§8.3 Hypothesis Testing Means

Recall The Rare Event Rule

If the probability of seeing an observed event is unusual (Statistically Significant, p-value $< \alpha$) Under a given assumption (H_0), Then the assumption is probably wrong!

If p-value is low the Null must go!

Hypothosis Test for Means

① Requirements: SRS, $n > 30$ or Population is Normal
then CLT ⇒ dist of Sample Means is a t-dist

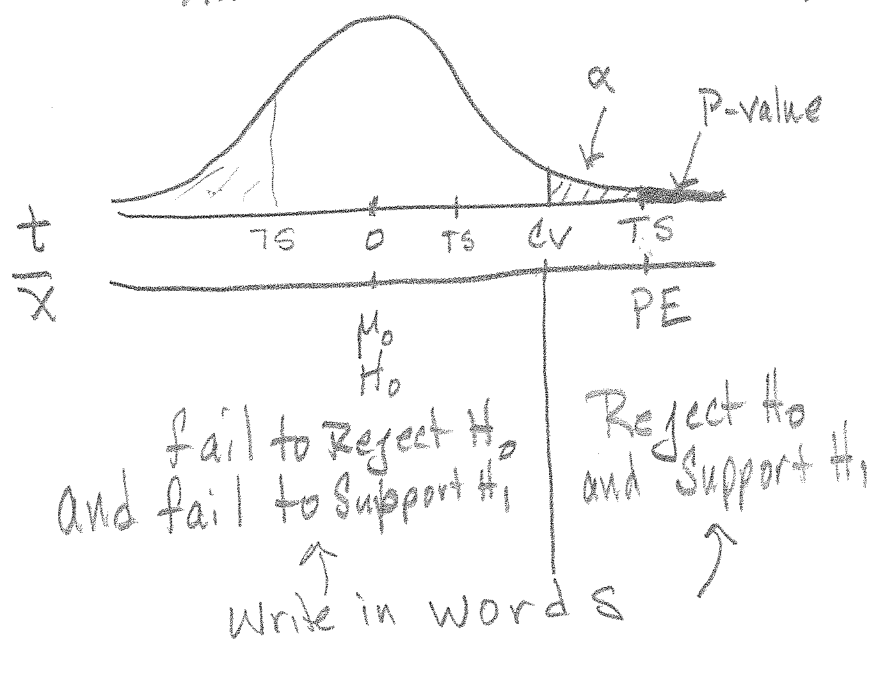
② Claim: $\mu = , \geq, \leq \mu_0 \Rightarrow H_1: \mu \neq, <, > \mu_0$ opposite Claim
$\mu \neq, >, < \mu_0 \Rightarrow H_1: \mu \neq, >, < \mu_0$ Same as claim

③ CV: $t^* = \text{inv}\,t(\text{area}, df)$ $H_1: <, >, \neq$
Area $= \underset{Lt}{\alpha}, \underset{Rt}{1-\alpha}, \underset{\text{Two Tailed}}{1-\frac{\alpha}{2}}$

④ PE: $\bar{x}$ = Sample Mean is Given or use 1-VarStats
Also Need $S = S_x$ n = Sample size

⑤ TS: $t = \dfrac{\bar{x} - \mu}{S_x/\sqrt{n}}$ σ - is unknown so Approximate $\sigma/\sqrt{n}$ with Standard error $= \frac{S}{\sqrt{n}}$

⑥ Draw distribution



⑦ P-value = Look at H_1
Lt = $t\text{cdf}(-\infty, TS, df)$
Rt = $t\text{cdf}(TS, \infty, df)$
Two = Mult Above by 2 < 1

