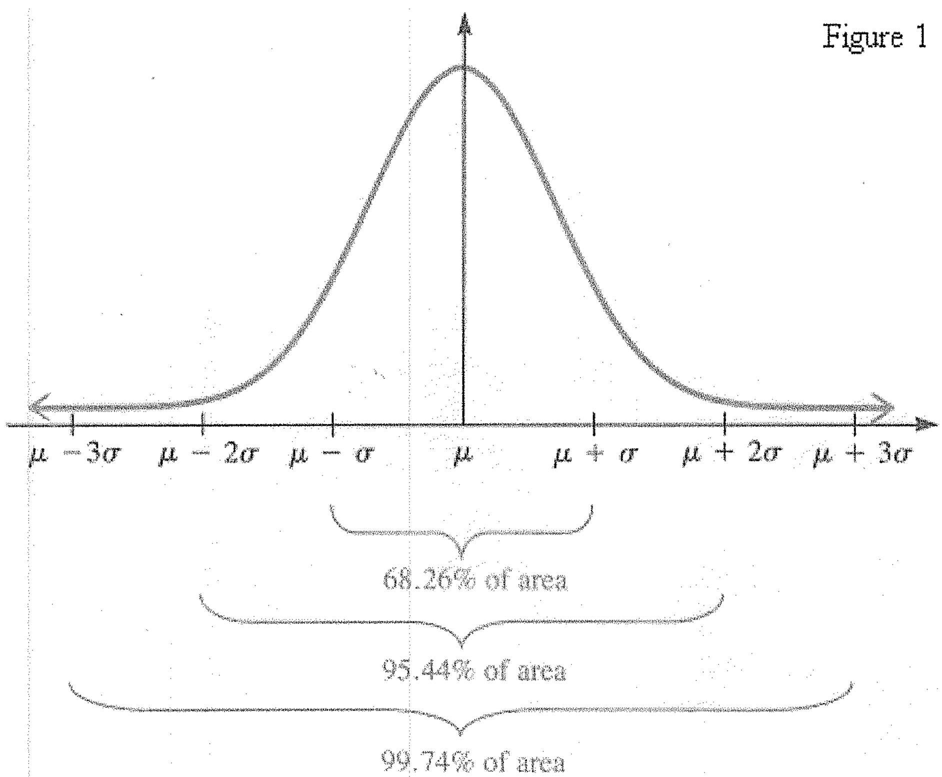
Geometric Distribution



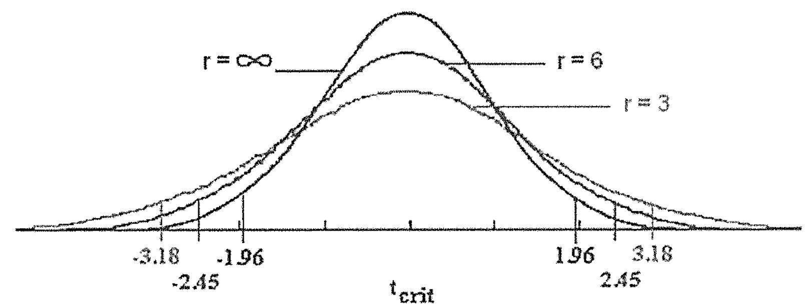


Continuous Probability Distributions

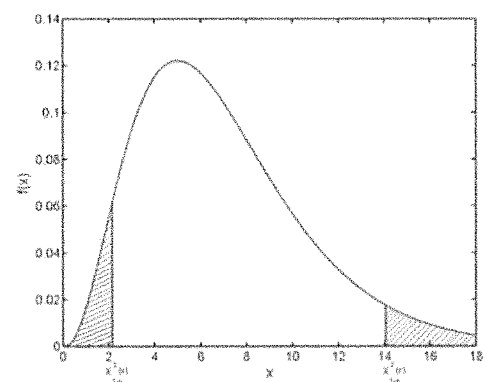
- Normal Distribution



T-distribution



Chi Square Distribution



2 3

Ex/a) Find $P(-1 < z < 1) = \text{normalcdf}(-1, 1, 0, 1)$

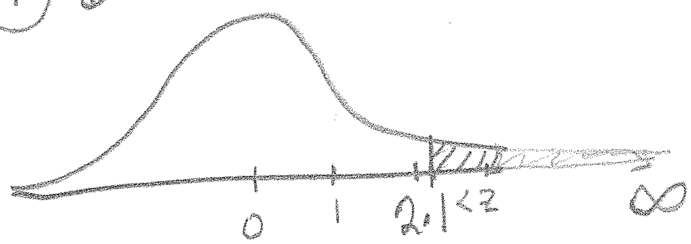
1 Draw 2 ↑ 3 r

$= \boxed{.6827}$ ← Round to 4 decimal places

b) Find $P(z > 2.1) = \text{normalcdf}(2.1, 9999, 0, 1)$

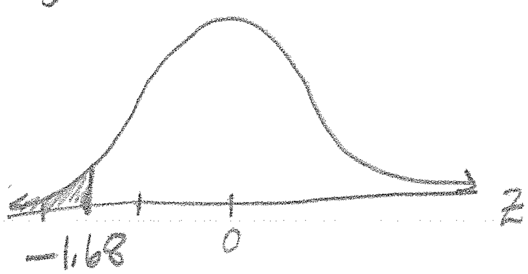
$= \boxed{.0179}$

1 Draw



c) Find $P(z \leq -1.68) = \text{normalcdf}(-9999, -1.68, 0, 1)$

$= \boxed{.0465}$



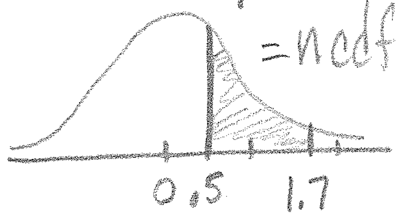
$P(a \le z \le b) = \text{normalcdf}(a, b, 0, 1)$

Find Probability as Area Under Curve.

Find Probability that a Z-Score is between .5 and 1.7.

Prob. Notation | Graph | Calc. | Ans. 4 decimals

$P(.5 \le Z \le 1.7) =$

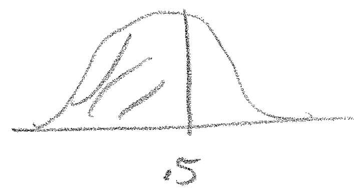


