§8.1 Hypothosis Testing

A hypothosis Test is a very structured Argument.

(2) The Null Hypothesis, H_0, Is a claim about a population parameter.

Claim: $\mu = 118gm$, $\mu = 98.6$, $p = .04$

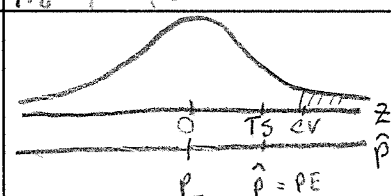
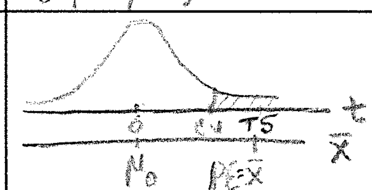
$\mu < 118gm$ $p > .04$

→ Claim Refers to the Whole population

Biologist claims the frogs mean weight is less than 118gm.

~~Consum~~ Buyer → Defect rate is greater than 4%.

Steps for Hypothesis testing

	Steps	Proportions	Means
1	Check requirements SRS	np>5 and nq>5	n>30 or parent poulation is normal
2	**Ho and H1** Write null and alternate hypothesis: Rules Ho always = If Claim: $<, >, \neq$ Then H_1: Same If Claim: $\leq, \geq, =$ Then H_1: $>, <, \neq$	$H_0: p = p_0$ $H_1: p < p_0, p > p_0,$ OR $p \neq p_0$	$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0, \mu > \mu_0$ or $\mu \neq \mu_0$
3	**PE: Point Estimate** Our best guess of the population parameter based on our sample summary statistics	$\hat{p} = \frac{x}{n}$ $\hat{q} = 1 - \hat{p}$ n = Sample Size X = # of Successes	$\bar{x}, s, n$ Given or 1-VarStat
4	**CV: Critical Values** TS further from zero than CV are statistically significant	CV: $z^* = $ invnorm(Area Left, 0, 1) Area Left $= \begin{cases} \alpha & \text{Left} \\ 1-\alpha & \text{Right} \\ 1-\frac{\alpha}{2} & \text{Two Tails} \end{cases}$	CV: $t^* = $ invT(Area Left, df) $df = n - 1$ area $= \alpha$ (Lt), $1-\alpha$ (Rt), $1-\frac{\alpha}{2}$ (Two)
5	**TS: Test Statistics** The T-score or Z-score of the PE assuming that Ho is true	TS: $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$ $H_0: p = p_0$	TS: $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$ $H_0: \mu = \mu_0$,
6	**Draw** Sampling Distribution Include: Ho, CV, TS, PE Shade critical Region		
7	**P-Value** Probability of being more extreme than Test Statistic	P-Value = LT = ncdf(-9999, TS, 0, 1) RT = ncdf(TS, 9999, 0, 1) Two = 2 · Smaller of LT & RT	P-Value = LT = Tcdf(-9999, TS, df) RT = Tcdf(TS, 9999, df) Two = 2 · Min(LT, RT)
8	**Initial Conclusion** Reject Ho or Fail to Reject Ho	fail to Reject that $H_0: p = p_0$	Reject that $H_0: \mu = \mu_0$
9	**Final Conclusion**: Use description of population given in the Question.	TINSE to Reject that (pop. prop.) is equal to p_0, and we can't support that $p > p_0$	TISE to Reject that (pop. Mean) equals μ_0, We can Support $\mu > \mu_0$.

Overview of Some pieces/Main Idea of HT

Ex A Company Claims that their Defect Rate is 4%

(2) $H_0: p = .04 \Rightarrow$ Center of Sampling Dist.

$H_1: p \neq .04$

(3) We collect some data, In 100 batteries 5 are defective.

