Practice Test 3 Math 215 Jones Name Solutions

You may work with classmates and get help at the Math Lab on this quiz.

Use the given degree of confidence and sample data to construct a confidence interval for the population proportion p. 90%

1) Of 367 randomly selected medical students, 30 said that they planned to work in a rural community. Find a ~~95%~~ 90% confidence interval for the true proportion of all medical students who plan to work in a rural community.

d) (2 Points) What is the critical value needed to calculate a 90% confidnece interval?

$CL = .90$
$\alpha = .10$

$CV: Z_{\alpha/2} = \text{invnorm}(1 - .10/2, 0, 1) = \boxed{1.645 = Z_{\alpha/2}}$

e) (2 Points) What is the point estimate for the population proportion? $\hat{p} = \frac{30}{367} = .0817$

f) (2 Points) Show the formula and the values used to calculate the margin of error

$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.645\sqrt{\frac{.0817 \cdot .9183}{367}} = \boxed{.0235 = E}$

E=

g) (2 Points) Find a 90 percent confidence interval for the proportion of doctors who plan to work in rural communities..

$\hat{p} - E < p < \hat{p} + E$ $\quad .0817 - .0235 < p < .0817 + .0235$

$.0582 < p < .1052$

h) (4 Points) State the meaning of this confidence interval.

We are 90% Confident that the proportion of doctors who plan to work in Rural communities is between .0582 and .1052 or 5.82% and 10.52%.

Provide an appropriate response.

2) Apply the Central Limit Theorem. Samples of size n = 800 are randomly selected from the population of numbers (~~0~~ through 9) produced by a random-number generator. digits that are multiples of 3

a) If the proportion of ~~odd numbers~~ is found for each sample what type of distribution is the distribution of the sample proportions? What is it's mean and what is it's standard deviation?

$S = \{ ,1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $\quad E = \{3, 6, 9\}$ $\quad p = \frac{3}{9} = \frac{1}{3}$

Normal, $\mu_{\hat{p}} = \frac{1}{3}$, $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{1/3 \cdot 2/3}{800}} = .0167$

b) If the mean of the 800 values is found for each of the samples what type of distribution is the distribution of sample mean? What is the mean and what is the standard deviation of the distribution of sample means? (please use correct notation.)

Normal, $\mu_{\bar{x}} = \mu = \frac{1+2+\cdots+9}{9} = 5$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} =$

1-Varstat $L_1 = \{1, 2, \ldots, 9\}$ gives $\sigma = 2.582$

So $\sigma_{\bar{x}} = \frac{2.582}{\sqrt{800}} = .0913$

Solve the problem.

3) A newspaper article about the results of a poll states: "In theory, the results of such a poll, in 99 cases out of 100 should differ by no more than 5 percentage points in either direction from what would have been obtained by interviewing all voters in the United States." Find the sample size suggested by this statement.

$CL = .99$ $\quad E = .05$
$\alpha = .01$

$n = \frac{Z_{\alpha/2}^2 \cdot .25}{E^2} = \frac{2.576^2 \cdot .25}{.05^2}$

$Z_{\alpha/2} = \text{invnorm}(1 - .01/2, 0, 1)$
$Z_{\alpha/2} = 2.576$

$\boxed{n = 663.6}$ $\quad \boxed{n = 664}$

Provide an appropriate response.

3) ~~4~~ Tell whether the following statistic is a biased or unbiased estimator of a population parameter:
Sample proportion used to estimate a population proportion. Unbiased

Use the given data to find the minimum sample size required to estimate the population proportion.

4) ~~5~~) Margin of error: 0.044; confidence level: 95%; $\hat{p}$ and $\hat{q}$ unknown $n = \frac{1.96^2 \cdot .25}{(.044)^2} = 496.07$ $\boxed{497}$

5) ~~6~~) Margin of error: 0.005; confidence level: 99%; from a prior study, $\hat{p}$ is estimated by 0.166.

$n = 2.576^2 \cdot .166 \cdot .834 / (.005)^2 = 36{,}747.3$ $\boxed{36748}$

6) ~~7~~) (5 points) Margin of error: 0.008; confidence level: 99%; from a prior study, $\hat{p}$ is estimated by 0.139.

$n = 2.576^2 \cdot .139 \cdot .861 / (.008)^2 = 12408.8$ $\boxed{12{,}409}$

b) Does the size of the population effect the size of the sample needed to make this confidence interval?