$= \text{ncdf}(.5, 1.7, 0, 1) = \boxed{.2640}$

↑ Better use this

$=$

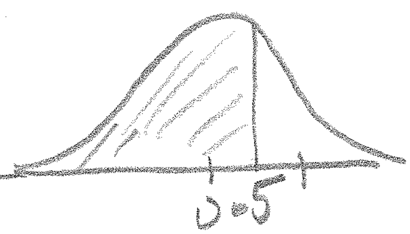
$-$



Book & MML

$= .9554 - .6915 = .2639$

$P(X < .5) =$



$-\infty$

↑

-9999

Value in Table

↓

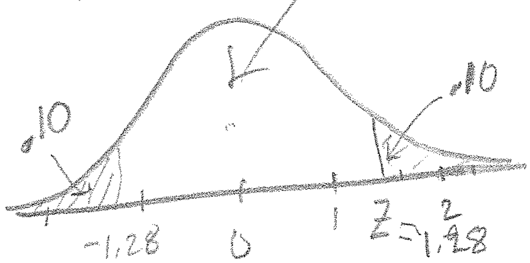
$= \text{ncdf}(-9999, .5, 0, 1) = \boxed{.6915}$

Given probability, proportion or percent
Find the Z-Score

<u>Ex</u> What Z-Score Separates the Top 10%
of all Z-Scores?

Area to the left of Z is 90%
or .90

Draw



$Z = 1.28$

$Z = \text{invnorm}(\text{Area left}, \mu, \sigma)$
$= \text{invnorm}(.9, 0, 1)$
$= 1.28$

<u>A Critical Value</u> is a Z Score
that Separates an area α in
Right Tail.

Critical Value $\boxed{Z_\alpha = \text{invnorm}(1-\alpha, 0, 1)}$

$Z_{.10} = 1.28 = P_{.90}$

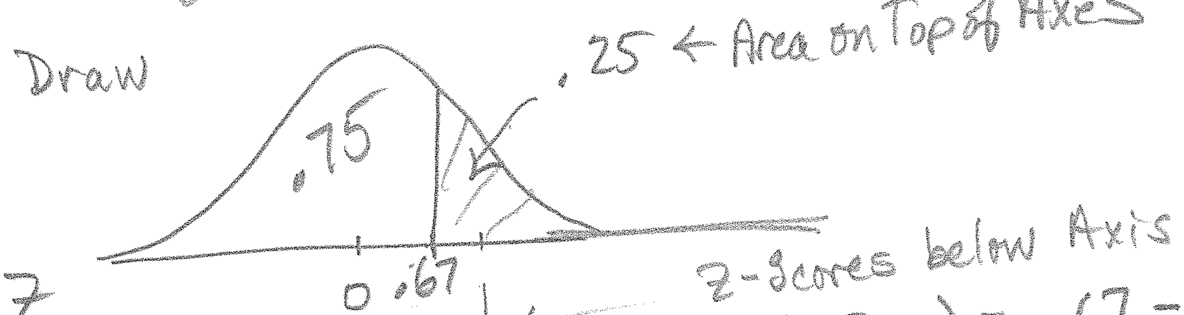
↑ 90^{th} Percentile

Class Try

1. Find the Z-Score that corresponds to Q_3.

Q_3 Separates the Top 25% from Bottom 75%.

