§3.3 Measure of Position

$$Z\text{-Score} = \frac{x - \mu}{\sigma} = \frac{x - \bar{x}}{s}$$

The z-score is the Number of standard deviations x is from the mean.

x - Individual score → given Data Value

μ = Mean of whole population

σ = pop. Standard deviation

Parameters

$\bar{x}$ = Sample Mean

s = Sample Standard deviation

Statistics

Ex If $\mu = 69$ inches and $\sigma = 2.8$ inches for the heights of men then Eddie is 74 inches tall. $x = 74$

So $$Z = \frac{x - \mu}{\sigma} = \frac{74 - 69}{2.8} = 1.79 = \text{Eddies Z-Score}$$

Round z to Two decimals

Women $\mu = 63.6$ $\sigma = 2.5$

§3.3 Measures of Position

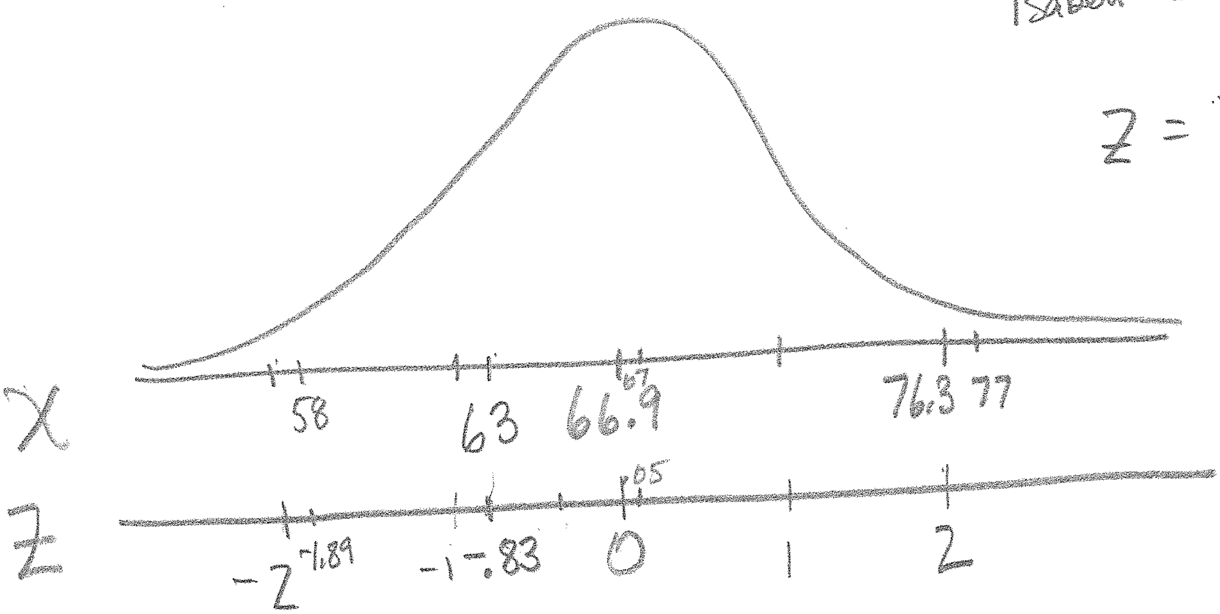
$$Z = \frac{x - \mu}{\sigma} = \frac{x - \bar{x}}{s} =$$ # of standard deviations a data value x is Above or Below the Mean

Ex Tyler $= 77 = x$, $\bar{x} = 66.9$ $s = 4.7$

$$Z = \frac{x - \bar{x}}{s} = \frac{(77 - 66.9)}{4.7} = 2.15$$

isabell $= Z = \dfrac{63 - 66.9}{4.7}$

$$Z = \frac{-3.9}{4.7} = -.83$$



$$Z_{67} = \frac{x - \bar{x}}{s} = \frac{67 - 66.9}{4.7} = \frac{.1}{4.7} = .05$$

§3.3 Measures of Position

Z-Score = The Number of standard deviations a data Value is above or below the mean