No, As long as the population size is much larger than the Sample Size

8) a) (2 Points) Define confidence interval. An interval of values that is likely with a confidence to contain the population parameter.

b) (2 Points) Define margin of error. the maximum likely difference between a sample statistic like $\hat{p}$ or $\bar{x}$ and a population parameter like p or μ.

b) (2 Points) Suppose a confidence interval is 0.12< p < 0.20. Find the sample proportion $\hat{p}$ and the error estimate E.

$E = \frac{UB - LB}{2} = \frac{.2 - .12}{2} = \frac{.08}{2} = .04$ $\hat{p} = \frac{.2 + .12}{2} = .16$

Use the given degree of confidence and sample data to construct a confidence interval for the population mean [].

9) A laboratory tested 80 chicken eggs and found that the mean amount of cholesterol was 213 milligrams with s = 12.8 milligrams. Construct a 95 percent confidence interval for the true mean cholesterol content, μ, of all such eggs.

$n = 80$
$\bar{x} = 213$
$s = 12.8$
$CL = .95$
$\alpha = .05$
$t_{\alpha/2} = \text{invT}(1 - .05/2, 79) = 1.990$

$E = t_{\alpha/2} \cdot 12.8/\sqrt{80} = 2.848$ use $\boxed{2.8 = E}$

$\bar{x} - E < \mu < \bar{x} + E$

$213 - 2.8 < \mu < 213 + 2.8$

$\boxed{210.2 < \mu < 215.8}$

We are 95% Confident that the True Mean Cholesterol Content of eggs is between 210.2 and 215.8mg.

Provide an appropriate response.

10) What assumption about the parent population is needed to use the t distribution to compute the margin of error when n<30 ?

the parent population must be Approximately Normal.

Solve the problem.

11) The sample data below consists of the heights of 30 randomly selected adults. You wish to use the data to obtain a confidence interval estimate of the population mean.

a) Does the data set include any outliers? Yes 682 is an outlier

b) How could you handle the outlier in this case? Explain your answer.

Report Results with and without the Outlier.
Check to see if it is a typo. Probably should be 68.2.

d) Calculate the confidence interval with and without the outlier. T Interval

With Outlier (45.4, 129.36)

Without (65.143, 68.63)

e) Are confidence interval limits sensitive to outliers? Yes The interval without the outlier has much lower center and Narrower.

60.1	66.9	70.4	73.2	65.2	64.1
68.5	69.2	64.0	62.4	66.9	71.2
682	61.4	65.7	72.5	74.0	70.0
65.8	69.3	60.4	72.4	58.1	68.3
60.5	66.4	60.5	71.3	67.8	73.2

~~f) Find the confidence interval for the standard deviation of the heights of men.~~

Use the degree of confidence and sample data to construct a confidence interval for the population proportion p.

12) When 306 college students are randomly selected and surveyed, it is found that 115 own a car. Find the point estimate for the proportion of college students who own a car, and find a 99% confidence interval for the true proportion of all college students who own a car.

What is the point estimate of the population proportion? $\hat{p} = \frac{x}{n} = \frac{115}{306} = \boxed{.376}$

What is the critical value? $Z_{.005} = 2.576$

What is the margin of error? E=________

Explain the meaning of the confidence interval.

$n = 306$

$x = 115$

$CL = .99$

$\alpha = .01$

$Z_{\alpha/2} = Z_{.005} = \text{invnorm}(1 - .01/2, 0, 1) = 2.576$

$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.576 \cdot \sqrt{\frac{.376 \cdot .624}{306}}$

$E = .0713$

$\hat{p} - E < p < \hat{p} + E$

$.376 - .071 < p < .376 + .071$

$\boxed{.305 < P < .447}$

We are 99% confident that the proportion of all college students who own a car is between .305 and .447.

Find the minimum sample size you should use to assure that your estimate, p, will be within the required margin of error around the population p. How large should their Sample be?

13) A political action committee is interested in finding out what kind of popular support they might expect on an environmental initiative. Similar issues have gotten 91% support. The committee will set up a polling program to assure 95% confidence that the margin of error is less than 0.07.

