Practice Test 2 Math 215 Fall 2019 Jones Name Key

You may get help from me, classmates and the Math Lab to complete this Practice Test.

1) Sometimes probabilities derived by the relative frequency method differ from the probabilities expected from classical probability methods. How does the law of large numbers apply in this situation? Look up in book.

As an experiment is repeated many times, the relative frequency of an event approaches the classical probability

Find the indicated probability.

2) A class consists of 69 women and 68 men. If a student is randomly selected, what is the probability that the student is a woman? 10) __________

$$\frac{69}{69+68} = .504$$

3) If you pick a card at random from a well shuffled deck, what is the probability that you get a face card or a spade? 11) __________

$$P(☺ \text{ or } ♠) = P(☺) + P(♠) - P(☺ \text{ and } ♠)$$
$$= \frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{22}{52} = .423$$

4) A sample of 100 wood and 100 graphite tennis rackets are taken from the warehouse. If 5 wood and 10 graphite are defective and one racket is randomly selected from the sample, find the probability that the racket is wood or defective. 12) __________

$$P(W \text{ or } D) = P(W) + P(D) - P(W \text{ and } D)$$
$$= \frac{100}{200} + \frac{15}{200} - \frac{5}{200}$$
$$= \frac{110}{200} = .55$$

5) A bag contains 7 red marbles, 4 blue marbles, and 1 green marble. Find P(not blue). 13) .666

$$P(B) = \frac{4}{12} \qquad P(\bar{B}) = 1 - \frac{4}{12} = \frac{8}{12} = \frac{2}{3}$$

Find the indicated probability.

6) A restaurant offers 9 entrees and 11 desserts. In how many ways can a person order a two-course meal? 14) $9 \cdot 11 = 99$

Find the indicated probability.

7) Describe an event whose probability of occurring is 1 and explain what that probability means. Describe an event whose probability of occurring is 0 and explain what that probability means. 15) __________

$P(E) = 0$, E can't happen E = {pigs fly today}

$P(E) = 1$, E is certain E = {sun rose today}

8) Skip

Find the indicated probability.

10

8) A batch consists of 12 defective coils and 88 good ones. 8) ____________

a) Find the probability of getting three defective coils when three coils are randomly selected if each selection is replaced before the next is made. Show method used to get answer.

$$P(X=3) = \left(\frac{12}{100}\right)^3 = \boxed{.001728}$$

$N=100$

$n=3$

$p = 12/100 = .12$

b) If X=the number of defective coils when 3 are selected. Make a probability distribution for the number of defective coils out of 3 when the selections are done with replacement.

Dist $\boxed{\text{Binomialpdf}(3, .12)}$ – defective or not, with replacement So Independent with unchanging probability of Success $= p = .12$ on each trial. The Number of trials is 3.

STR → L_2

x	P(x) =
0	.6815
1	.2788
2	.0380
3	.001728

c) Find the probability of getting at least one defective coil. You get extra credit if you can find both methods for solving this problem.

$$P(X \geq 1) = .2788 + .0380 + .0017 = 1 - .6815 = 1 - (.88)^3 = \boxed{.3185}$$

9

9) Among the contestants in a competition are 42 women and 28 men. If 5 winners are randomly selected, what is the probability that they are all men? 9) ____________

a) In how many ways can 5 people be selected from this group of 70? $_{70}C_5 = 12,103,014$

b) In how many ways can 5 men be selected from the 28 men? $_{28}C_5 = 98280$

c) Find the probability that the selected group that will consist of all men.

$$P(\text{All Men}) = \frac{_{28}C_5}{_{70}C_5} = \boxed{.00812} = \frac{28}{70} \cdot \frac{27}{69} \cdot \frac{26}{68} \cdot \frac{25}{67} \cdot \frac{24}{66}$$

Solve the problem.

11

10) 8 basketball players are to be selected to play in a special game. The players will be selected from a list of 27 players. If the players are selected randomly, what is the probability that the 8 tallest players will be selected? 10) ____________

only one group of the 8 tallest player, order Selected doesn't Matter

$$P(\text{Tallest } 8) = 1/_{27}C_8 = 1/2220075 = \boxed{.0000004504}$$

12

11) There are 9 members on a board of directors. If they must elect a chairperson, a secretary, and a treasurer, how many different slates of candidates are possible? 11) ____________

order Selected ⇒ Which job → order Matters ⇒ Permutations

$$_9P_3 = \boxed{504}$$

Solve the problem involving probabilities with independent events.

