

§8.1 Hypothesis Testing

A Hypothesis Test is a systematic form to make an argument.

② H_0 : The Null hypothesis is a claim about the population parameter.

$$H_0: \mu = \mu_0 \text{ or } p = p_0$$

$$H_0: \mu = 98.6^\circ \text{ or } p = .52$$

③ Take some data, and find a Sample Mean $\bar{x}$ or sample proportion $\hat{p}$ → These are your point estimates.

⑦ Calculate the likelihood of seeing sample statistic given that the Null hypothesis is true. This is the P-Value =

⑤ Test Statistic

$$TS: Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

The Test Statistic is z or t score of sample data assuming H_0 is True

(5) P-value = Area More extreme than test Statistic
 = probability of a sample as or more extreme than my sample given H_0 is true

(8) Conclusion

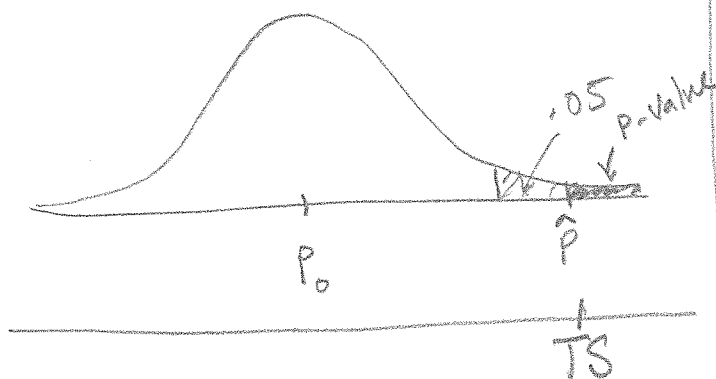
Reject H_0 $P = P_0$ or

fail to Reject H_0

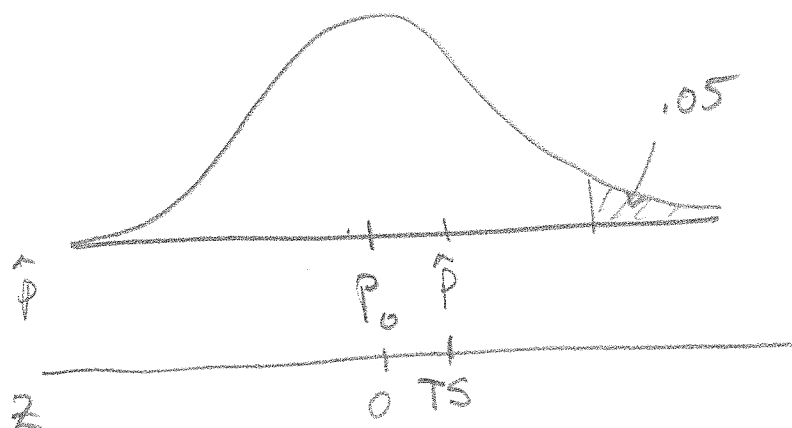
P-value is small

$$P\text{-value} < \alpha = .05$$

$$P\text{-value} > \alpha$$



$$TS > Z \rightarrow \text{Critical Value}$$



Ex A company claims that the defect rate for their batteries is 4%.

① $H_0: p = .04 = p_0 \leftarrow \text{value in } H_0$

$H_1: p \neq .04$ Alternate hypothesis

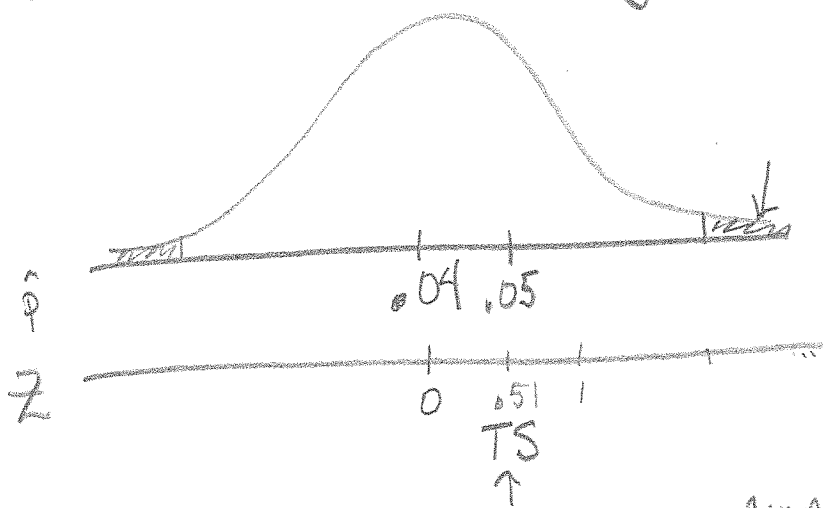
③ Take some sample data. In a shipment of 100 Batteries $\hat{p} = .05$ defect rate $\hat{p} = \frac{x}{n} = \frac{5}{100}$