$\hat{p} = .91 \quad CL = .95 \Rightarrow \alpha = .05 \Rightarrow Z_{\alpha/2} = Z_{.025} = \text{invnorm}(1 - .025, 0, 1) = 1.96$

$E = .07$ Find $n = \left(\frac{Z}{E}\right)^2 \cdot \hat{p}\hat{q} = \left(\frac{1.96}{.07}\right)^2 \cdot .91 \cdot .09 = 64.2$

The committee should Sample at least $\boxed{\text{65 people}}$

6.4

14) (21 Points) A machine in a saw mill cuts pieces of lumber to an average width of $\mu = 4$ inches with a standard deviation $\sigma = .05$ inches

a) (3 Points) Graph the distribution with both an x–axes and a z–axes. Show mean and standard deviation. Calculate the z–score of a 4.08 width for a piece of lumber and label on your graph.

b) (3 Points) What is the probability that width is at least 4.08 for a piece of lumber? Show all work. Use proper probability notation, calculator inputs and shade region with equal area on the graph above.

c) (3 Points) What width separates the widest 10% of cuts? Show on a new graph.

d) (3 Points) On a given day the inspector samples 16 boards, and finds the sample mean. Find the mean $\mu_{\bar{x}}$ and standard deviation $\sigma_{\bar{x}}$ of the population of sample means for samples of size n = 16.

e) (3 Points) Find the z–score of a sample mean that is at least $\bar{x}$ =4.08 inches in the distribution of sample means.

f) (6 Points) For a sample of size 16, what is the probability that the **mean** at least $\bar{x}$ =4.08 inches in the distribution of sample means? Graph the distribution of sample means when the sample size is 16 with both an $\bar{x}$–axes and a z–axes. Does the data indicate that the machine is working properly.

Use the given degree of confidence and sample data to construct a confidence interval for the population mean μ. Assume that the population has a normal distribution.

15) The principal randomly selected six students to take an aptitude test. Their scores were:

76.5 85.2 77.9 83.6 71.9 88.6 $= L_1$ Stat Tests TInterval

Determine a 90% confidence interval for the mean score for all students.

a)) What point estimate of the population mean does this sample give? $\boxed{80.6 = \bar{x}}$ $S_x = 6.23$

b) What is the margin of error? (Show work. Include critical value.)

$E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}} =$

d) Find the confidence interval.

Tinterval = (75.493, 85.74) $75.5 < \mu < 85.7$

e) Interpret the meaning of this confidence interval. Is the principal reasonable confident that the average of his students' scores is higher than the national average if the national average for the aptitude test is 70.

We are 90% confident that the true mean of all Students at this school is between 75.5 and 85.7.
The Pricipal can be reasonably confident that his students have a mean score above $\mu = 70$

Use the given degree of confidence and sample data to find a confidence interval for the population standard deviation σ. Assume that the population has a normal distribution.

16) The football coach randomly selected ten players and timed how long each player took to perform a certain drill. The times (in minutes) were:

7 10 14 15 15

5 12 15 11 11

What is the point estimate for the population ~~standard deviation~~ mean? mean, μ

Find a 95 percent confidence interval for the population ~~standard deviation σ~~.

Identify the null hypothesis, alternative hypothesis. Find and graph the point estimate for the population Proportion and test statistic,. Find the P-value. State your conclusion about the null hypothesis, and final conclusion that addresses the original claim. Another 1-Prop Z Test

17) According to a ~~recent~~ poll 53% of Santa Rosans would vote for the incumbent president. However a random sample of 100 people results in 45% who would vote for the incumbent, test the claim that the actual percentage is 53%. Use a 0.10 significance level.

5

$p = .53$ $n = 100$ $\hat{p} = .45$

$\alpha = .10$

(3 Points) State claim, null and alternate hypothesis.

(3 Points) Find the critical value and graph and shade the critical region.

(3 Points) Find the point estimate of the population proportion and it's test statistic.

(3 points) Label these values on your graph.

(5 Points) Clearly state your initial conclusion and your final conclusion so that it is understandable without knowing statistics.

(5 Points) Find and explain the meaning of the P-value. Shade a graph showing the area equal to the p-value.

a) Claim: $p = .53$

$H_0: p = .53$

$H_1: p \neq .53$

b) $\alpha = .10$

Two Tails

$z_{\alpha/2} = \text{invnorm}(1 - \frac{\alpha}{2}, 0, 1)$

$z_{\alpha/2} = \pm 1.645$

c) PE: $\hat{p} = .45$

TS: $z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{(.45 - .53)}{\sqrt{\frac{.53 \cdot .47}{100}}} = -1.60$

18) A poll of 1,068 adult Americans reveals that 513 of the voters surveyed prefer the Democratic candidate for the presidency. At the 0.05 level of significance, test the claim that at least half of all voters prefer the Democrat.