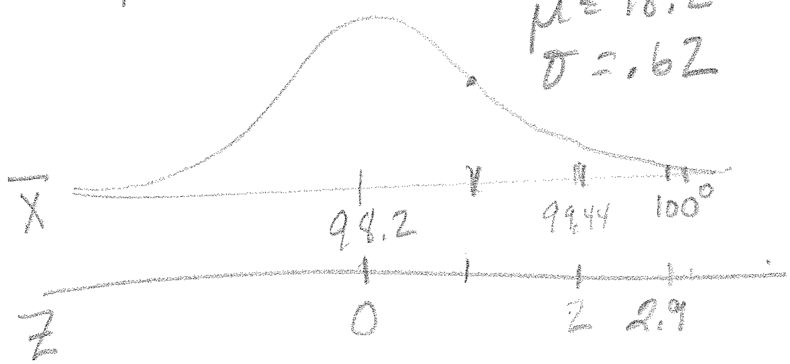
$$Z = \frac{x - \bar{x}}{S} \qquad Z = \frac{x - \mu}{\sigma}$$

Ex what is Z-score of a temp of $x = 100°$. $\mu = 98.2$, $\sigma = .62$

$$Z = \frac{x - \mu}{\sigma} = \frac{(100 - 98.2)}{.62}$$

$$Z = \frac{1.8}{.62} = 2.90$$

↑ Round to 2-decimals



Class Try

Men $\bar{x} = 69$ $S = 2.8$

Women $\bar{x} = 64$ $S = 2.5$ inches

$X =$

Find your Z-Score

$$Z = \frac{x - \bar{x}}{S} =$$

Ex Compare Test Scores

a) On a Math test the mean was 80 and the standard deviation was 7. You scored a 67. Find your Z-score.

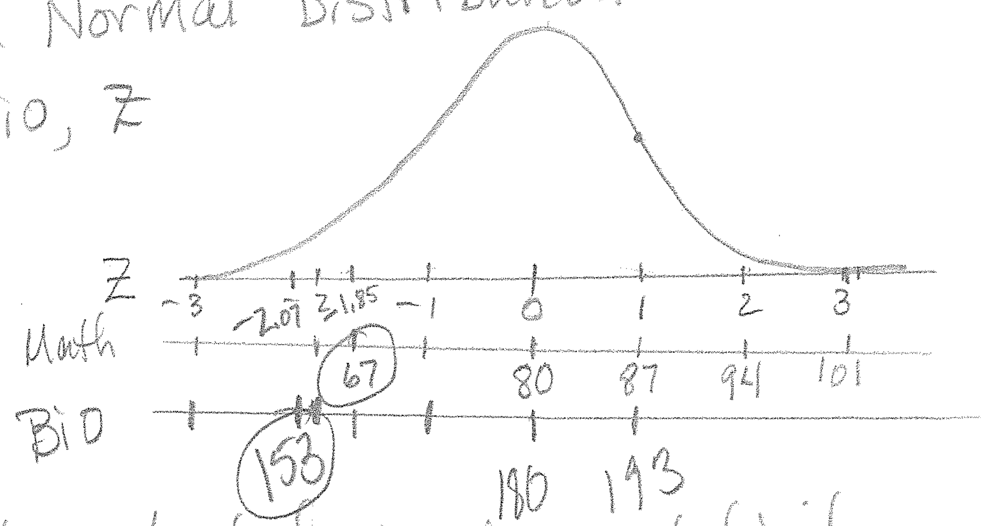
Math test $\bar{X}_1 = 80$, $S = 7$, $X = 67$ $\quad Z = \frac{X - \bar{X}}{S} = \frac{67 - 80}{7} = \frac{-13}{7} =$

$Z_M = -1.85$

b) On Biology Test, $X = 153$, $\bar{X} = 180$, $S = 13$

$$Z_B = \frac{X - \bar{X}}{S} = \frac{153 - 180}{13} = -2.07$$

c) Graph a Normal Distribution with 3 Axes
Math, Bio, Z



d) On which test did you do relatively better?