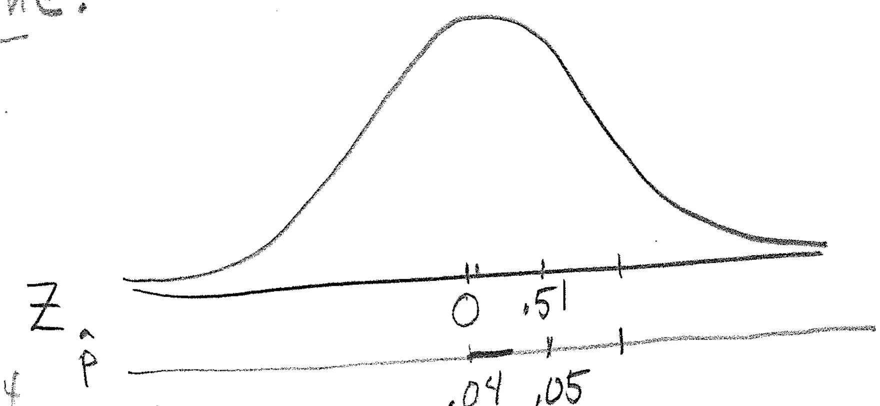
PE: $\hat{p} = \frac{X}{n} = \frac{5}{100} = .05$

(6) Test Statistic is the z or t score of my sample data given that H_0 is true.

Use CLT

$\mu_{\hat{p}} = .04$

$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.04 \cdot .96}{100}} =$



TS: $Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{.05 - .04}{\sqrt{\frac{.04 \cdot .96}{100}}} = .51$

(2) The Null and Alternate Hypothosis

Rule I H_0 Always Contains $=$ $H_0: \mu = 118$ or $p = .04$

Rule II H_1; the Alternate Hypothosis Always Has a Strict inequality, $<$, $>$, $\neq$

Case 1: If Claim: $\mu \neq 118$ $<$ or $>$, Then H_1 is the Same
then $H_1: \mu \neq 118$gm, $p < .04$, $\mu > 30$mpg

Case 2: If Claim has: $\leq$, $\geq$, $=$ then H_1: $>$, $<$, $\neq$
(Atmost, at least, equal)
then Alternate Hypothosis has the opposite inequality
Claim: Defect rate is at most .04, Claim: $p \leq .04$ $H_1: p > .04$
Claim: Weight is 118gm, Claim: $\mu = 118$gm $H_1: \mu \neq 118$gm

③ Critical Values

α = Significance level of Test

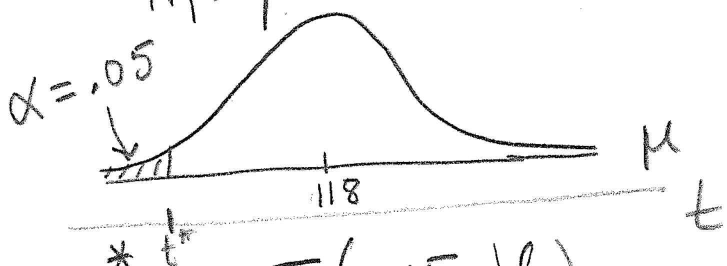
Separate Statistically Significant, TS, from Values that are Not Statistically Significant

Further from Zero than Critical Value.

The critical Region → Shaded Tail of Distribution

Left Tailed

$H_1: \mu < 118$



$t^* = \text{invT}(.05, df)$

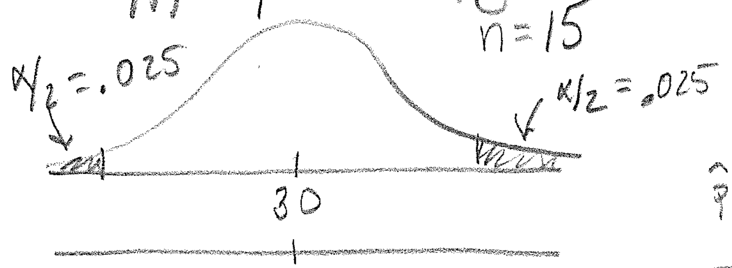
$CV: t^* = \text{invT}(.05, 19) = -1.729$