⑥ Draw Sampling Distribution for every HW

⑤
TS: $Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} \quad q_0 = 1 - p_0$

$$Z = \frac{(.05 - .04)}{\sqrt{\frac{.04 \cdot .96}{100}}}$$

$$Z = 0.51$$



↑
Z-score of my sample

$\hat{p} = .05$ on dist.

of sample proportions

Mean of

$$\mu_{\hat{p}} = .04$$

$$\sigma_{\hat{p}} = \sqrt{\frac{.04 \cdot .96}{100}}$$

CLT for prop.

③ SKIP to conclusion
fail to Reject H_0

⑨ We can't prove beyond a doubt that the Overall proportion of Defects is different from 4%

b) Same claim: $p = .04$

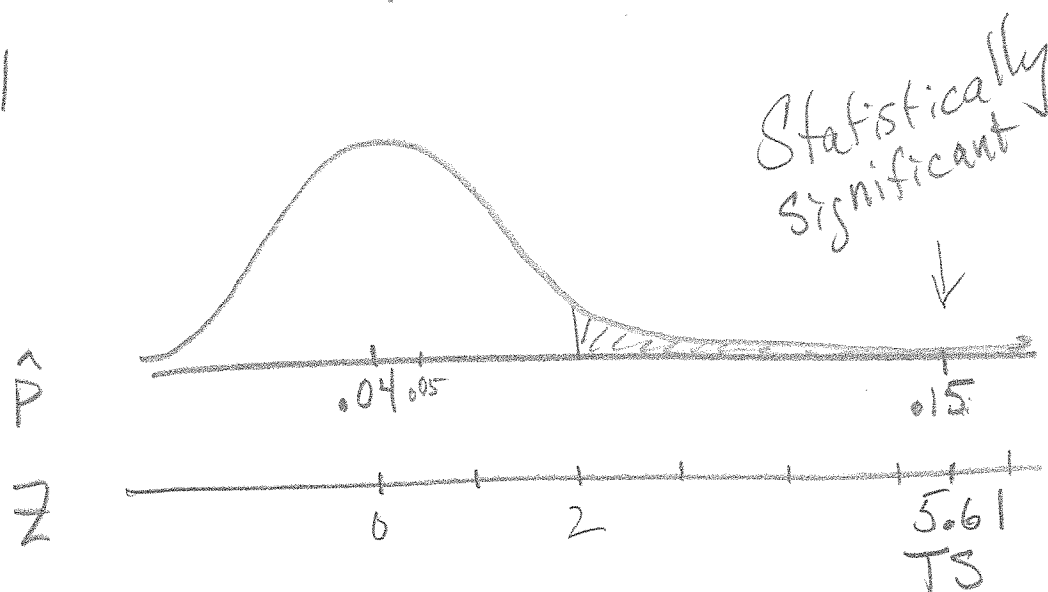
② $H_0: p = .04$
 $H_1: p \neq .04$

③ Take data $\hat{p} = \frac{15}{100} = .15$

⑤ TS: $Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.15 - .04}{\sqrt{\frac{.04 \cdot .96}{100}}}$

$Z = 5.61$

⑥ Graph



⑧ Conclusion

If Reject H_0 then ~~reject~~ Support H_1

⑨ Reject that proportion of defects is 4% and

Support that the Defect rate is Not 4%.
Looks higher

Step 2 Writing H_0 and H_1

Rule 1 $H_0: p = p_0$ or $\mu = \mu_0$ is Always Equal

Rule 2 $H_1: p < p_0$ $\mu > \mu_0$ or $p \neq p_0$
is Always a strict inequality