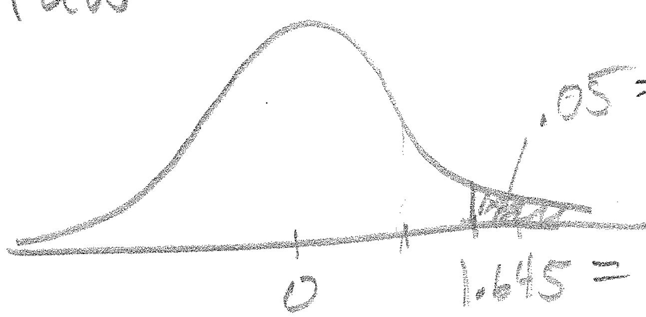
Draw



$z = \text{invnorm}(.75, 0, 1) = .67 = Q_3 = Z_{.25}$

2. Find $Z_{.05}$ the critical value that separates an an area of .05 in Right Tail

Draw

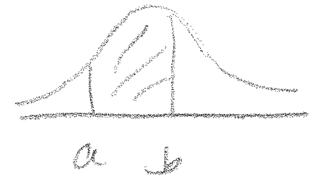


$1.645 = Z_\alpha = \boxed{Z_{.05}}$ Critical Value

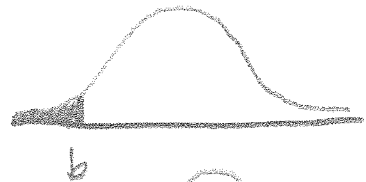
Z-Score that separates usual from unusual Values Above $Z_{.05}$ are Statistically Significant

Formula Page

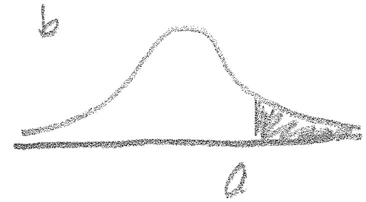
$P(a \le z \le b) = \text{normalcdf}(a, b, \mu, \sigma) =$



$P(z \le b) = \text{normalcdf}(-9999, b, \mu, \sigma) =$



$P(z \ge a) = \text{normalcdf}(a, 9999, \mu, \sigma) =$



Inverse Normal

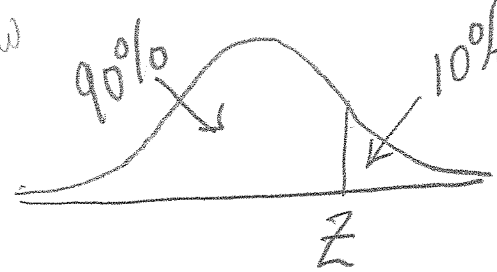
invnorm(Arealeft, μ, σ)

Given the area, %, proportion, or probability

1. Draw picture
2. Find Area left of defined z
3. Use invnorm

Ex What z-score separates the top 10% of z-scores. .9 =

1) Draw



② $z = \text{invnorm}(.9, 0, 1) = 1.28$ ← Area left

$\boxed{z_{.10} = 1.28 = \text{critical value}}$

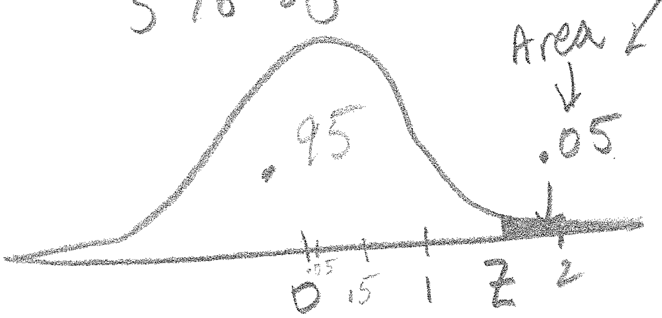
The z-score that separates the top 10% of z-scores

Inverse Normal function

If we are given a probability and we want the associated z-score

Ex What z score separates the Top 5% of all z-scores?

Just estimate to get rough picture



Calculator Needs Area to left of the z-score that you want to find.