$t^* = \text{invT}(.05, 35) = -1.690$

Two Tailed

$H_1: \mu \neq 30\text{mpg}$

$n = 15$

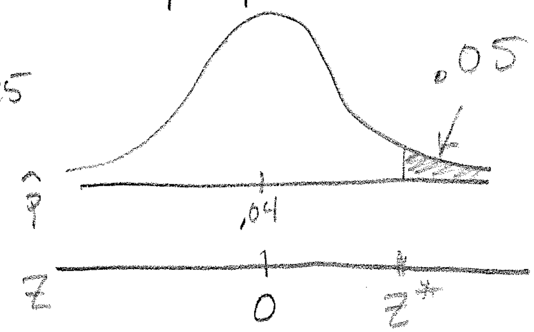


$CV: t^* = \pm \text{invT}(1 - \frac{\alpha}{2}, \overset{df}{14})$

$t^* = \pm 2.045$

Right Tailed

$H_1: p > .04$



$CV: z^* = \text{invnorm}(1 - .05, 0, 1)$

$z^* = 1.645$

⑥ Find Test Statistic

However if we got 15 Defective.

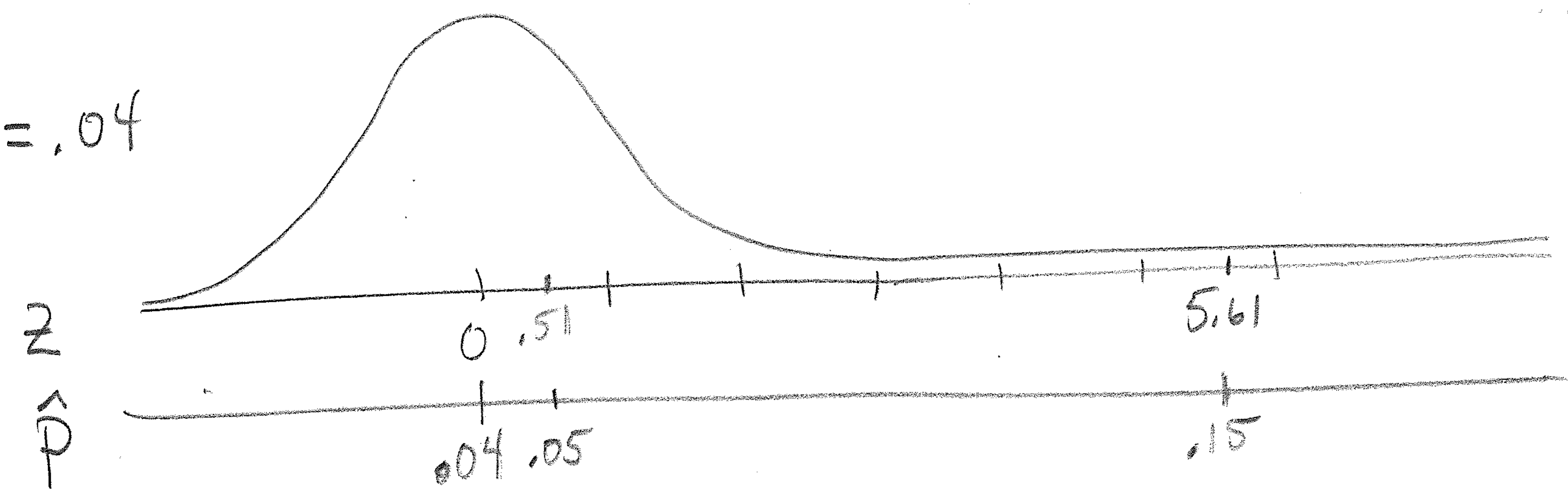
$\hat{p} = .15$

$p = .04$

$n = 100$

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{(.15 - .04)}{\sqrt{\frac{.04 \cdot .96}{100}}} = 5.61$$