Rule 3 If claim is $<, >, \neq$ then
Case 1 the claim is the Alternate Hypothesis

Ex Claim: The defect rate is greater than 4%

Translate to math $p > .04$

Contains $> \Rightarrow H_1: p > .04$

Case 2 If claim is $\leq, \geq, =$ then,

the Alternate Hypothesis
has opposite inequality $>, <, \neq$

Ex Claim: Company says the Defect rate is at most 4%

Claim: $p \leq .04$

then $H_1: p > .04$

Step 9 Write final Conclusion to Address initial Claim.

If Claim: $p > .04$ $\leftarrow$ has strict inequality

Conclusion $\rightarrow$ Support H_1

Reject H_0 : Support that prop. of defects is greater than 4%
 $\hat{p} = .15$

fail to Reject H_0

fail to Support H_1 : There is not enough data to Support that the proportion of defects is above 4%
 $\hat{p} = .05$

If Claim: $p \leq .04$ $\left\{ \begin{array}{l} H_0: p = .04 \\ H_1: p > .04 \end{array} \right.$

Reject H_0

Reject that proportion of Defects is at most 4%

$\hat{p} = .15$

fail to Reject H_0

fail to Reject that the Proportion of defects is at Most 4%

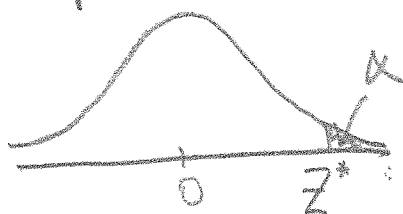
Step ③ Critical Values

CV: Z_α or $Z_{\alpha/2}$ is the Z-Score that separates statistically significant Test Statistics. α = Significance level of the Test = Given or .05