13

12) A single die is rolled twice. Find the probability of getting a 2 the first time and a 2 the second time. 12) ____________

$$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} = \boxed{.0278}$$

12) 21) There are 9 members on a board of directors. If they must elect a chairperson, a secretary, and a treasurer, how many different slates of candidates are possible? 21) ______

$_9P_3 = 504$

BCA
CBA

olve the problem involving probabilities with independent events.

13) 22) A single die is rolled twice. Find the probability of getting a 2 the first time and a 2 the second time. 22) ______

$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

Find the indicated probability.

14 23) The following table contains data from a study of two airlines which fly to Small Town, USA. 23) ______

	Number of flights which were on time	Number of flights which were late
Podunk Airlines	33	6
Upstate Airlines	43	5
	76	11

87

a) If one of the flights is randomly selected, find the probability that the flight selected arrived on time given that it was an Upstate Airlines flight.

$\frac{43}{48} = .8958$

b) If one of the flights is randomly selected, find the probability that the flight selected arrived on time and was an Upstate Airlines flight.

$\frac{43}{87} = .4943$

c) If one of the flights is randomly selected, find the probability that the flight selected arrived on time or was an Upstate Airlines flight.

$\frac{76}{87} + \frac{48}{87} - \frac{43}{87} = \frac{81}{87} = .9310$

d) If two flights were randomly selected find the probability that both flights were on time. Calculate this probability with and without replacement.

With replacement

$P(\text{1st On time and 2nd On time}) = \frac{76}{87} \cdot \frac{76}{87} = .763$

Without replacement

$P(O_1 \& O_2) = \frac{76}{87} \cdot \frac{75}{86} \stackrel{3}{=} .762$

PRACTICE TEST 2 Math 15 STATISTICS Jones ~~Spring 2012~~ Fall 2018 Name Key

Show all work! Draw a normal distribution when needed.

Answer the question.

(16) Suppose that computer literacy among people ages 40 and older is being studied and that the accompanying tables describes the probability distribution for four randomly selected people, where x is the number that are computer literate. 1) ______

a) is this a probability distribution yes

x	P(x)
0	0.16
1	0.25
2	0.36
3	0.15
4	0.08

$\sum p(x) = 1$

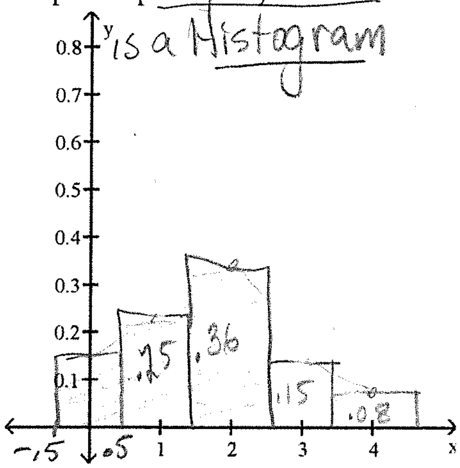
$0 \le p(x) \le 1$

Not Binomial

Is it unusual to find four computer literates among four randomly selected people? (WHY?) No, $P(X=4) = .08 > .05$ so Not Unusual

What is the probability of getting 2 or fewer people out of the 4 who are computer literate?______ $P(X \le 2) = .36 + .25 + .16 = \boxed{.77}$

Graph this probabillity distribution.



Find the mean of this probability distribution. 1-Var stat (L_1, L_2)

$\mu = \bar{x} = \boxed{1.74 = \mu}$

Find the Standard deviation of this distribution. $\sigma = \sigma_x = \boxed{1.14}$

Skip

~~What kind of probability distribution is this? (circle one)~~

~~Normal Distribution~~ ~~Other Continuuous Distribution Distribution~~ ~~Binomial Distribution~~ ~~Discrete~~

$np = 4 \cdot p = 1.74$
$\Rightarrow p = .435$ but binomialpdf(4, .435) is Not the one given

Solve the problem.