$H_0: p = .04$



#5 Claim $p > .5$

$H_0: p = .5$

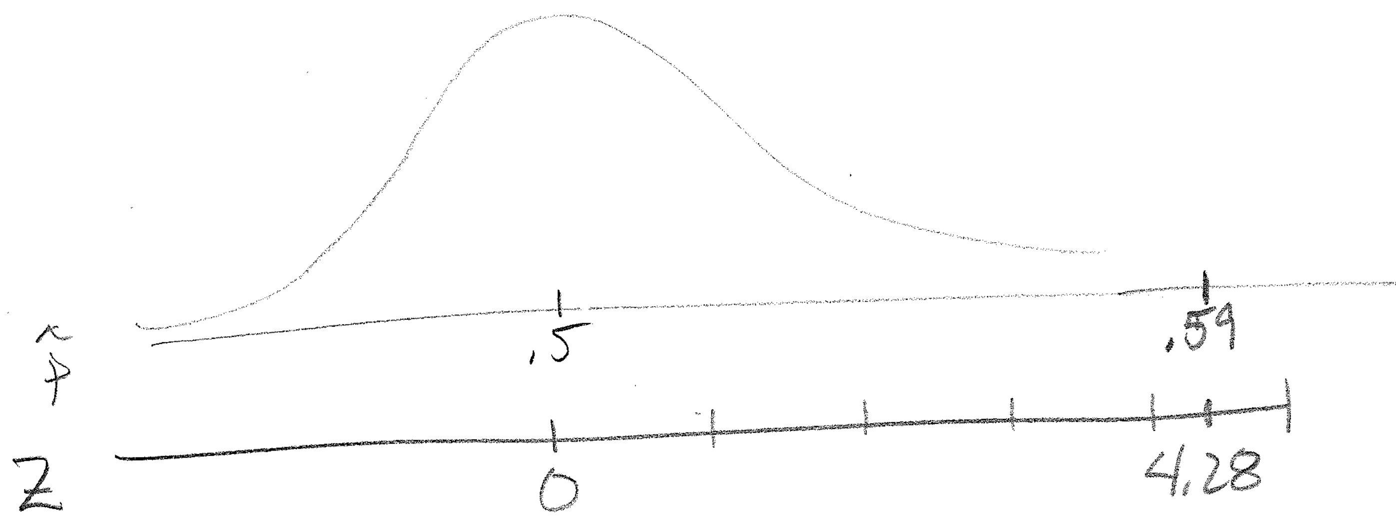
$H_1: p > .5$

#13 TS: $Z = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}} = \dfrac{(.59 - .5)}{\sqrt{\dfrac{.5 \cdot .5}{565}}} = 4.28$

$\hat{p} = .59$

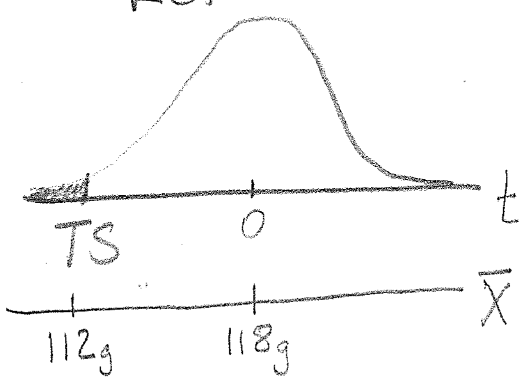
$n = 565$

$q = 1 - p$



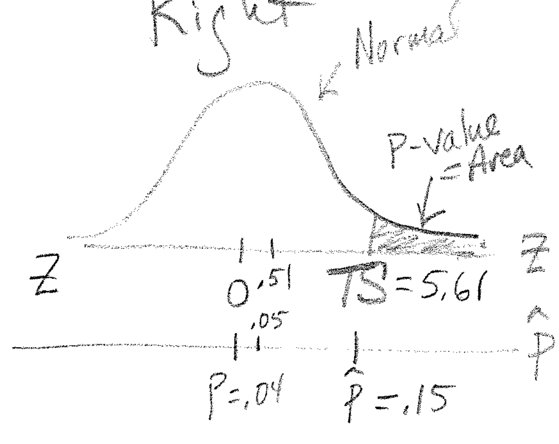
⑦ The P-value is the probability of being More extreme than the Test Statistic.