Three Cases

Right Tailed

$$H_1: p > p_0$$

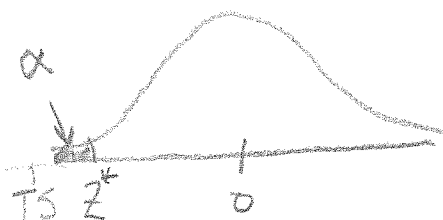


Proportions

$$CV: Z^* = \text{invnorm}(1-\alpha, 0, 1)$$

Left tailed

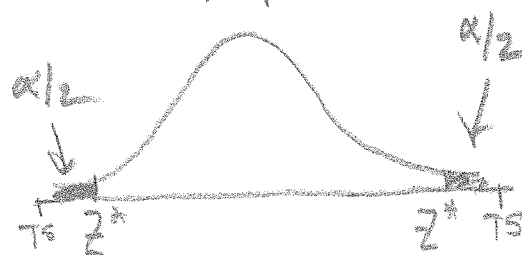
$$H_1: \mu < \mu_0$$



$$Z^* = \text{invnorm}(\alpha, 0, 1)$$

Two Tailed

$$H_1: p \neq p_0$$



$$Z^* = \pm \text{invnorm}(1-\frac{\alpha}{2}, 0, 1)$$

Same as CI

Means

$$t^* = \text{inv}t(1-\alpha, df)$$

$$t^* = \text{inv}t(\alpha, df)$$

$$t^* = \pm \text{inv}t(1-\frac{\alpha}{2}, df)$$

Reject H_0

$$TS < Z^*$$

Reject H_0

$$TS < -Z^* \text{ or } TS > Z^*$$

$$TS: Z > Z^* CV$$

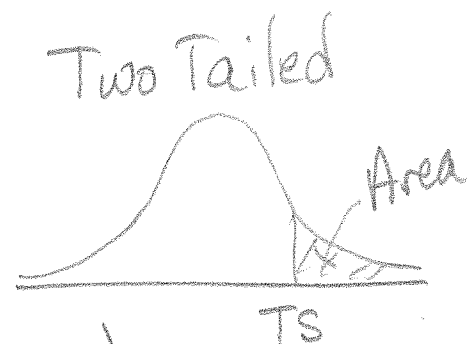
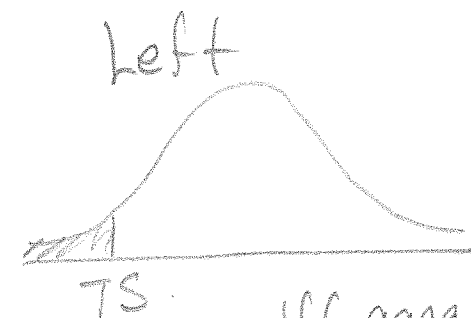
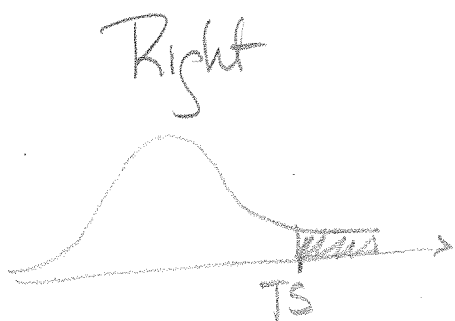
then

$\hat{p}$ is statistically significant

Reject H_0

Assuming H_0 is True

⑦ P-value = probability of seeing a Sample Statistic as or more extreme than the point estimate.
= Area