⑧ Initial Conclusion

⑨ final Conclusion Sentence

Test at $\alpha = .02$ level

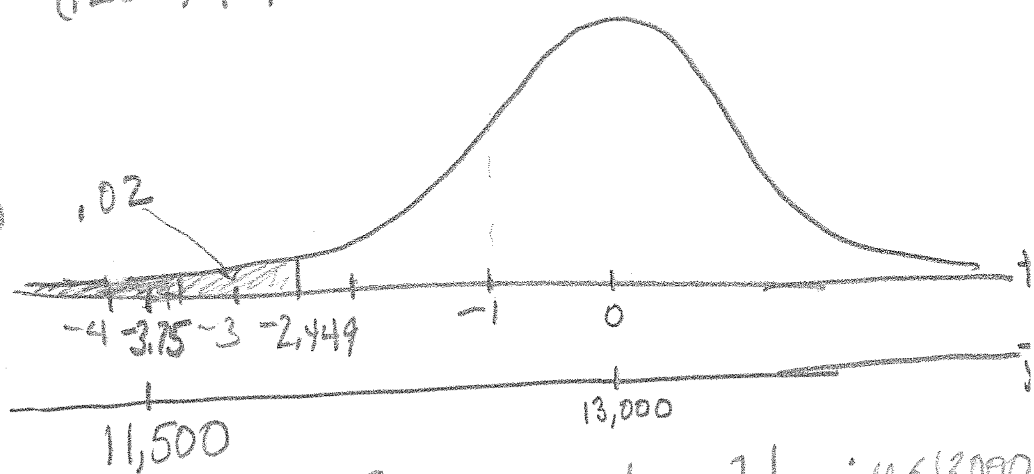
<u>Example</u> Claim: Oakmont Residents drive less than <u>average</u> of 13,000 ~~miles~~, Always Don't Know

Nationally $\mu = 13{,}000$ and $\cancel{\sigma = 1{,}200}$

Sample $\bar{x} = 11{,}500$ miles and $s = 1{,}200$ miles $n = 9$

Driving distances have a <u>Normal Distribution</u>.

1. If skewed $\Rightarrow$ Stop but since Normal ok to do a HT
2. Claim: $\mu < 13{,}000$ $\quad H_0: \mu = 13{,}000 \quad H_1: \mu < 13{,}000$
3. CV: $t^* = \text{invt(area, df)} = \text{invt}(.02, 8) = -2.449$
4. PE: $\bar{x} = 11{,}500$
5. TS: $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{(11{,}500 - 13{,}000)}{(1200/\sqrt{9})} = -3.75$
6. Draw



7. P-value $= P(\bar{x} \overset{TS}{<} 11{,}500)$

$= \text{tcdf}(-999, -3.75, 8)$

$= \boxed{.002812} < .02$

8. Reject H_0 and Support $H_1: \mu < 13{,}000$

We Reject that Oakmont residents drive an average of 13,000 miles

$H_0: \mu = 13{,}000$

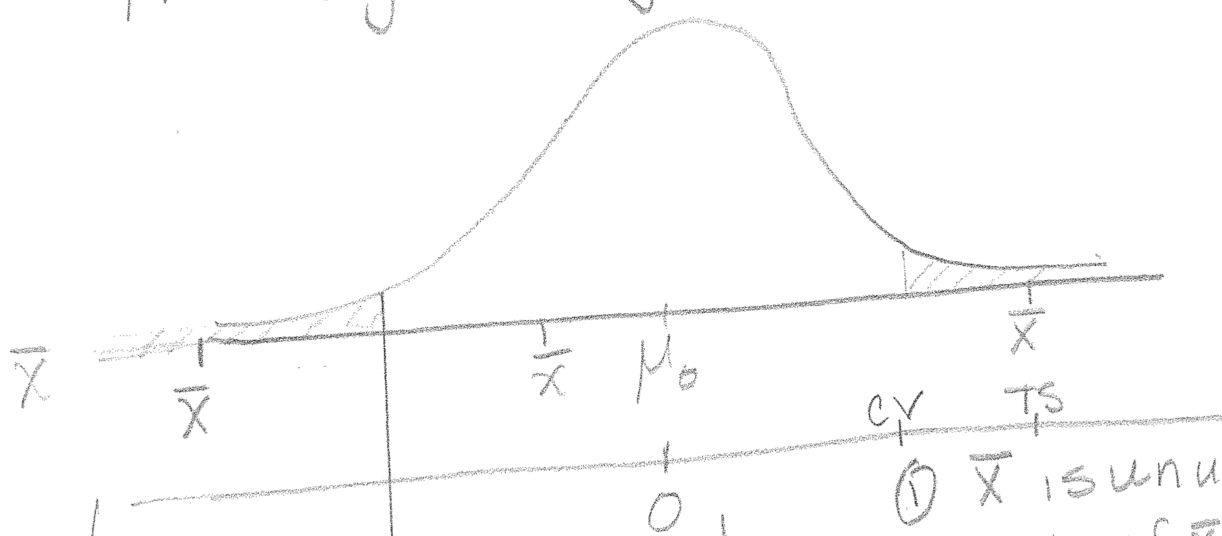
TISE to Support that the mean miles driven by Oakmont residents is less than 13,000 miles.