Left



P-Value
$= Tcdf(-9999, TS, df)$
$= Tcdf(-9999, -2, 35)$
$= \boxed{.0267 = \text{p-value}}$

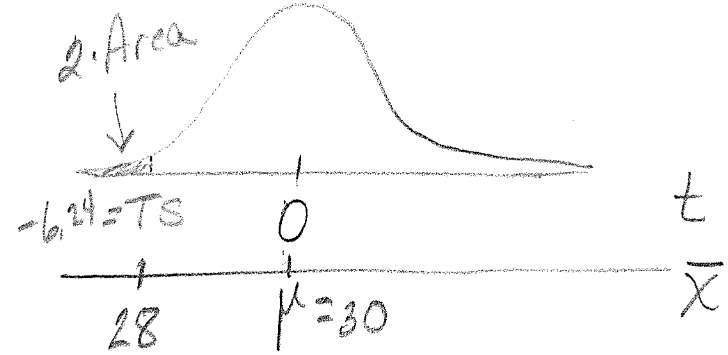
Right



P-value = normalcdf(TS, 9999, 0, 1)
= normalcdf(5.61, 9999, 0, 1)
$= 1.01 \times 10^{-8} = .0000000101$
$\approx 0 < .0001 < \alpha$

P-value = ncdf(.51, 9999, 0, 1)
$= .3050 > \alpha$

Two



$H_0: \mu = 30$ mpg
$\bar{x} = 28 \quad n = 14 \quad s = 1.2$

P-value =
= 2 · Area left
= 2 · Tcdf(−9999, −6.24, 13)
$= 3.02 \times 10^{-5}$
= .0000302

8 Initial Conclusion

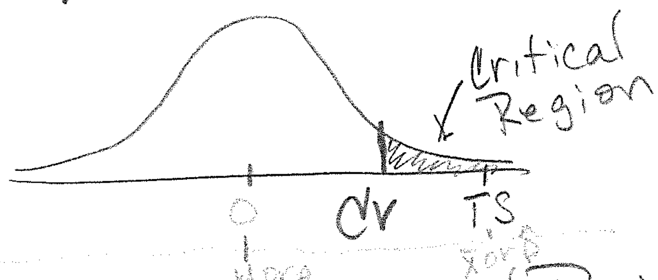
Reject H_0: $p=p_0$, $\mu=\mu_0$ or **Fail to Reject H_0**

P-value Method

P-value $< \alpha$ | P-value $> \alpha$

Traditional Method

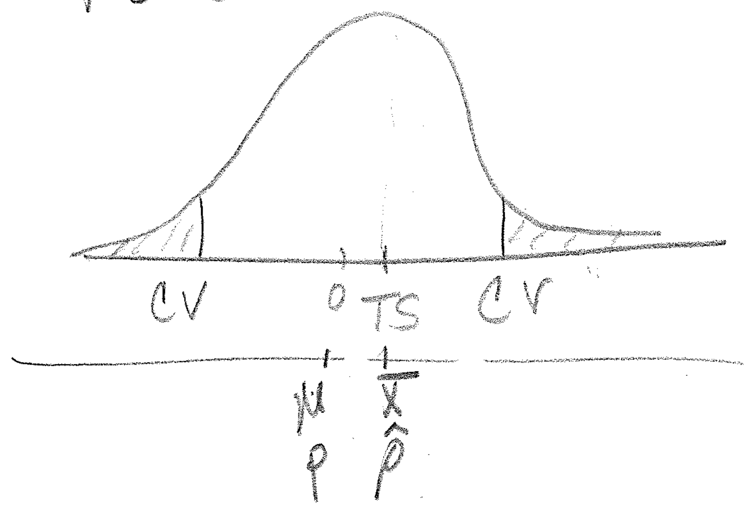
$|TS| > |CV|$



TS is in critical Region

$|TS| < |CV|$

TS closer to zero than CV



Final
(9) Conclusion

$\boxed{\text{Reject } H_0}$ $\hat{p} = .15$ This is statistically Significant

We Reject that the prop. of defects is 4%.

AND we Support $H_1: p > .04$

The Data Supports that the overall defect rate is greater than 4%

$\boxed{\text{fail to Reject } H_0}$: $\tilde{p} = .05$ Not Significantly Different ~~from~~ .04

there is Not Sufficient evidence to Reject that the Overall defect rate is .04

and $H_0: p = .04$

We can't Support that the Overall defect rate is greater than .04.