Special

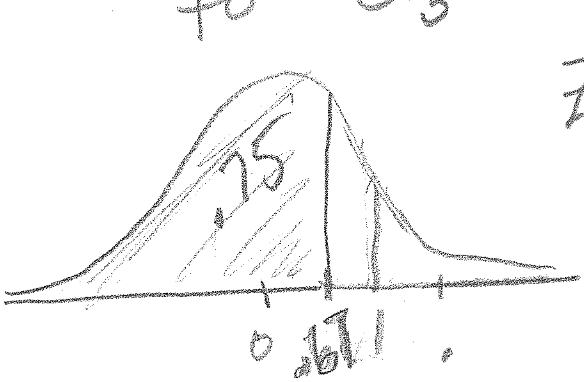
$$z = \text{invnorm}(1-.05, 0, 1) = \boxed{1.645}$$

Class Try

Ex Find the z-score that corresponds to Q_3 or P_{75} the 75^{th} percentile

$$z = \text{invnorm}(.75, 0, 1) = \boxed{0.67}$$

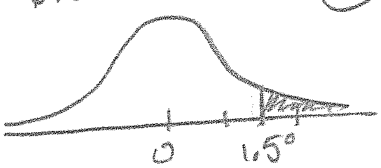
Just Two decimals on z-scores



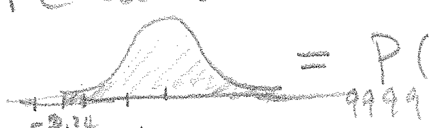
Ex A Freezer has a Temperature of 0°.
I make thermometers. They have a $\mu = 0°$ and $\sigma = 1°$

a) Find the probability (Find Area) that a randomly selected thermometer reads above 1.5°

① Draw ② $P(Z > 1.5)$ = ③ normalcdf(1.5, 9999, 0, 1)
$= \boxed{0.0668}$

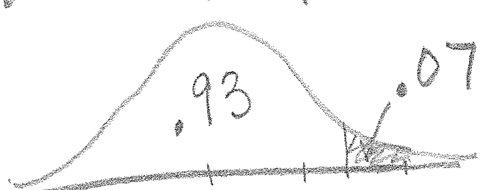


b) What percentage (Find Area) of thermometers read above −2.34 degrees



$= P(Z > -2.34)$ = ncdf(−2.34, 9999, 0, 1)
$= \boxed{.9904}$

c) The top 7% of thermometers ~~that~~ Are rejected. Area = .07
What Temp Separates rejected from accepted



Area Left

Z = invnorm(.93, 0, 1) = $\boxed{1.476 = Z}$

$\boxed{Z_{.07} = 1.476}$

Still 6.2

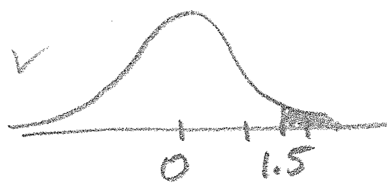
Ex thermometers $\mu = 0^\circ$ and $\sigma = 1^\circ$ & Normal

a) Find the probability that a randomly Selected thermometer reads above 1.5.

Prob $\Leftrightarrow$ find Area

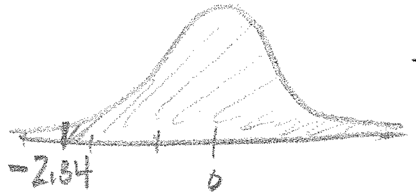
✓ $P(x > 1.5) = \text{normalcdf}(1.5, 9999, 0, 1)$

✓



$= \boxed{.0668}$

b) What percentage of thermometers read above -2.34 degrees



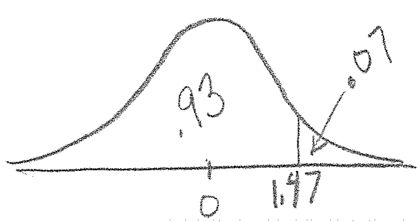
$= P(X > -2.34) = \text{ncdf}(-2.34, 9999, 0, 1)$

$= .9903$ or $\boxed{99.03\%}$

$\boxed{\text{Prob} \quad \text{Proportion or percentage} \quad \text{Find Area} \equiv \text{Normalcdf}}$

c) 7% of thermometers are rejected for reading too high. What Temperature Separates the rejected thermometers from the accepted.

