§4.1 Probability

Goal: Understand Vocabulary of Probability

Probability of an Event = % of Time the event occurs = Proportion of occurences out of the Number of Trials

Percent

① Relative frequency of an event

the probability of event E = $P(E) = \frac{\#\text{ of occurences of } E}{\#\text{ of Trials}}$

Ex If a coin is flipped 100 times then $n = 100$ $X = 48$ heads occur.

$P(H) = \frac{48}{100} = .48$

$P(E) = \frac{X}{n}$

The Law of Large Numbers

In the long run the relative frequency will Approach the theoretical probability.

Classical

§ 4.1 Probability

Goal: Use probability to Interpret Statistics.

Three Definitions of the Probability of an event

① Relative frequency $= \frac{x}{n} = \frac{\#\text{ of occurences of } E}{\#\text{ of Observations}}$

Notation $P(E) = \frac{x}{n}$

Read "the probability of the event E"

Ex a) $H = \{$Hit by lightning$\}$ $n = \#$ of lightning Strikes

$$P(H) = \frac{350 \text{ hits}}{5,467,848 \text{ Strikes}} = 6.40106 \times 10^{-5}$$

$E-5$

$= .00006401$

Very unlikely

b) Flip a coin 100 times, $H = \{$flips show Head$\} = 52$

$$P(H) = \frac{x}{n} = \frac{52}{100} = .52 = \hat{p}$$

= proportion of Sucesses in a sample

Classical Definition of Probability

Let S = Sample Space = the set of all possible equally likely Outcomes.

If a family has 3 children, what genders

~~{ggg, ggb, gbb, bbb}~~ $\neq S$

Need All orders

{ggg, ggb, gbg, bgg, gbb, bgb, bbg, bbb} $= S$

$n(S) = 8$

$P(ggg) = \frac{1}{8}$

$P(2 \text{ girls}) = \frac{3}{8}$

Counting → HW $\underset{\uparrow}{2} \cdot \underset{\uparrow}{2} \cdot \underset{\uparrow}{2} \cdot \underset{\uparrow}{2} = 16 = 2^4$

Method 2

(2) Theoretical Probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{\#\text{ elements of } E}{\text{size of Sample Space}}$$

S = Sample Space = Set of all possible Outcomes

Ex Flip a coin 3 times. Write Sample Space

$S = \{TTT, HTT, THT, TTH, HHH, HTH, HHT, THH\}$

a Each outcome is equally likely

a) $P(\text{No heads}) = \frac{1}{8}$ $\quad E = \{TTT\}$, $n(E) = 1$

b) $P(\text{one head}) = \frac{3}{8}$ $\quad G = \{HTT, THT, TTH\}$

c) $P(\text{At least one head}) = \frac{7}{8} = 1 - \frac{1}{8}$

$F = \{\text{At least one head}\}$ $n(F) = 7$

$= \{HTT, THT, TTH, HHH, HTH, HHT, THH\}$

$F = E^c$ = the complement of E

= Everything Not in E

Prob of 1 girls in 14 births

$E = \{BBGBBBBB, GBBBBBBB, \quad \}$ $j = 7$

$n(S) = 2^7 = 128$

$$P(\text{one girl in 7 Births}) = \frac{7}{128} = \boxed{.0547}$$

Rule: Round probabilities to 4 decimals

Dice

a) Find the Probability of a 7 if two dice are rolled.

$n(S) = 6 \cdot 6 = 36$

$S = \{11, 12, 13, 14, 15, 16, 21, 22, 25, 34, 43, 46, 52, 55, 56, 61, 64, 65, 66\}$

$E = \{61, 52, 43, 34, 25, 16\}$

$P(\text{Sum}=7) = \frac{6}{36} = \frac{1}{6} = \boxed{.1667}$

b) Find the prob of a Sum of 3 or 7.

$P(F) = P(\text{Sum}=3) = \frac{2}{36} = \frac{1}{18} = .0556$

$P(F \text{ or } E) = .0556 + .1667 = \boxed{.2222}$

c) Prob of a Sum of at least 10?

$P(\text{Sum} \geq 10) = \frac{6}{36} = \boxed{.1667} = P(G)$

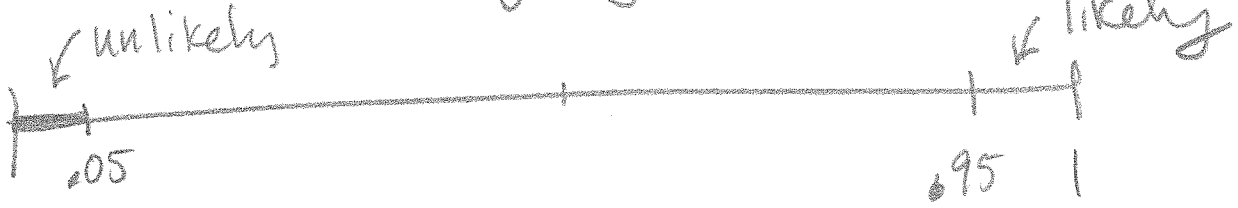
d) Prob of a Sum less than 10?

Complement of an Event

$G^c = G$ does Not occur

$P(G^c) = 1 - P(G) = 1 - \frac{6}{36} = \frac{30}{36} = \boxed{.8333}$

Probability Language



$0 \leq P(E) \leq 1$

$P(E) = 0$ → Never, impossible

$P(E) = 1$ → Always, certain

$P(E) < .05$ → unlikely

$P(E) < .001$ → Really unlikely
Almost impossible

$P(E) > .95$ → Likely

The law of large Numbers

"The casino always wins"

As an experiment or procedure is repeated the relative frequence of an observed event approaches the classical probability

10 times	100	1000	P(H)
$\frac{4}{10}$	$\frac{47}{100}$	$\frac{493}{1000}$	
.4	.47	.493	→ .5

Sample Space for 10 flips?

$S = \{HHHHHHHHHH, TH\ldots H, HTH\ldots H, \ldots\}$ (10)

$n(S)$ = the size of $S = 2^{10}$ Multiplication principle of counting 4.3

$n(S) = 1024$

$P(\text{one head}) = \frac{10}{1024} = .00977$

Def An event is statistically Significant if the probability of that event is Small (usually, $P(E) < .05$)

Ex Is the event that a family with 4 children is all boys statistically Significant?

$$P(\text{4 boys}) = P(bbbb) = \frac{1}{2^4} = .0625$$

$$\text{HWS} = \{bbbb, gbbb, bgbb, ggbb\}$$

No, 4 boys is Not statistically Significant

Ex If the bus was on time 35 out of 40 times ~~in the last~~ Recently. What is the probability that it will be on time today?

a) $P(\text{On time}) = P(E) = \frac{35}{40} = 0.875$

b) Prob. it is late. (Complement)

$P(\bar{E}) = 1 - .875 = .125$

Is it unusual for bus to be late? No

$P(\text{Late}) = .125 > .05$

↑ Always justify Answer ↑

Ex A class has 46 women & 81 men What is the probability of randomly selecting a women?

$$P(W) = \frac{46}{46+81} = \frac{46}{127} = .362$$