$H_1: p > .04$

5 Frogs Test Statistic

$\bar{x} = 112 \quad n = 36 \quad s = 18 \quad H_0: \mu = 118$

$H_1: \mu < 118$

TS: $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{112 - 118}{18/\sqrt{36}} = \dfrac{-6}{3} = -2$

Car Test Statistic $\quad \bar{x} = 28 \quad s = 1.2 \quad \mu = 30 \quad n = 14$

TS: $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{(28 - 30)}{\left(\frac{1.2}{\sqrt{14}}\right)} = -6.24$

Project

Ask 30 men & 30 women a yes or No Question.

Exchange Email Addresses   Name

Person	M/F	Dist To School	St/Faculty	Yes/No
1				
2				

Put into a google sheet
Share with group
Share with Me

Type I and Type II $\boxed{\text{Error}}$

	H_0 is True	H_0 is False
Reject H_0	Type I Error $P(\text{Type I}) < \alpha$	Correct
fail to Reject H_0	Correct	Type II Error $P(\text{Type II}) = \beta$

Write a sentence Explaning Errors

If Population Defect Rate is 4% and we get 15% in our Sample

A type I Error means this is a very unusually high sample, but the true rate is 4% overall.

We rejected $p = .04$ when we should Not have would be a Type I Error

If Sample was $\hat{p} = .05$ rate, and defect rate $p > .04$

But we didn't collect enough data to be sure.

We can't prove $p > .04$ even though it is.

Do a full 9 Step Hypothesis Test

Ex A researcher claims that the asthma rate in her town is greater than the National Average of 11%.

In a Random Sample of 167 people, 37 have Asthma.

Test at $\alpha = .05$ level of Significance.

(1) SRS $np = 18.4$ $nq = 148.6$

(2) Claim: $p > .11$ $H_0: p = .11$ $H_1: p > .11$

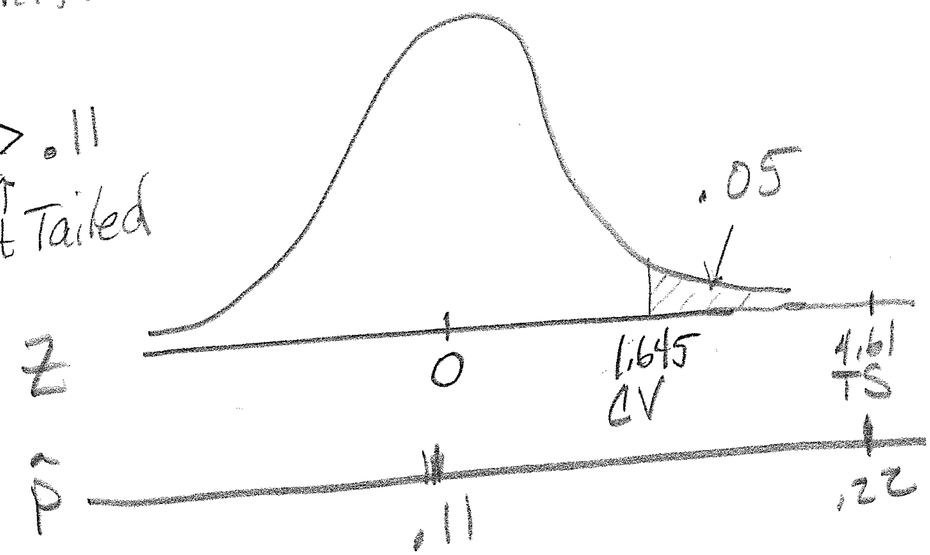
Rt Tailed

(3) PE: $\hat{p} = \frac{x}{n} = \frac{37}{167} = .2216$

(4) CV: $Z^* = \text{invnorm}(1-\alpha, 0, 1)$

$Z^* = \text{invn}(.95, 0, 1) = 1.645$

(5) TS: $Z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \dfrac{(.2216 - .11)}{\sqrt{\frac{.11 \cdot .89}{167}}} = 4.61$



Class Project Part 1 Jones Fall 2019 Names____________________

The data should be collected by Wednesday, October 30,
The final version of part 1 of your project will be due on Wednesday, November 19,
You may work in groups of at most 4 people.