§8.3 HT Means

Rare Event Rule

If the probability of seeing an observed event is unusual given an assumption, then the assumption is probably wrong.

$H_0: \mu = \mu_0$



① $\bar{x}$ is usual
② Prob. of getting a sample $\bar{x}$ when μ is true is high $\equiv$ p-value > α
③ TS is Not in Critical Region. TS is close to 0.
④ fail to Reject $\mu = \mu_0$
fail to Support H_1
Can't Support $\mu < \mu_0$, $\mu > \mu_0$ or $\mu \neq \mu_0$

① $\bar{x}$ is unusual
② prob. of $\bar{x}$ given μ is small $\equiv$ p-value < α
③ TS is in Critical Region
④ Reject H_0 $\quad \mu \neq \mu_0$
⑤ Support H_1
$\mu > \mu_0$
$\mu \neq \mu_0$

⑤ Support H_1:
$\mu < \mu_0$
$\mu \neq \mu_0$

Stat Tests $\boxed{\text{TTest}}$

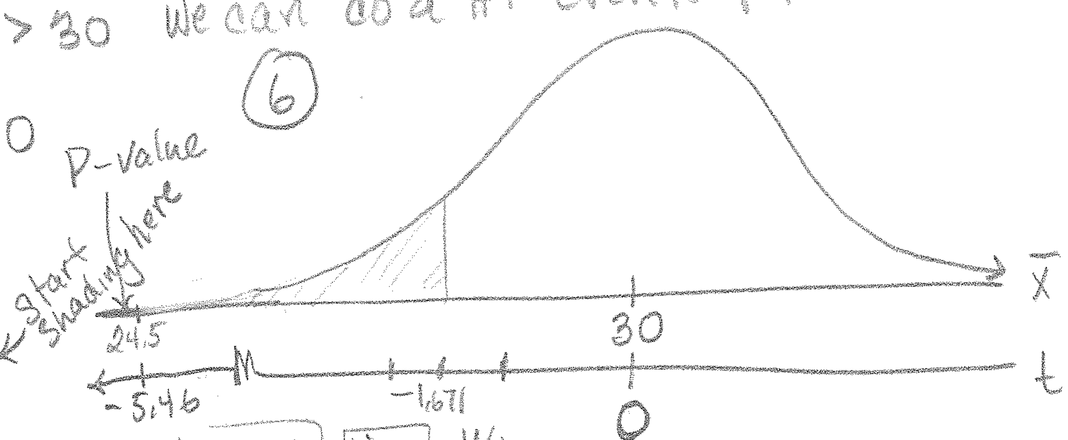
Ex Gas Milage. The average gas milage of SRJC students cars is at least 30mpg.

Sample Data: $\bar{x} = 24.5$ $s = 7.8$ $n = 60$

Test claim at $\alpha = .05$ level.

(1) Since $n = 60 > 30$ We can do a HT even if pop. is not Norm

(6)



(2) Claim: $\mu \geq 30$

$H_0: \mu = 30$

$H_1: \mu < 30$

(4) CV: $t^* = -1.671$

$\boxed{\text{ALPAA}}$ $\boxed{Y=}$ u/d

(3) PE: $\bar{x} = 24.5$

(5) TS: $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{(24.5 - 30)}{(7.8/\sqrt{60})} = -5.46$

(7) P-value $= \text{tcdf}(-9999, \underset{\text{TS}}{-5.46}, \underset{\text{df}}{59}) = 4.99 \times 10^{-7}$

$= .000000499$ $\boxed{\text{TTest}}$

(8) Reject $H_0: \mu = 30$ → Support $H_1: \mu < 30$

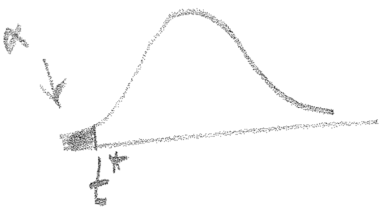
We Reject that the gas milage is at least 30 mpg.