Math Z is $-1.85 >$ Bio Z at -2.07

$$Z_M = -1.85 > Z_B = -2.07$$

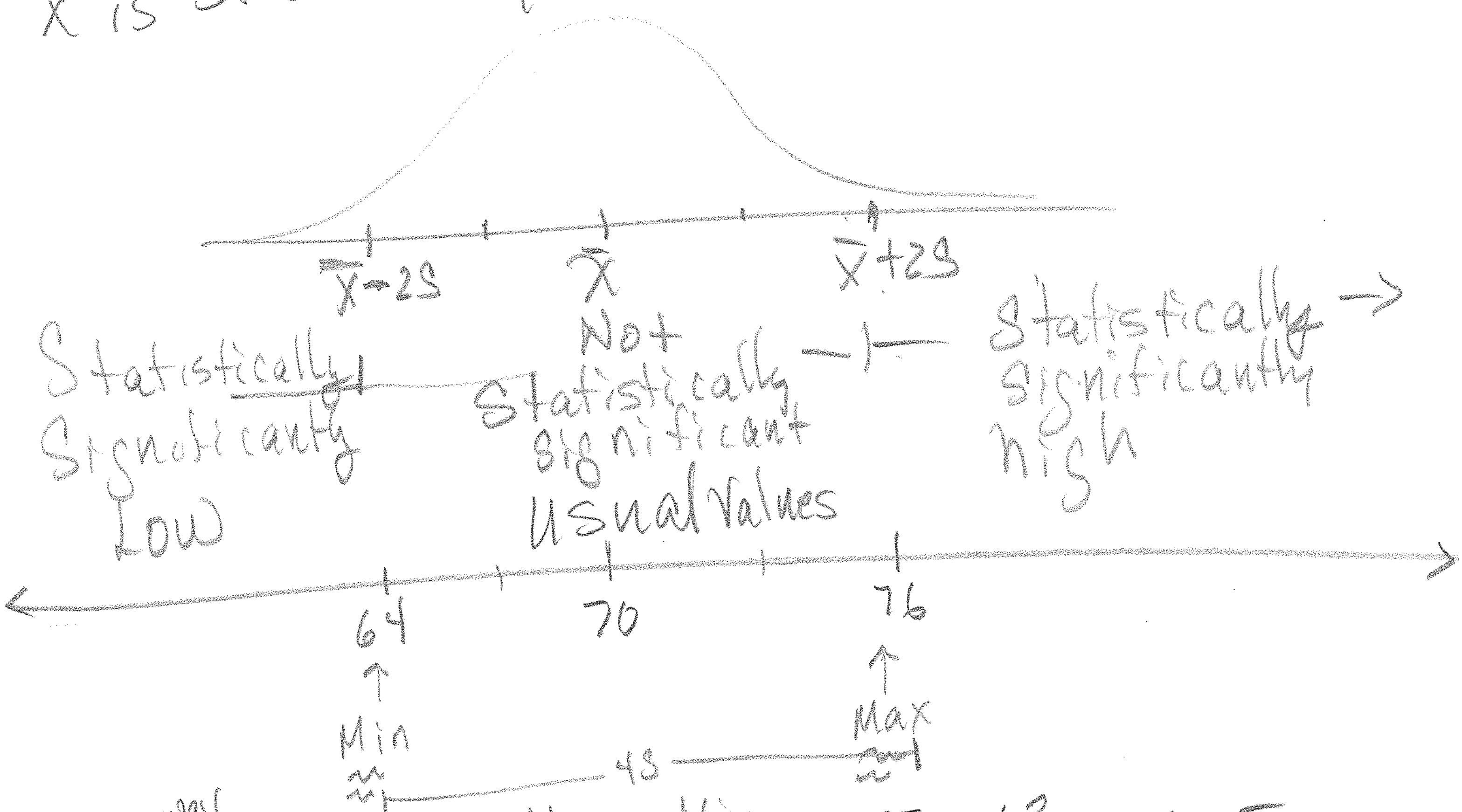
$\boxed{\text{Did Better on Math Test}}$

Range Rule of Thumb

Usual Data or Statistically insignificant Data lies between $\bar{x} - 2s$ and $\bar{x} + 2s$

