7.2 Estimating a Mean

Make a Confidence Interval for a Mean

(1) Check Requirements $n > 30$ OR if $n < 30$ Check if population is Approximately Normal, histogram, Box Plot, QQ Plot

If $n < 30$ & Not Normal STOP

(2) PE = $\bar{x}$ = Sample Mean = best guess for population parameter

(3) ~~$z_{\alpha/2}$~~ CV: $t_{\alpha/2} = \text{invT}(1 - \frac{\alpha}{2}, df)$ $df = n - 1$ $\alpha = 1 - CL$

(4) $E = t_{\alpha/2} \frac{S}{\sqrt{n}}$ $S = S_x$ ← from 1-Var Stats

(5) CI: $\bar{x} - E < \mu < \bar{x} + E$

(6) We are ____ Confident that The Mean is All ____ the population mean

CL

is between $\bar{x} - E$ and $\bar{x} + E$

The T-Distribution

(1) A continuous Probability Distribution

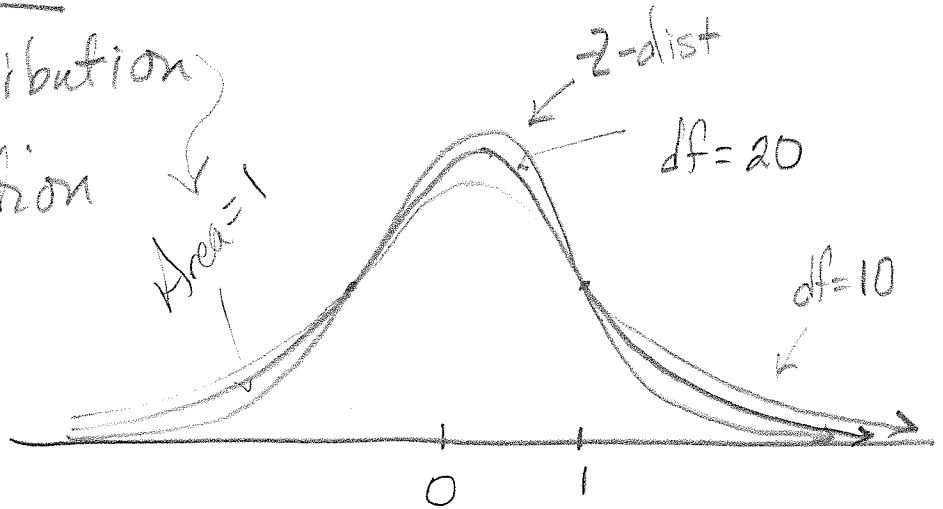
(2) Bell shaped like a Z-Distribution

(3) Tails are wider

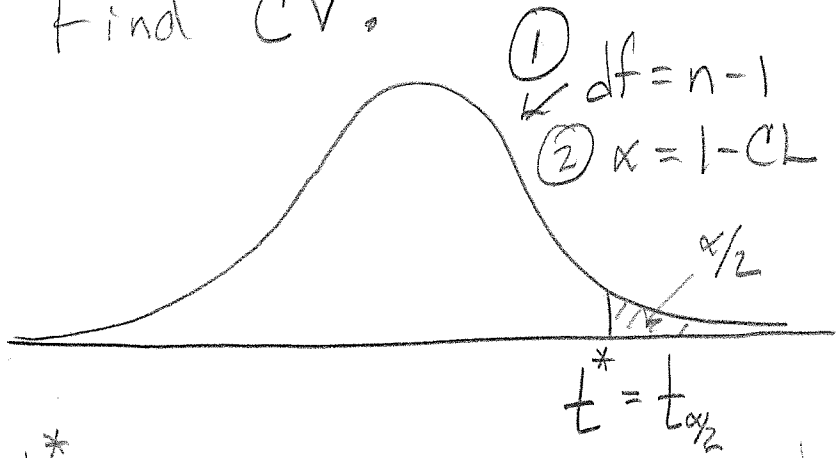
(4) T-Dist. is Different for every value of df

(5) $df = n - 1 = \text{sample size} - 1$
degrees of freedom (df)

(6) As n increases T-dist gets closer to Z-distribution.



Find CV:



t^* is the t-score that separates an area of $\frac{\alpha}{2}$ in Right Tail

(2) $t_{\alpha/2} = t^* = invT\left(1 - \frac{\alpha}{2}, 0, 1\right)$

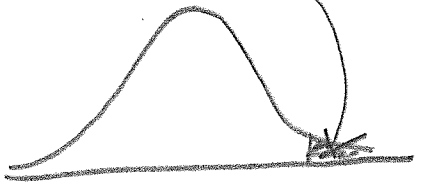
Ex 1 Find the CV: t^* for a 95% confidence interval when $n = 20$.