We Support that the gas milage is less than 30mpg

Finding Critical Values

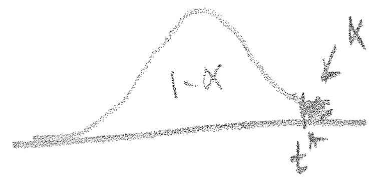
CV: $t^* = \text{invt}(\underset{\text{Left}}{\text{Area}}, df)$

$H_1: \mu < \mu_0$



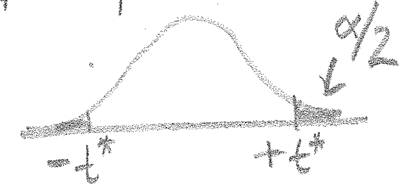
$t^* = \text{invT}(\alpha, df)$

$H_1: \mu > \mu_0$



$t^* = \text{invT}(1-\alpha, df)$

$H_1: \mu \neq \mu_0$



$t^* = \pm\text{invT}(1-\frac{\alpha}{2}, df)$

Ex① claim: $\mu < 112$ $n=16$ $\bar{x}=100$ $S=12$

Significance level $= \alpha = .01$, $df = n-1 = 16-1 = 15$

$H_0: \mu = 112$ CV: $t^* = \text{invT}(.01, 15) = -2.602$

$H_1: \mu < 112$

Left

Ex② Claim Gas milage is <u>at least</u> 30 mpg

Claim: $\mu \geq 30$ $n=60$ $\bar{x}=24.5$ $S=7.8$ $\alpha=.05$

$H_0: \mu = 30$ $t^* = \text{invT}(.05, 59) =$

$H_1: \mu < 30$ $= -1.671$

Ex Pulse Rates

Claim: My Pulse Rate is less than the class Average of All my classes Ever. $\alpha = .05$

Mine $\mu_0 = 46$ bpm

70 76 80 60 72 68 62 74 76 83 74
66 70 70 66 68 84 110 76 60 88

(1) heart Rates are Normally distributed so even though $n < 30$ we can use CLT. & do a test

(2) $H_0: \mu = 46$

$H_1: \mu > 46$

(3) CV: $t^* = \text{invt}(1-.05, 20)$

CV: $t^* = 1.725$

(4) PE: $\bar{X} = 73.95$

$S = 11.2$

$n = 21$

(7) P-value = tcdf(11.43, 9999, 20)

$= 1.598 \times 10^{-10} \approx 0$

(5) TS: $t = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}} = \dfrac{(73.95 - 46)}{(11.2/\sqrt{21})} = 11.43$

(6) Draw

(8) Reject H_0

(9) We Reject that the class average pulse is 46

We Support that the Average pulse is greater than 46

What does the P-value mean?

P-value < .0001

It is very unlikely to see a sample mean of 73.95 from a population with a mean of 46. So we reject that the student mean is 52bpM and support that the mean is greater than 52bpM.

The Confidence Interval Method

Create a CI based on Sample.

If μ is in CI $\Rightarrow$ Fail to reject H_0

μ is Not in CI $\Rightarrow$ Reject H_0

CI $(64.9, 72.0)$

Since $\mu = 52$ is Not in CI

We reject that the Student mean pulse is equal to 52bpm.

Math 15

Lecture 8.3

Hypothosis Test Means

With Data

Given

Claim: $\mu > 56$

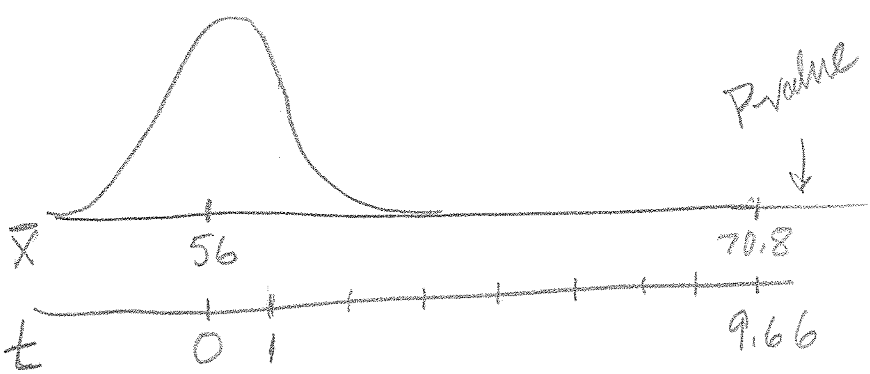
$\alpha = .05$

$\bar{x} = 70.8$

$n = 28$

$S = 8.09$

- $H_0: \mu = 56$
- $H_1: \mu > 56$



Find Test

$df = 27$

TS: $t = 9.66$

P-value $< .0001$ $= 1.41 \times 10^{-10}$

TS: $t = \dfrac{\bar{x} - \mu_{\bar{x}}}{s/\sqrt{n}}$

$s = S_x$ Not σ_x on Calculator

Given TS:

P-value = tcdf(9.66, 9999, 27)

$= 1.48 \times 10^{-10}$

DIST

Meaning of p-value → the probability is very small that the population of all students ~~has a mean of 56.~~

Frog problem

Test the claim that the mean weight of the frogs is less than 118 gm.

$n = 36$, $\bar{x} = 110$ gm, $s = 18$ gm, at $\alpha = .05$

Graph

1. Claim: $\mu < 118$

   $H_0: \mu = 118$

   $H_1: \mu < 118$

2. CV: $t^* = 1$

3. PE: $\bar{x}$

   TS: $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

4. p-value

5. Initial Conclusion

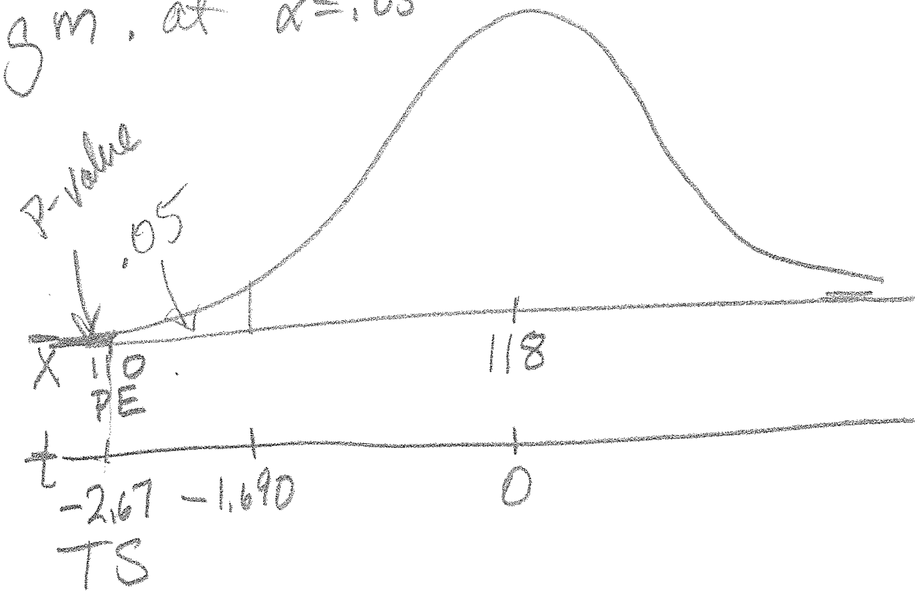
6. Final Conclusion

Frog problem

Assume that $\mu = 118gm$ & $s = 18gm$ $\bar{X} = 110gm$
$n = 36$

Test claim that the mean weight of frogs is less than 118gm. at $\alpha = .05$

claim: $\mu < 118gm$



① $H_0: \mu = 118$
$H_1: \mu < 118$

② CV: $t^* = invt(.05, 35)$
$\boxed{t^* = -1.690}$

③ $\boxed{PE: \bar{X} = 110}$

TS: $t = \dfrac{\bar{X} - \mu}{(s/\sqrt{n})} = \dfrac{110 - 118}{(18/\sqrt{36})} = \dfrac{-8}{3} = \boxed{t = -2.67}$ TS

p-value = tcdf$(-\infty, -2.67, 35)$ =
$\boxed{\text{p-value} = .0057 < \alpha}$

Critical value method: If $|TS| > |cv| \Rightarrow$ Reject H_0
P-value Method: If p-value $< \alpha \Rightarrow$ Reject H_0

Conclusion: TISE to Support that the mean weight of the frogs is below 118gm.

Solve the problem.