(3 Points) State claim and the null and alternate hypothesis. (3 Points) Graph and shade the critical region.

(3 Points) Find the critical value, point estimate of the population proportion and it's test statistic. (3 points) Label these values on your graph.

(5 Points) Clearly state your initial conclusion and your final conclusion so that it is understandable without knowing statistics.

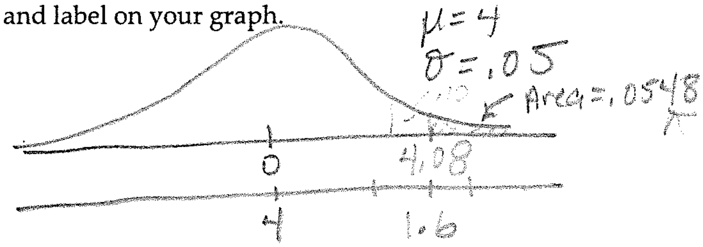
14) (21 Points) A machine in a saw mill cuts pieces of lumber to an average width of $\mu = 4$ inches with a standard deviation $\sigma = .05$ inches

a) (3 Points) Graph the distribution with both an x-axes and a z-axes. Show mean and standard deviation. Calculate the z-score of a 4.08 width for a piece of lumber and label on your graph.

6.4

$z = \frac{x-\mu}{\sigma} = \frac{4.08-4}{.05} = \frac{.08}{.05} = 1.6$

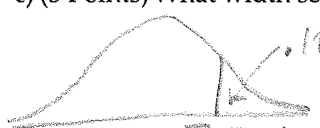
$\mu = 4$
$\sigma = .05$



b) (3 Points) What is the probability that width is at least 4.08 for a piece of lumber? Show all work. Use proper probability notation, calculator inputs and shade region with equal area on the graph above.

$P(X \geq 4.08) = $ ncdf$(4.08, 9999, 4, .05) = .0548$

c) (3 Points) What width separates the widest 10% of cuts? Show on a new graph.



$z =$ invnorm$(.9, 0, 1) = 1.28 = z$

$x =$ inv norm$(.9, 4, .05) = 4.06$

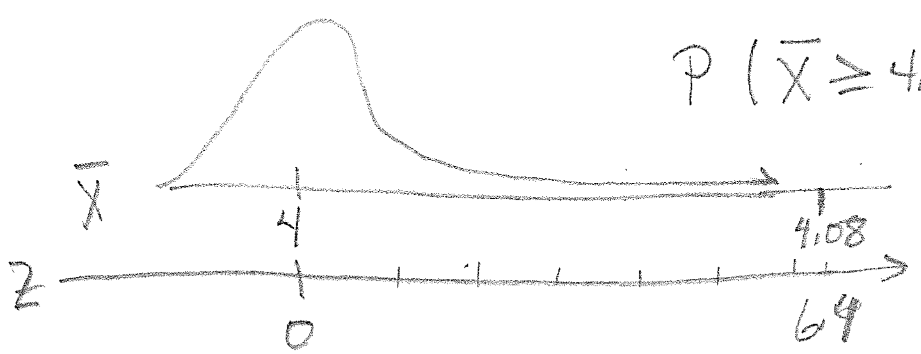
d) (3 Points) On a given day the inspector samples 16 boards, and finds the sample mean. Find the mean $\mu_{\bar{x}}$ and standard deviation $\sigma_{\bar{x}}$ of the population of sample means for samples of size n = 16.

Normal, $\mu_{\bar{x}} = \mu = 4$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{.05}{\sqrt{16}} = .0125$

e) (3 Points) Find the z-score of a sample mean that is at least $\bar{x} = 4.08$ inches in the distribution of sample means.

$z = (\bar{x} - \mu_{\bar{x}}) / (\sigma/\sqrt{n}) = (4.08-4)/(.0125) = 6.4$

f) (6 Points) For a sample of size 16, what is the probability that the **mean** at least $\bar{x} = 4.08$ inches in the distribution of sample means? Graph the distribution of sample means when the sample size is 16 with both an x-axes and a z-axes. Does the data indicate that the machine is working properly. No



$P(\bar{X} \geq 4.08) = $ ncdf$(4.08, 9999, 4, .0125)$

$= 7.8 \times 10^{-11} \approx 0 < .0001$

It is very unlikely that we would get a sample mean of 4.08 inches if the machine was actually cutting boards with a population mean length of 4 inches. The machine does not appear to be working correctly.