Overview: You will need to collect, organize, summarize and present two pairs of comparable representative samples. Each group member should ask 5 faculty and 20 students how far they live from campus.

Format: The report should be typed with 14 point bold headings. The report should include these headings in this order. It should be turned in stapled in the upper left hand corner (a folder is not required or desired.)

1. **Sampling method/ collection of data**
2. **Data summary**
3. **Graphs, charts and tables**
4. **Discussion and conclusions**
5. **Confidence intervals**
6. **Hypothesis**
7. **Appendix**

1. Describe how each of you **collected** your data to avoided bias. Each member should report.

2. **The Data Summary** includes the sample size n, Mean, median, mode, standard deviation, range, minimum, maximum and quartiles. Just use StatCrunch Summary Stats. Use all data from the whole class for the faculty data. Use all data from your group for the Student data.

3. **The Graphs Charts and Tables** section will include statistical graphs of your data. Be sure to include frequency distributions, cumulative frequency distributions, histograms, qqplots and side by side box plots of your data. Your histograms should use the same classes as your frequency distributions. Be sure to start bins at the same values. Copy and paste these graphs and charts into your report.

4. **The Discussion** describes the shape of your mean data and any outliers. Write a sentence describing each graph.

5. **Make two confidence intervals**. One for Students and one for Faculty.

6. **Write a hypothesis** about the difference between mean distance to school for students and faculty.

7. **Appendix** should have a hand written copy of the data collected by each group member.
Grading will be based on how well you answer every part of the questions in complete sentences that demonstrate an exemplary understanding of the concepts with graphs and tables correct, neat and clearly labeled.

Class Project Part II Math 15 Jones Name____________________

Data is due November 18.
Draft is due Nov. 25.
The final report is due Thursday, Nov. 27.

Overview: You will be a hypothesis test and making confidence intervals based on the data you collect. Each group member will ask 30 men and 30 women a yes or no questions of the group's choice.

Individual Part: Each group member will hand write the entire graph for each of the hypothesis tests, the final conclusions and the meanings of p-value for each test. This should be stapled to the back of the group's final project. Also, attach a handwritten copy of the data as it was collected.

Format: The report should include these headings in this order for your proportion data. It should be turned in stapled in the upper left-hand corner (a folder is not required nor desired.)

Proportion comparison

- **Title** for proportions
- **Data summary**. Include n1, n2, x1, x2, p1-hat and p2-hat. These calculations are done by hand.
- Assumptions and **Requirements** for confidence intervals and hypothesis test.
- Make 3 **Confidence Intervals** for each of your proportions and for the difference between your proportions. You'll need a confidence interval for p1, p2, and p1-p2. Write a sentence or two that explains what each confidence interval means
- Write a **Question** about your proportions looks like the questions in section 9.2 Determine if there is a significant difference between the men and the women at the 0.05 level of significance.
- Preform a **Hypothesis Test** of your claim. Which Test are you using? What level of significance will give the same critical value as your confidence interval?
  1) State the claim, null and alternate hypothesis.
  2) Graph and shade the critical region on a hand drawn normal distribution.
  3) Find the critical value. Label it on your graph.
  4) Use your calculator to find the point estimate of the difference between the sample proportions and its test statistic.
  5) Label the Test statistic and point estimate on your graph.
  6) Make an initial conclusion.

Class Project Part III Math 15 Jones Name____________________

Draft is due Nov. 26. Final Project Dec. 2.

Overview: You will do a hypothesis test for two independent means. Test the claim that the average distance to SRJC is the same for both faculty and students. Use the class data collected for Project Part I for the faculty, and your groups data for the students.

Individual Part: Each group member will hand write the graph of the sampling distribution for the hypothesis test. In addition, each member must hand write the final conclusions and the meanings of p-value for the test in nontechnical terms. A sheet from each group member should be stapled to the back of the group's final project.

Format: The report should include these headings in this order for **both** your mean data and your proportion data. It should be turned in stapled in the upper, left hand corner (a folder is not required nor desired.)

Means comparison

Preform a Hypothesis Test the above claim about population mean distance to SRJC.