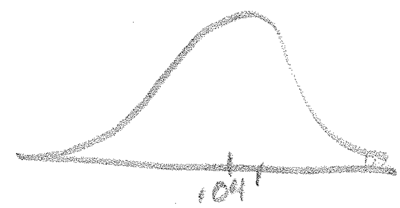
15) What do you conclude about the claim below? Do not use formal procedures or exact calculations. Use only the rare event rule and make a subjective estimate to determine whether the event is likely. 15) ____________

Claim: $p = .04$

$H_0: p = .04$

$H_1: p \neq .04$

Claim: A company claims that the proportion of defectives among a particular model of computers is 4%. In a shipment of 200 such computers, there are 10 defectives.



$\hat{p} = \frac{10}{200} = .05$ is close to $P = .04$

There is Not Sufficient evidence to Reject the claim that $p = .04$

Formulate the indicated conclusion in nontechnical terms. Be sure to address the original claim.

16) The owner of a football team claims that the average attendance at games is over 727, and he is therefore justified in moving the team to a city with a larger stadium. Assuming that a hypothesis test of the claim has been conducted and that the conclusion is failure to reject the null hypothesis, state the conclusion in nontechnical terms. 16) ____________

Claim $\mu > 727$

$H_0: \mu = 727$

$H_1: \mu > 727$

There is $\boxed{\text{Not}}$ Sufficient evidence to $\boxed{\text{Show}}$ that the average attendence is greater than 727.

17) Carter Motor Company claims that its new sedan, the Libra, will average better than 19 miles per gallon in the city. Assuming that a hypothesis test of the claim has been conducted and that the conclusion is to reject the null hypothesis, state the conclusion in nontechnical terms. 17) ____________

Claim $\mu > 19$

$H_0: \mu = 19$

$H_1: \mu > 19$

Reject H_0 and We Show that $H_1: \mu > 19$.

there is suffecient evidencl to Show that the Mean miles per gallon is better than 19 miles per gallon in the libra

Assume that the sample has been randomly selected from a population with a normal distribution.

6) The Test Prep company claims that the mean SAT score among a large population of students taking their course is greater than 600. This claim applies to all three sets of sample data given below. Test the claim at the 5% level of significance.
State the claim, null and alternate hypothesis.

Claim: $\mu > 600$

$H_0: \mu = 600$

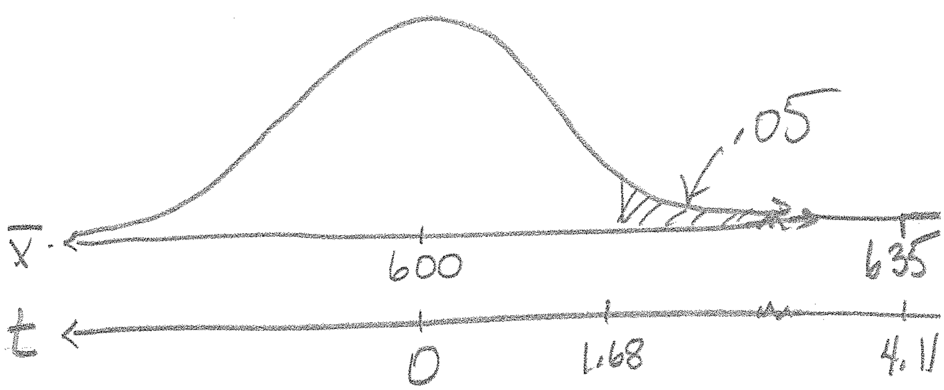
$H_1: \mu > 600$

use t for all prob in 8.5

a) If a random sample of students who have taken their course has size 50, a mean of 635, and a standard deviation of 60.2. Find the critical value, test statistic and p-value. Label these on a graph of the sampling distribution of means. $n = 50$

$\bar{x} = 635$

$s = 60.2$



CV: $t^* = invt(1-.05, 49)$

$t^* = 1.68$

TS: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{(635-600)}{(60.2/\sqrt{50})} =$

$t = 4.11$

P-value $= tcdf(LB, UB, \underline{df}) = tcdf(\overset{TS}{4.11}, 9999, 4$

$= .000075$

Clearly state your final conclusion.

TISE To Support the claim that Student who take this class have a <u>Mean</u> score above 600.