X is Statistically high $X \geq \bar{x} + 2s$

x is Statisically low $x \leq \bar{x} - 2s$



Easy way To get an estimate of standard Deviation

$$S \approx \frac{Max - Min}{4} = \frac{72 - 63}{4} = 2.25$$

Ex Human body Temperatures

have a mean $\mu = 98.2^\circ F$ and $\sigma = .62^\circ$

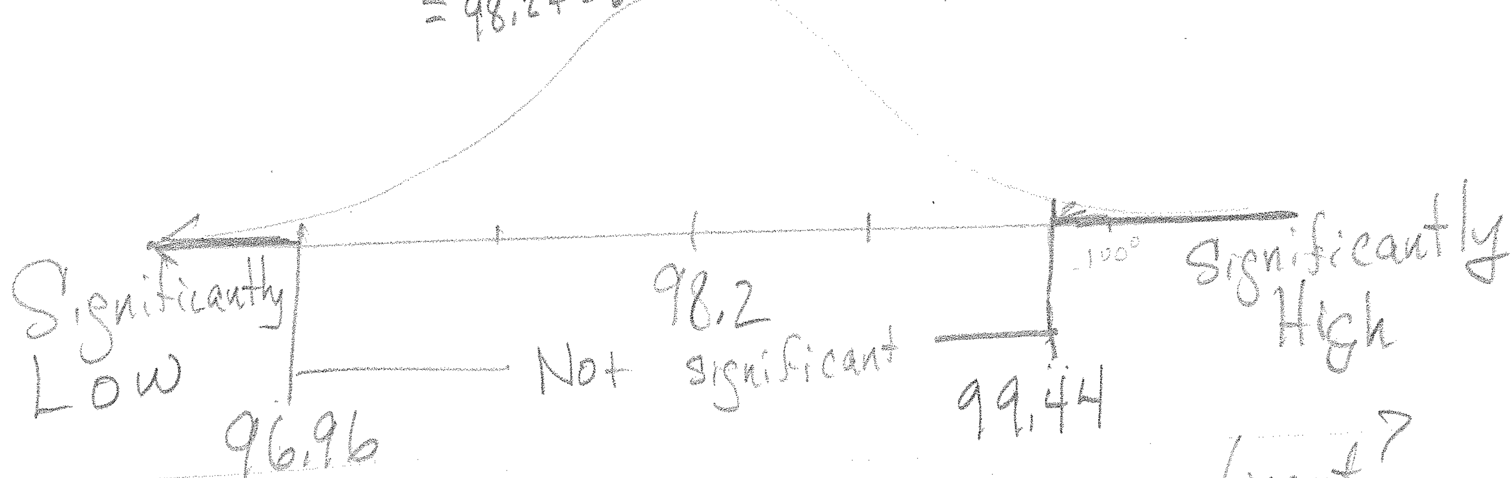
a) Calculate

Min Usual $= \mu - 2\sigma$ = value so all lower values are significantly low

$\bar{x} - 2s$ $= 98.2 - 2(.62) = 96.96$

Max usual $= \mu + 2\sigma$ = larger values are significantly high

$\bar{x} + 2s$ $= 98.2 + 2 \cdot .62 = 99.44$



b) Is a temperature of 100° significant?

yes $100^\circ >$ $\boxed{99.44 = \text{max usual}}$

So 100° is significantly high

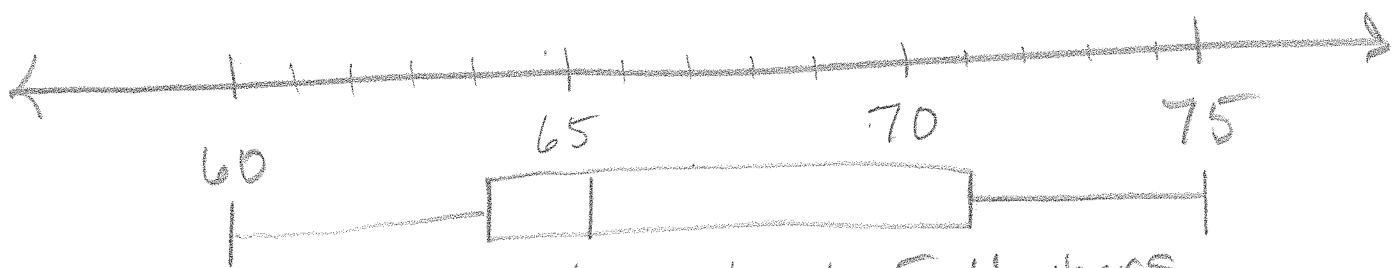
Box-Plots

Plot of the 5-Number Summary

1-VarStat → Min, Q_1, Med, Q_3, Max

Test Scores Min=60 Q_1=64, Med=65.5, Q_3=71, Max=75

① Draw Numberline, Start ≤ Min Stop ≥ Max



② Put Tick marks at each of 5 Numbers

③ Box Middle 3 - Wiskers to Max & Min

Box Plots

Bank waiting lines
Jefferson Vally Bank (JV) uses one line
Bank of Providence uses 5 lines