① $df = n - 1 = 20 - 1 = 19$

$df = 19$

② $\alpha = 1 - CL = 1 - .95 = .05$

$\frac{\alpha}{2} = \frac{.05}{2} = .025 = \frac{\alpha}{2}$



③ $t^* = invT(1 - \frac{\alpha}{2}, df)$

$t^* = invT(1 - .025, 19)$ ← DISTR

CV: $t^* = 2.093$ ← Round to 3 decimals

Ex 2 Construct a 90% Confidence Interval for the ~~Mean~~ Life of all LED light bulbs if you have a Sample with a mean of 441 hrs and Standard deviation of 40 hrs, with 105 light bulbs.

Rewrite in Math
CL = .90 for Mean
Find a T-Interval
$\bar{x} = 441$
$s = 40$
$n = 105$

① Requirements $n > 30$ ✓

② PE: $\bar{x} = 441$

③ CV: $t_{\alpha/2} = invT(1 - \frac{\alpha}{2}, df)$
$= invT(1 - \frac{.10}{2}, 104)$
$t^* = 1.660$

④ $E = t^* \cdot \frac{s}{\sqrt{n}} = 1.660 \cdot 40/\sqrt{105} = 6.48$

One more decimal than ↓ Summary stats

$6.5 = E$

⑤ CI $\bar{x} - E < \mu < \bar{x} + E$
$441 - 6.5 < \mu < 441 + 6.5$
$434.5 < \mu < 447.5$

(6) We are 90% confident that the true Mean Life of ALL LED light Bulbs
Description of pop. parameter
is between 434.5 hrs and 447.5 hrs

Find Confidence Interval for a Mean With Data

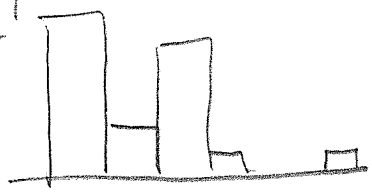
Ex3 Find a 95% CI for the MEAN distance to campas from home for all SRJC Students

3	12	11	20	9,8	13	11	10	5,2	3,9	9,5
4,2	12	4,9	6	3,2	5,2	11	2,3	8,5	2,9	

1-Var Stats gives $\bar{X} = 8.03$ $S = 4.5$ $n = 21$

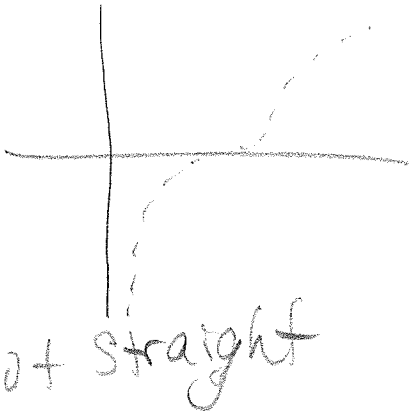
(1) Requirements $n < 30$ check if Normal

Hist



Not Normal
- Skew Rt
- Not Symmetric
- Starts high
- Gap

QQ Plot
▷ 5 times



Not straight

⇒ No CI Should be Made!

Pretend Data is close to Normal

(2) PE: $\bar{x} = 8.03$ CL = .95 n = 21 S = 4.5

(3) CV: $t^* = \text{invT}(1 - \alpha/2, df) = \text{invT}(1 - .05/2, 20) = 2.086 = t^*$

(4) $E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}} = 2.086 \cdot 4.5/\sqrt{21} = 2.048 = E$

(5) CI $\bar{x} - E < \mu < \bar{x} + E$

$8.0 - 2.05 < \mu < 8.0 + 2.05$

$5.95 < \mu < 10.05$

STAT
Tests
8: T Interval

(6) We are 95% confident that the true Mean Distance to school for SRJC Students is between 5.95 and 10.05 Miles.

Test 2 Corrections

- Due ~~Monday~~ Tuesday 10/29
- $\frac{1}{2}$ Credit per point Added to test Score.
  Only if you get it Right
- Extra help Tuesday & Wed. After class here
- Write Complete Solution
- Problem # and Score on problem 3) 2/6 +2
- Separate Paper!