Use the given degree of confidence and sample data to construct a confidence interval for the population mean μ. Assume that the population has a normal distribution.

T Interval
Data L_1
CL = .9

15) The principal randomly selected six students to take an aptitude test. Their scores were:

76.5 85.2 77.9 83.6 71.9 88.6

Determine a 90% confidence interval for the mean score for all students.

a)) What point estimate of the population mean does this sample give? $\bar{x} = 80.6$

b) What is the margin of error? (Show work. Include critical value.)

$S_x = 6.23$
$n = 6$
$t_{\alpha/2} = invT(1-.1/2, 5$

$$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 2.015 \cdot \frac{6.23}{\sqrt{6}} = 5.12$$

c) Find the confidence interval.

$$80.6 - 5.12 < \mu < 80.6 + 5.12$$

(75.5, 85.7)

$$75.5 < \mu < 85.7 \text{ pts}$$

d) Interpret the meaning of this confidence interval.

We are 90% confident that the mean score of all students will be between 75.5 and 85.7

e) Is the principal reasonable confident that the average of his students scores is higher than the national average of 70.

Yes, since 70 is not in this CI, and this CI lies above 70, the principal is confident that his students average score will be above 70

Identify the null hypothesis, alternative hypothesis. Find and graph the point estimate for the population Proportion and test statistic. Find the P-value. State your conclusion about the null hypothesis, and final conclusion that addresses the original claim.

8.1 1-Prop Z Test

16) In a random sample of 100 people in Santa Rosa, 45% said they would vote for the incumbent president. According to a politician, 53% of Santa Rosans would vote for the incumbent president. $\alpha = .10$
Use the above sample to test the claim that the actual percentage is 53%. Use a 0.10 significance level.

a) (3 Points) State claim, null and alternate hypothesis. a) $H_0: p = .53$ $H_1: p \neq .53$

b) (3 Points) Find the critical value and graph and shade the critical region.

c) (3 Points) Find the point estimate of the population proportion and it's test statistic.

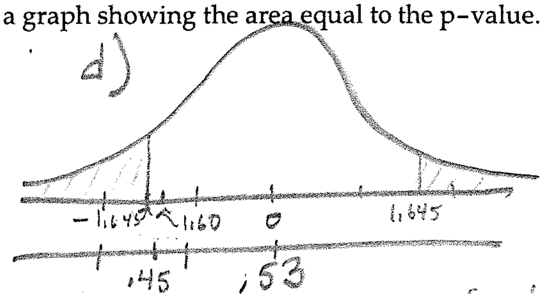
d) (3 points) Label these values on your graph.

e) (5 Points) Clearly state your initial conclusion and your final conclusion so that it is understandable without knowing statistics.

(5 Points) Find and explain the meaning of the P-value. Shade a graph showing the area equal to the p-value.

b) CV: $Z = \pm invnorm(1 - .10/2, 0, 1) = \pm 1.645$

d)



c) PE: $\hat{p} = .45$ TS: $z = \dfrac{.45 - .53}{\sqrt{\dfrac{.53 \cdot .47}{100}}} = -1.60$

e) fail to Reject H_0

TINSE to Reject that (53% voted for the incumbent president)

We can't show that the proportion who voted for the incumbent is Not 53% (Can't show H_1)

Provide the appropriate answer.

20) ~~18)~~ (4 Points) An entomologist writes an article in a scientific journal which claims that fewer than 19% of male fireflies are unable to produce light due to a genetic mutation. Identify the Type I error in this context.

§8.2 If the entomologist made a Type I error he showed that fewer than 19% of male fruit flies are unable to produce light when in fact the proportion is at least 19%.

Do one of the following, as appropriate: (a) Find the critical value $z_{\alpha/2}$, (b) find the critical value $t_{\alpha/2}$, (c) state that neither the normal nor the t distribution applies.

19) 90%; n = 9; σ = 4.2; population appears to be very skewed.

§8.2 $n < 30$ and population is Not Normal So Neither applies

20) 93%; n = 40; σ is known; population appears to be very skewed.

σ is known so use $Z_{\alpha/2} = \text{invnorm}(1-\alpha/2) = \text{invnorm}(.965) = 1.81$

21) 90%; n = 27; σ is unknown; population appears to be normally distributed.

σ is unknown Use $t_{\alpha/2} = \text{INVT}(.95, 26) = 1.706$

$1-\alpha/2 = .95$

Test the given claim by using the P-value method of testing hypothesis. Assume that the sample is a simple random sample selected from a normally distributed population. Include the hypothesis, the test statistic, the p-value, and your conclusion.