#26 Nicotine Patch

$x = 39 \quad n = 32 + 39 = 71 \quad \alpha = .05 \quad C$

a) Claim: $p > .5$

$H_0: p = .5$ ← Must be =

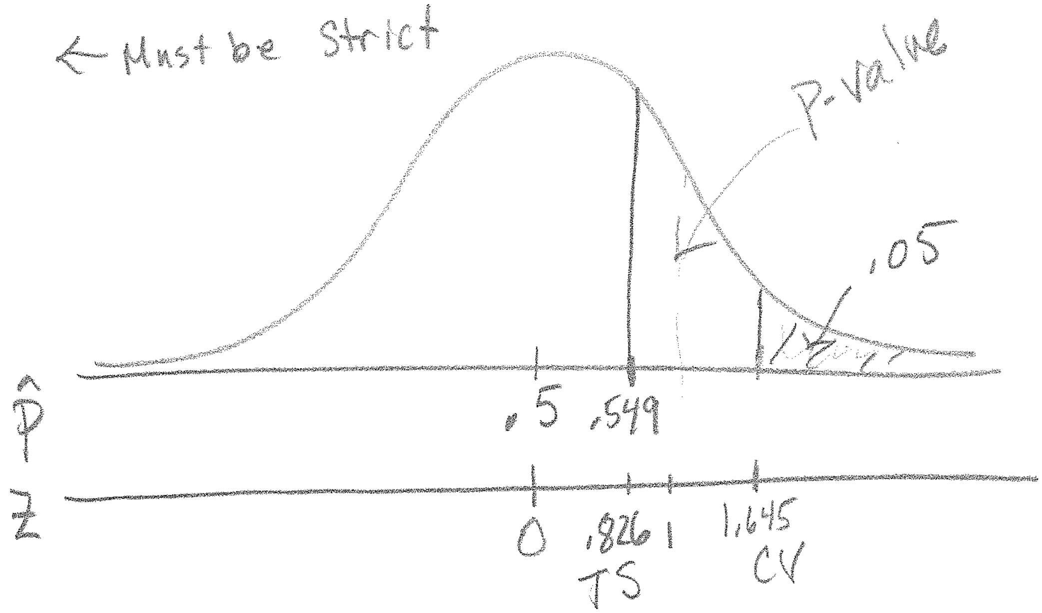
$H_1: p > .5$ ← Must be strict

↑

Right Tail

CV: $Z^* = \text{invN}(1-\alpha, 0, 1)$

$Z^* = 1.645$



b) TS, p-value, PE, Put on Graph

line up

PE: $\hat{p} = \frac{x}{n} = \frac{39}{71} = .549$

TS: $Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{.549 - .5}{\sqrt{\frac{.5 \cdot .5}{71}}} = .826$

P-Value = Area more extreme that TS

$= \text{ncdf}(TS, \infty, 0, 1) = \text{ncdf}(.826, 999, 0, 1)$

$= \boxed{.2033}$

We can not Support that the Majority are Not Smoking.

We can't show that the proportion who Quit is greater than .5

H_1: $p > .5$

Project Part II Due Thursday!
4 Confidence Intervals Typed!

Do you have a Major?

Male

$x_1 = \#$ yes from men

$n_1 = \#$ men $\Rightarrow$ Sum of all group members

$\hat{p}_1 = \frac{x_1}{n_1}$

CI_1 Prop of men who have a major

Female

$x_2 = \#$ yes women

$n_2 = 90$ women

CI_2

How many BOOKS?