§7.1 # 32 find n, $CL = .90$ $E = .02$

a) No guess for $\hat{p}$ $\Rightarrow$ $n = \frac{Z_{\alpha/2}^2 \cdot .25}{E^2} = \frac{(1.645)^2 \cdot .25}{(.02)^2} = \boxed{1692}$

Find $Z_{\alpha/2}$

(1) $\alpha = 1 - .90 = .10$ (2) $\frac{\alpha}{2} = .05$

(3) $Z_{\alpha/2} = \text{invnorm}(1 - \frac{\alpha}{2}, 0, 1) = \text{invnorm}(1 - .05, 0, 1)$

$Z_{\alpha/2} = 1.645$

b) $\hat{p} = .95$ $n = \frac{(Z_{\alpha/2})^2 \cdot \hat{p} \cdot \hat{q}}{E^2} = \frac{(1.645)^2 \cdot .95 \cdot .05}{(.02)^2}$

$\boxed{n = 322}$

c) Yes, Interviewing 322 people is much less than 1692 people

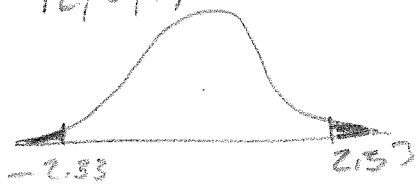
#24 (2) $\hat{p} = .7$ $n = 1002$ $p = .61$

a) Use (1) $\hat{p} = .7$ to make a 98% CI. (1) Requirements Met SRS, $n\hat{p} > 5$ $n\hat{q} > 5$ $700 > 5$ $300 > 5$

CL = .98 $\Rightarrow$ $\alpha = .02$

(3) $z_{\alpha/2} = \text{invnorm}(1 - \alpha/2, 0, 1) = \text{invnorm}(1 - .02/2, 0, 1) = 2.33$

$z_{\alpha/2} = 2.33$



(4) $E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.33 \cdot \sqrt{\frac{.7 \cdot .3}{1002}} = .034 = E$

(5) $\hat{p} - E < p < \hat{p} + E$

$.7 - .034 < p < .7 + .034$

$.666 < p < .734$

(6) We are 98% Confident that the prop. of People who say vote is between .666 and .734 or 66.6% and 73.4%

b) Inconsistent because 61% is Not in the Confidence Interval. It appears that People we lying about whether they voted.

How do I do Better on the Next Test?

Ask, Explain, Connect

No Cramming

Switch Topics

Words and Visuals

Examples

Recall what you know

Unbiased Mean Proportions Variances

$\mu_{\bar{x}} = \mu$ $\mu_{\hat{p}} = p$ $\mu_{s^2} = \sigma^2$

Biased Median, range, Standard deviation

Q_1

Using TI calculator to check

STAT ▶ TESTS ▽ T Interval

Inpt: Data | Stats ← Enter ← If $\bar{x}$, s, n given

Data ← Data in L_1

Stat ▶ TStats One Sample with Data

Select Data

CI

Confidence Intervals

	§7.1 Proportions	§7.2 Means
Population Parameter	p = population proportion	μ = population mean
Sample Statistic / Point estimate (2)	$\hat{p} = \frac{x}{n} = \frac{\#\text{Successes}}{\#\text{in Sample}}$	$\bar{x} = \frac{\Sigma x}{n}$, use 1-Var Stat (Needs s too)
Sampling Distribution	Z or Normal	t-distribution
Requirements (1)	Binomial and $np = \#\text{Successes} = x > 5$, $nq = \#\text{failures} > 5$	$n > 30$ or original distribution of population is Normal
(3) Critical Value	CV: $z_{\alpha/2} = \text{invnorm}(1-\alpha/2, 0, 1)$	CV: $t_{\alpha/2} = \text{invT}(1-\alpha/2, df)$
(4) Margin of Error	$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$	$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$
(5) Confidence Interval	$\hat{p} - E < p < \hat{p} + E$	$\bar{x} - E < \mu < \bar{x} + E$
Example	$.27 < p < .35$	$\$1.25 < \mu < \$1.75$
(6) Interpret we are CL Confident that	the proportion of people who drink coffee is between .27 and .35	the mean amount spent by all coffee drinkers is between \$1.25 and \$1.75
(7) Calculator [STAT] ▷▷ [TEST]	A: 1 PropZINT	8: TInterval

Ex 3 CI for a Mean age of the Class

19, 19, 21, 44, 19, 24, 19, 20, 18, 19, 19, 20
19, 19, 19, 18, 19, 18, 18, 21, 21, 17, 21

Summary Stats $\boxed{\bar{x} = 20.48}$ $\boxed{S = 5.33}$ $\boxed{n = 23}$

Make a 98% CI. So $\boxed{\alpha = .02}$

(1) Requirements? No $n < 30$ & $x = 44$ is an outlier
But we'll make the interval anyway - with & without

(2) PE: $\boxed{\bar{x} = 20.48}$ $\alpha = 1 - CL$ $df = n - 1$

(3) CV: $t_{\alpha/2} = \text{invT}(1 - .02/2, 22) = \boxed{2.508 = t_{\alpha/2}}$