7.2

JV	6.5	6.6	6.7	6.8	7.1	7.3	7.4	7.7	7.7	7.7
BP	4.2	5.4	5.8	6.2	6.7	7.7	7.7	8.5	9.3	10.0

Summary Statistics
1-Var Stats

	JV	BP
$\bar{x}$	7.15	7.15
$s = S_x$	.47668	1.82
Min	6.5	4.2
Q_1	6.7	5.8
Med	7.2	7.2
Q_3	7.7	8.5
Max	7.7	10.0
Mode	7.7	7.7
Midrange	7.1	7.1
range max-min	1.2	5.8
Variance $= S^2$	.227	3.318

$$\text{Midrange} = \frac{\max + \min}{2} = \frac{6.5 + 7.7}{2} = 7.1$$

$$\text{range} = \text{Max} - \text{Min} = 7.7 - 6.5 = 1.2$$

$$\text{Variance} = S^2 = S_x^2$$

VARS
5: Statistics
3: S_x

Middle is the same for Both Wait Times

Spread is much larger at BP

JV has happier customers

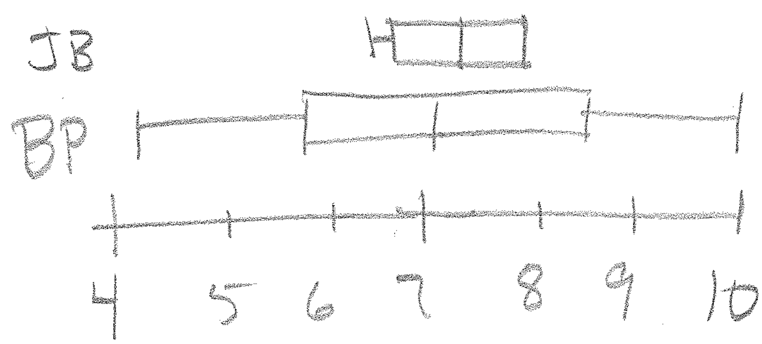
Compare Jefferson Bank to the Bank of Providence. Wait time

JV	6.5	6.6	6.7	6.8	7.1	7.3	7.4	7.7	7.7	7.7
BP	4.2	5.4	5.8	6.2	6.7	7.7	7.7	8.5	9.3	10

a) Report Summary Statistics

	JV	BP
$\bar{x}$	7.15	7.15
S_x	.47	1.82
Min	6.5	4.2
Q_1	6.7	5.8
Med	7.2	7.2
Q_3	7.7	8.5
Max	7.7	10

Draw Side-By-Side Box plot



b)

Ex a) Would a 9 minute Wait be Unusual at the Bank of Providence? (Statistically long) High

Method 1

Max usual $= \bar{x} + 2s = 7.15 + 2(1.82) = 10.79$

Not Statistically high.

Method 2 $z = \frac{x - \bar{x}}{s} = \frac{9 - 7.15}{1.82} = 1.02 < 2$

So $x = 9$ is No statistically High.

b) Would a 9 minute wait be unusually high at Jefferson Bank.

$\bar{x} = 7.15 \quad s = .47$

Max usual $= 7.15 + 2(.47) = 8.09 < 9$

$z = \frac{x - \bar{x}}{s} = \frac{9 - 7.15}{.47} = 4.00 > 2$

$\Rightarrow$ 9 is unusually or Statisticly long Wait time.

Find Measures of Center	JV	BP
$\bar{X}$ =	7.15	7.15
Med. =	7.2	7.2
Midrange = $\frac{Max + Min}{2}$	7.1	7.1
Mode =	7.7	7.7

JV Midrange = $\frac{7.7 + 6.5}{2}$

The center of both dist. is very close to the Same. Average wait is the Same.

Measures of Spread

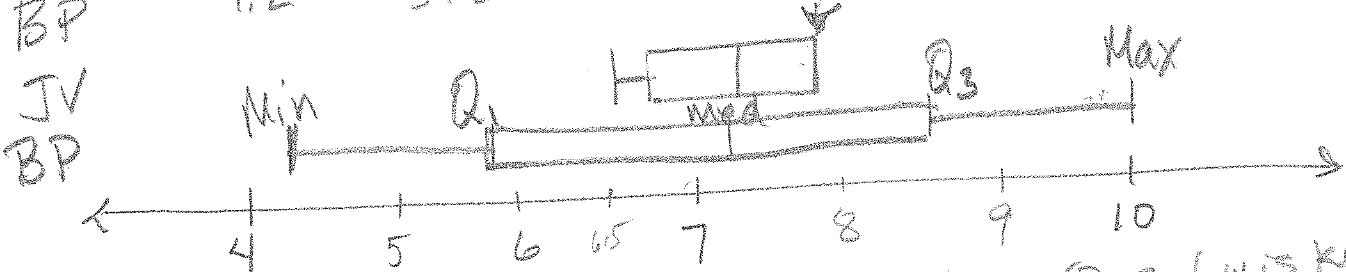
	JV	BP
S	.47	1.82
Var. = S^2	.23	3.318
range = Max − Min	1.2	5.8

the Spread of the Data Value is Much larger at BP than at JV

Varience of BP is larger than the variation at JV

A Box Plot is graph of the 5-Number Summary.