$$P\text{-value} = \text{ncdf}(-9999, TS, 0, 1)$$

Proportions

$$P\text{-value} = \text{normalcdf}(TS, 9999, 0, 1)$$

$$P\text{-value} = 2 \cdot \text{Area}$$

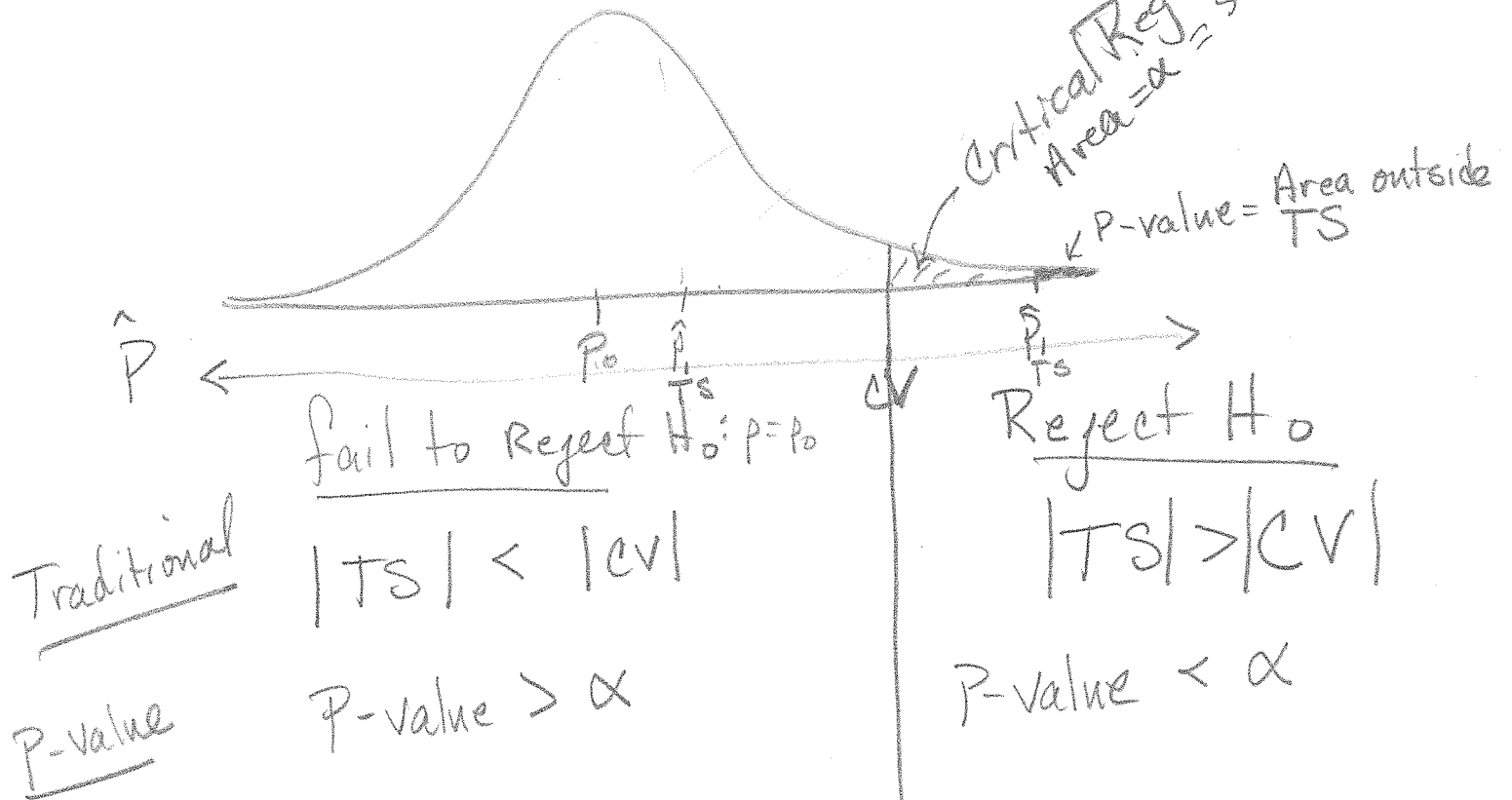
area < .5

Means

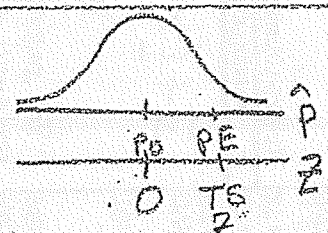
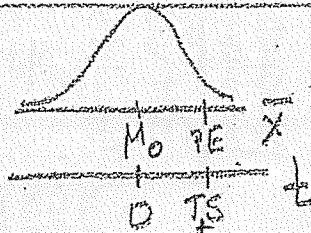
$$= \text{tcdf}(TS, 999, df) = \text{tcdf}(-999, TS, df)$$

$$2 \cdot \text{Area}$$

Final Conclusions



Steps For a hypothesis Test

	Proportions	Means
① Check Requirements SRS	$np > 5$ and $nq > 5$	$n > 30$ or Population is Normal
② Write Null and Alternate Hypotheses Claim: $<, >, \neq \Rightarrow H_1$ is claim Claim: $\leq, \geq, = \Rightarrow H_0$ is opposite	$H_0: p = p_0$ $H_1: p <, >, \neq, p_0$	$H_0: \mu = \mu_0$ $H_1: \mu <, >, \neq \mu_0$
③ Point Estimate and the Summary Statistics	$\hat{p} = \frac{x}{n}$	$\bar{x}$ from 1-var stat, n or given
④ Find Critical Values	$CV: Z^* = \text{invnorm}(\text{Area})$	$CV: t^* = \text{invT}(\text{Area}, df)$
⑤ Test Statistic is Score of point estimate	$TS: Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$ use p in H_0	$TS: t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$ use μ in H_0 and s from 1-var stat
⑥ Draw Sampling Distribution Vertically line up PE and TS		
⑦ P-value is the probability of being more extreme than PE	P-value $Rt = \text{normalcdf}(TS, 999)$ $Lt = \text{normalcdf}(-999, TS)$ Two Tail = 2 * smaller of Lt or Rt	P-value $Rt = \text{tcdf}(TS, 999, df)$ $Lt = \text{tcdf}(-999, TS, df)$ Two Tail = 2 * smaller of Lt or Rt
⑧ Make Initial Conclusion Reject H_0 or fail to Reject H_0	$H_0: p = p_0$	$H_0: \mu = \mu_0$
⑨ Write Final Conclusion describing what the population Parameter is in Words.		

Project collect Data

4 Samples Everyone collect

30 from each population

Two Questions

M/F	Yes/No	Number
	$\hat{p}_1 = \frac{x_1}{n_1}$	$\bar{x}_1 = \text{Mean \# of hrs}$ Summarize
	$\hat{p}_2 = \frac{x_2}{n_2}$	$n_1 = \# \text{ Men that play}$
	$n_1 = \# \text{ of Men Asked}$	Population
Population	All SRJC Men	= SRJC Men who play video games
		$\bar{x}_2 = \text{Mean hrs Women video gamers play}$