(4) $E = t_{\alpha/2} \frac{S}{\sqrt{n}} = 2.508 \cdot \frac{5.33}{\sqrt{23}} = \boxed{2.79 = E}$

(5) CI: $\bar{x} - E < \mu < \bar{x} + E$

$20.48 - 2.79 < \mu < 20.48 + 2.79$

$\boxed{17.69 < \mu < 23.27}$

(5)' Check with 8: T Interval STAT ▸ TESTS ✓

(6) The mean age of Stats Students is between 17.69 and 23.27

Without Outlier (18.6, 20.2)

Ex 5 Find a 93% CI for the Average age of an SRJC Statistics Student (Daytime).

$L_1 = 24\ 18\ 19\ 25\ \ldots$ ✓ $n \geq 30$

$n = 30$

PE: $\bar{x} = 20.13$ $S_x = 2.417$

$t_{\alpha/2} = \text{invT}(1 - \alpha/2, df) = \text{invT}(1 - .07/2, 29)$

$\alpha = 1 - CL$ $df = n - 1$

$\boxed{t_{\alpha/2} = 1.881}$

$E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}} = 1.881 \cdot \frac{2.417}{\sqrt{30}} = .83$

$\bar{x} - E < \mu < \bar{x} + E$

$20.13 - .83 < \mu < 20.13 + .83$

$19.3 < \mu < 20.96$

We are 93% confident that the Average age of an SRJC stats student is between 19.3 & 20.9 years

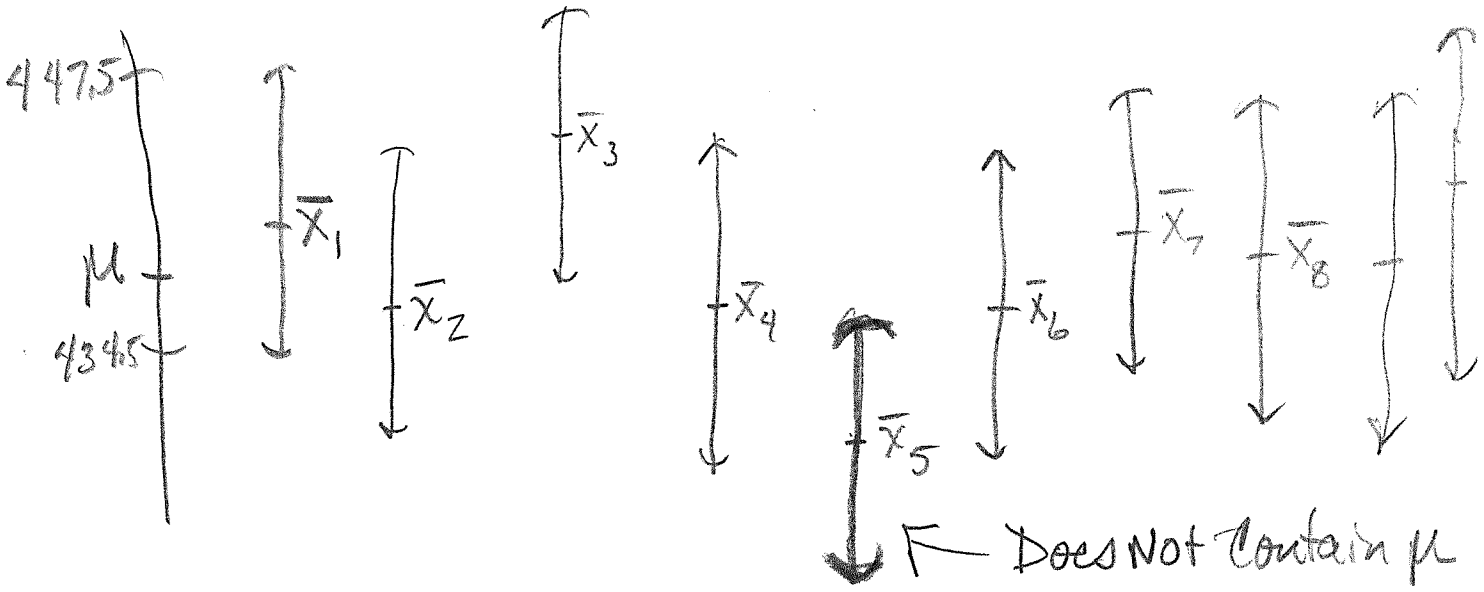
Check TI T Interval Data

Interpreting Confidence Interval

Be careful

Not There is a 90% chance μ will be in interval

Not 90% of data values will be in Confidence Interval.



Does Say 9 out 10 Confidence intervals will contain the population Mean