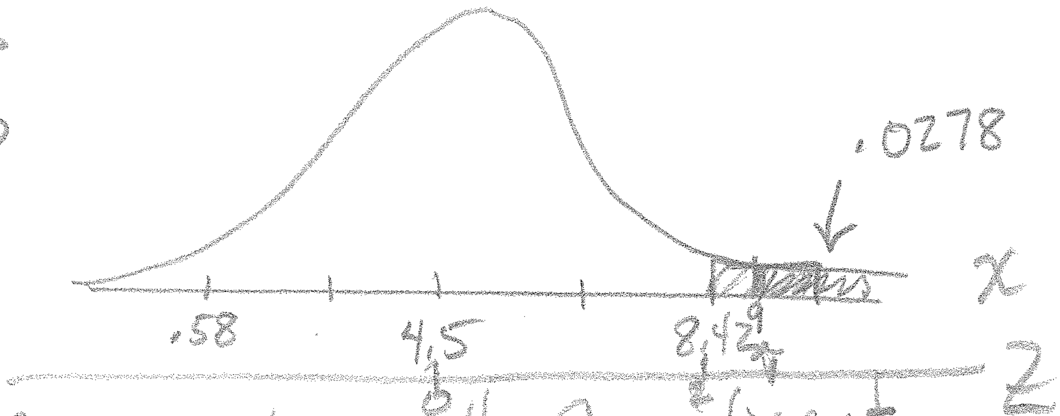
Note: Asks for Temp = z-Score $\Rightarrow$ use InvNorm



$Z = \text{invnorm}(.93, 0, 1)$

$= \boxed{1.47^\circ}$

Houses out of 30 with LED



c) 9 is Statistically Significant

(1) $P(X \geq 9) = .0278 < .05$
So the probability of getting 9 houses or more with LED's is unusually low

(2) Since $9 > 8.42$, 9 is above the expected max usual of 8.42 for a sample of $n=30$. So 9 is statistically high Number of Homes with LED lights.

d) What can you conclude about Santa Rosans use of LED Bulbs.

the proportion of Santa roseens that use LED's is greater than the National average of 15%

$$\hat{P} = \frac{9}{30} = .3 = 30\%$$

5.2 #38, 36

#36 $n = 50$ $\quad P(X \leq 2)$ $\quad p = .02$ $\quad N = 2000$

Since $n = 50 < 5\%$ of 2000
$50 < 100$
So we can Treat as Binomial

$P(\text{Accepted}) = P(X \leq 2)$

$= \text{binomialcdf}(\overset{n}{50}, \overset{p}{.02}, \overset{r}{2})$

$= \boxed{.9216}$

92% will be accepted, most of the time accepted.
Only 8% will be rejected.

#38 Binomial? $n = 41$, $p = .5$
yes if Not cheating

a) sig. high $\geq \mu + 2\sigma = 20.5 + 2(3.2) = 26.9$
sig. low $\leq \mu - 2\sigma = 20.5 - 6.4 = 14.1$

$\mu = np = 41 \cdot .5 = 20.5$

$\sigma = \sqrt{npq} = \sqrt{41 \cdot .5 \cdot .5} = 3.2$

40 is Significantly high because it is greater than 26.9.

$$\text{38b) } p(X=40) = \text{bpdf}(\overset{n}{41}, \overset{p}{.5}, \overset{r}{40}) = 1.86 \times 10^{-11}$$

$$\text{c) } p(X \geq 40) = 1 - \text{bcdf}(\overset{n}{41}, \overset{p}{.5}, \overset{r-1}{39}) = 1.91 \times 10^{-11}$$

d) Part c) tells us it is very unlikely to get at least 40, is relavent. So 40 Dem. is significanlly high

e) Clerk did cheat, very little chance of seeing this outcome if line spots are randomly selected.

you dont get it back

Q4 #3 pay \$2 to play
get back \$4 > Net winnings = \$2

	win	Lose
X	2	-2
P(x)	2/6	4/6

$E(x) = 2 \cdot \frac{2}{6} + -2\left(\frac{4}{6}\right) = \frac{-4}{6} =$

$\boxed{E(x) = -.67}$

#38 $n=41$ $p=.5$ (n, p, r)

b) $P(X=40) = \text{binpdf}(41, .5, 40) = 1.86 \times 10^{-11}$

a) Range Rule $\mu = np = 41 \cdot .5 = \boxed{20.5}$ $\sigma = \sqrt{npq} = \sqrt{41 \cdot .5 \cdot .5} = \boxed{3.2}$

Significantly high $\geq \mu + 2\sigma = 20.5 + 2(3.2) = 26.9$

yes, 40 is way above 26.9 so it is Statistically high

c) $P(X \geq 40) = 1 - \text{bcdf}(41, .5, 39)$

$= 1.91 \times 10^{-11}$

d) Staticaly high means 40 or More so

Part c) is relevant.

More extreme

Means observed value or more extreme.

e) Yes Clerk is cheating