TTest Stats $\mu_0 = 30{,}000$

21) ~~22)~~ Test the claim that for the adult population of one town, the mean annual salary is ~~given by μ = $30,000.~~ less than $30,000
§8.5 Sample data are summarized as n = 17, $\bar{x}$ = $22,298, and s = $14,200. Use a significance level of α = 0.05.
a) State the claim, null and alternate hypothesis.
b) Graph and shade the critical region. Find the critical value, point estimate of the population mean, and test statistic. Label these values and areas on your graph above.
c) Explain the meaning of the P-value. Shade a graph showing the area equal to the p-value.
d) Clearly state your initial and final conclusion.

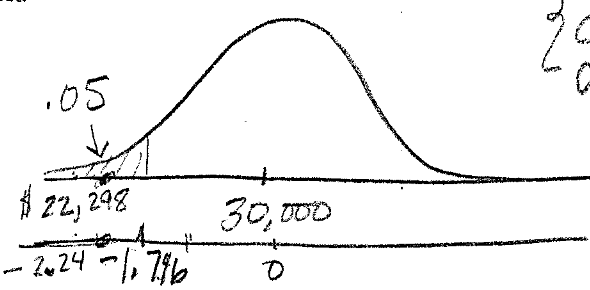
Lt Tail
df = 16
α = .05

Claim: $\mu < 30{,}000$

$H_0: \mu = 30{,}000$ one Tail

$H_1: \mu < 30{,}000$ α = .05, df = 16

CV: $t = -1.746 = \text{invt}(.05, 16)$



TS: $t = \dfrac{(\bar{x}-\mu)}{s/\sqrt{n}} = \dfrac{(22298 - 30{,}000)}{(14200/\sqrt{17})} = -2.24$

c) P-Value = .01996 = tcdf(−999, −2.24, 17)

~~Incorrect~~

There is only a 2% chance of seeing a Sample Mean of $22,298 or less, if the true mean is at least 30,000.

d) Reject H_0 there is Sufficient evidence to show the mean Salary is below 30,000.

Interpret the confidence interval.

17) ~~19~~) A random sample of clients at a weight loss center were given a dietary supplement to see if it would promote weight loss. The center reported that the 100 clients lost an average of 43 pounds, and that a 95% confidence interval for the mean weight loss this supplement produced has a margin of error of ±9 pounds.

We are 95% Confident that the Mean weight loss for those on this dietary Supplement will be between 43-9=34lb and 43+9=52lb.

Provide an appropriate response.

18) ~~20~~) (4 Points) A survey investigation whether the proportion of employees who commute by car to work is higher than it was five years ago finds a P-value of 0.011. Is it reasonable to conclude that more employees are commuting by car? Explain the meaning of this P-value.

Yes p-value = .011 < .05, It Would be unlikely (p=.011) to see the sample proportion observed today this much lower if the population proportion had Not changed lower.

19) ~~21~~ (4 Points) Hannah selected a simple random sample of all adults in her town and, based on this sample, constructed a confidence interval for the mean salary of all adults in the town. However, the distribution of salaries in the town is not exactly normal. Will the confidence interval still give a good estimate of the mean salary?

As long as the Sample Size is greater than 30, the Confidence Interval will be accurate even though Salaries are Not Normal the distribution of Sample Means will be.

Provide the appropriate answer.

20) ~~22~~) (4 Points) An entomologist writes an article in a scientific journal which claims that fewer than 19% of male fireflies are unable to produce light due to a genetic mutation. Identify the Type I error in this context.

Type I Means that The entomologist's Study indicates that fewer than 19% of Male fruitflies produce Light When in fact the True proportion is 19%.

Test the given claim by using the P-value method of testing hypothesis. Assume that the sample is a simple random sample selected from a normally distributed population. Include the hypothesis, the test statistic, the p-value, and your conclusion.

21) Test the claim that for the adult population of one town, the mean annual salary is less than μ = \$30,000. Sample data are summarized as n = 17, $\overline{x}$ = \$22,298, and s = \$14,200. Use a significance level of $\alpha = 0.05$.

a) State the claim, null and alternate hypothesis.

b) Graph and shade the critical region. Find the critical value, point estimate of the population mean, and test sta Label these values and areas on your graph above.

c) Explain the meaning of the P-value. Shade a graph showing the area equal to the p-value.

d) Clearly state your initial and final conclusion.