	Min	Q_1	Med	Q_3	Max
JV	6.5	6.7	7.2	7.7	7.7
BP	4.2	5.8	7.2	8.5	10.0



① Put I Above each # in 5 Number Summary
② Put a Box Around Middle 3 Q_1, Med, Q_3
③ Put wiskers out Min and Max

#15 §3.3

$\bar{x} = 6.5$ $\boxed{s = 3.5}$ range $= 15 - 4 = 11$ Variance $= (3.5)^2 = 12.2$

$\boxed{\text{Maximum Usual} = \bar{x} + 2s} = 6.5 + 2(3.5) = 13.5$

Not unusual to take 12 years because 12 is between the Min & Max usual

Min usual $= 6.5 - 2(3.5) = -.5$ or 0

There is one unusual value $x = 15$

On Stat crunch use $\boxed{\text{Shift}}$ to Select both data sets for one Side by Side Box plot of both data sets

§3.2 & 3.3

$$\text{Coefficient of Variation} = \frac{S}{\bar{X}} \cdot 100\% \text{ or } \frac{\sigma}{\mu} \cdot 100\%$$

The Standard deviation relative to the Mean

#24 Single Line

$\bar{X} = 4.29$

$\boxed{S = 28.6}$

$n = 10$

$\boxed{Var = S^2 = 818}$

Range = Max − Min
= 462 − 390

$\boxed{\text{Range} = 72}$

$$\text{Coeff of Variation} = \frac{S}{\bar{X}} = \frac{28.6}{429}$$

$= .0667$

$= \boxed{6.67\%}$

Individual Lines

$\bar{X} = 429$

$S = 109.3$

Var =

$$\text{Coff of variation} = \frac{109.3}{429} = .2548$$

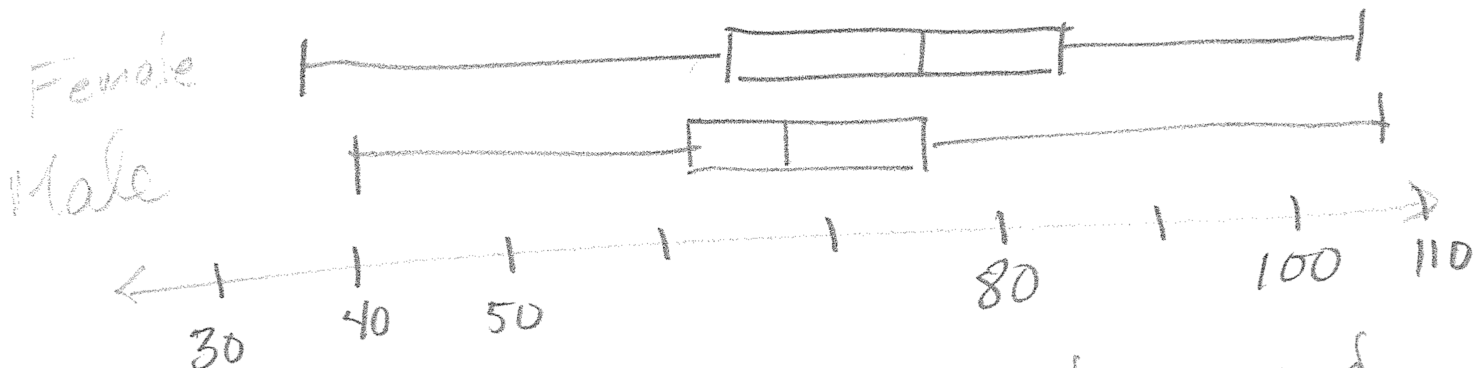
$= \boxed{25.48\%}$

The variation (Spread) in service times for Individual lines is much greater that when a single line is used.

Happier Customers when a Single line is used.

#33

Pulse Rates	Min	Q_1	Med	Q_3	Max
Female	36	64	74	82	104
Male	40	62	68	76	107



Females have have higher center and a slightly large spread than males.