- Project Part III, your name(s), Math 15, and my name (Jones).
- **Data summary** should be summary statistics copied from StatCrunch.
  Formatted so it is easy to read.
- Check Assumptions and **requirements** for a hypothesis test using Project Part I.
  Does data appear to come from a normal population? Can we still do a hypothesis test.
- **Hypothesis test** for the difference between two means.
  1) State the claim, null and alternate hypothesis.
  2) Graph and shade the critical region on a hand drawn t-distribution.
  3) Find the critical value. Label it on your graph.
  4) Use StatCrunch to find the point estimate of the difference between the samples means (independent 9.2). Copy and paste the hypothesis test output into your project. Label the test statistic and point estimate on your hand drawn graph of the t-distribution.
  5) Give initial conclusion.
- Discussion and **final conclusion** about population means. This should be included in the typed portion of the project. (However, each member must hand write this part as well.)
  1) Give justification of conclusion using the P-value and meaning of p-value.
  2) Graph the test statistic and critical value and explain their meanings. Explain how they justify your conclusion.
  3) Make a **confidence interval** for the difference between your population means. Choose your confidence level so the Critical values will be the same as those used for your hypothesis test.
  4) Explain how the confidence interval justifies your conclusion.
- **Appendix** giving your mean data collection lists from each group member.

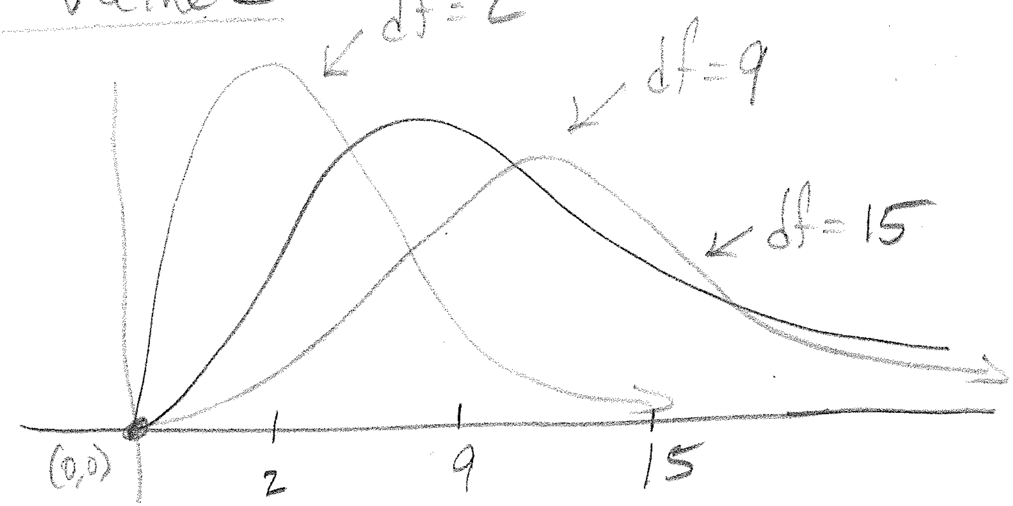
Skip 6.5 — Get program before Dec. 4

Find χ^2 - Critical Values

① Skewed Rt

② Different for each value of degrees of freedom

$df = n-1$



③ High Point or Mode is just Left of df

④ Always Positive

χ is Chi Say "Kai"

IR88

Answers to My Turn!

1. $z \approx 9.19$
2. $t \approx 5.59$
3. $\chi^2 \approx 61.7$

Practice Problems

1. Evaluate the formula

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$

when $\hat{p} = \frac{x}{n}$, $x = 90$, $n = 150$, $p = 0.82$, and $q = 1 - p$. Round your answer to the nearest hundredth.

2. Evaluate the formula

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

when $\mu = 44$, $\bar{x} = 45.2$, $n = 80$, and $s = 3.1$. Round your answer to the nearest hundredth.

~~3.~~ Evaluate the formula

$$\chi^2 = \frac{(n-1) \cdot s^2}{\sigma^2}$$

when $\sigma = 11.3$, $n = 160$, and $s = 11.4$. Round your answer to the nearest tenth.

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