Men L_1
2
4
1
5
⋮

← StatCrunch Summary Stats

$\bar{x}_1$

s_1

n_1

CI_3 for mean # of books read by men

L_2 = Women
7
4
3
8

$\bar{x}_2 =$

$s_2 =$

$n_2 =$

CI_4

One Project Per group

HT on Calculator For Means

STAT Test 2 $\boxed{\text{Stats}}$ T Test

HT with StatCrunch

Claim: My pulse rate is less than the mean pulse rate of all my students

beats per minute $= 48 < \mu$

HW §8.2 #26

$\alpha = .05$ Claim: $p > .5$ p = prop who are still smoking

39 are smoking and 32 are Not smoking

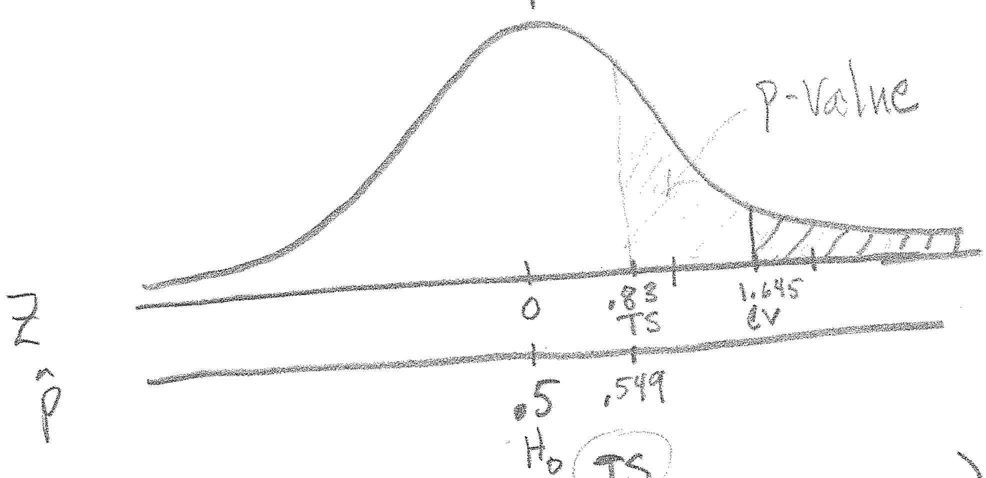
$n = 71$ $x = 39$ $\boxed{PE: \hat{p} = \frac{39}{71} = .549}$ of Sample are still smoking.

$H_0: p = .5$ CV: $Z = \text{invnorm}(1 - \overset{.05}{\alpha}, 0, 1) = \boxed{1.645 = Z^*}$ CV

$H_1: p > .5$ rt-tailed

TS: $Z = \dfrac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \dfrac{.549 - .5}{\sqrt{\frac{.5 \cdot .5}{71}}} = \boxed{.83 = Z}$ TS

Graph



P-value = normalcdf(.83, 9999, 0, 1) = .203 > .05

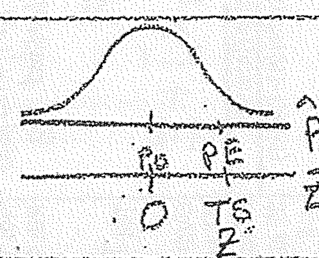
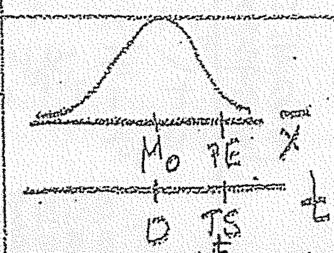
fail to Reject H_0 and we fail to Support H_1

We fail to Reject ~~that~~ ~~the~~ prop who quit is 50%

TINSE to
We fail to Suppot that a (majority) are still Smoking. (More than 50%)

Since it is So hard to Quit getting 45% to Quit is Good.

Steps For a hypothesis Test

	Proportions	Means
① Check Requirements SRS	$np > 5$ and $nq > 5$	$n > 30$, or Population is Normal
② Write Null and Alternate Hypothesis Claim: $<, >, \neq \Rightarrow H_1$ is claim Claim: $\leq, \geq, = \Rightarrow H_1$ is opposite	$H_0: p = p_0$ $H_1: p <, >, \neq, p_0$	$H_0: \mu = \mu_1$ $H_1: \mu <, >, \neq \mu_0$
③ Point Estimate and the Summary Statistics	$\hat{p} = \frac{x}{n}$	$\bar{x}$ from 1-varstat s, n or given
④ Find Critical Values	CV: z^* = invnorm(area)	CV: t^* = invT(area
⑤ Test Statistic is Score of point estimate	TS: $z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$ use p in H_0	TS: $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$ use μ in H_0 and s from 1-varstat
⑥ Draw Sampling Distribution Vertically line up PE and TS		
⑦ P-value is the probability of being more extreme than PE	P-value Rt = normalcdf(TS, 9999) Lt = ncdf(-999, TS) Two Tail = 2 · smaller of Lt or Rt	P-value = tcdf(LB, UB, df
⑧ Make Initial Conclusion Reject H_0 or fail to Reject H_0	$H_0: p = p_0$	$H_0: \mu = \mu_0$
⑨ Write Final Conclusion describing what the population parameter is in words.		