(17) Suppose you buy 1 ticket for \$1 out of a lottery of 1,000 tickets where the prize for the one winning ticket is to be \$500. What is your expected value? X = Net winnings 2) ______

	X	P(X)
Win	500	1/1000
Lose	−1	999/1000

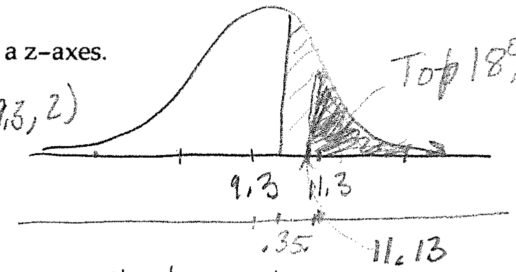
$E(x) = 499 \cdot \frac{1}{1000} + (-1) \cdot 999/1000$

$= -.50$

one outcome should be Negitive

(18) 3) Suppose that replacement times for washing machines are normally distributed with a mean of 9.3 years and a standard deviation of 2 years. 3) ____________

a) Draw this distribution showing an axes for the age of the machine and a z–axes.



$Z = \frac{10-9.3}{2} = .35$

normalcdf(10, 9999, 9.3, 2)

$P(X>10) = P(Z>.35) = \boxed{.3632}$

b) What proportion of washing machines last more than 10 years?

c) Find the replacement time that separates the top 18% from the bottom 82%. → Shade region

X = invnorm(.82, 9.3, 2) = 11.13

d) If you sell 9 washing machines and insure them for 10 years. Wht is the probability that all 9 will last more than 10 years?

P(All last more than 10) = P(1st does) · P(2nd does) · ... · P(9th does)

$= (.3632)^9 = .000110$ 3 significant digits

It is very unlikely that all 9 will last more than 10 years.

(5) **Then use the Binomial Theorem to find the probability exactly.**

4) An engineer thinks that she had improved the quality of the circuit boards that she is designing. The defect rate has been 14%. But in the last sample of 50 parts she found that only 4 were defective. Is this conclusive proof that she improved her design or is this sample usual to see when the defect rate is 14% and more data is needed to be sure that the defect rate really has decreased. Assume that many thousands of parts are being produced. 4) ____________

p = .14
n = 50
X = 4
q = 1 − p
= 1 − .14
= .86

$\hat{p} = \frac{4}{50} = .08$ defect rate in Sample

Indepenence

$\boxed{\sigma = \sqrt{npq}}$

a) What is the mean and standard deviation of the binomial ~~distribution used for this~~ problem. $\boxed{\mu = n \cdot p} = 50 \cdot .14 = \boxed{7}$ $\sigma = \sqrt{50 \cdot .14 \cdot .86} = \boxed{2.45}$

b) How many do we expect to be defective? $\boxed{7 = E(X)}$ X = # of defective circuit boards in a Sample of 50.

c) What is the proability that we see a sample with at most 4 when the defect rate is 14%? Use the binomial Distribution. Draw the disribution and shade the rectangles with area corresponding to the probability that we are finding.

n p X

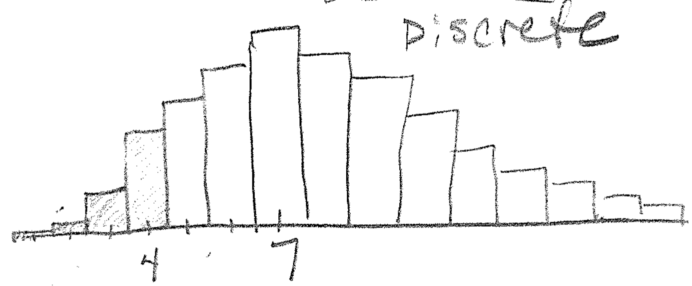
$P(X \le 4) = \text{binomialcdf}(50, .14, 4) = \boxed{.1528} > .05$

≤ 4

Discrete

$\boxed{P(X \le r) = \text{bincdf}(n, p, r)}$

X
0
1
2
3
4
5
⋮
50



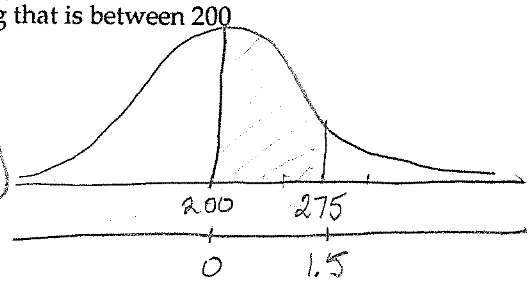
d) Does this sample verify her claim that the defect rate has been lowered?

No, it would No be unusual to See a Sample of 50 with only 4 defective pens. However her results are promising. A larger Sample with this rate would be conclusive.

Find the indicated probability.

(19) 5) A bank's loan officer rates applicants for credit. The ratings are normally distributed with a mean of 200 and a standard deviation of 50. $Z = (275-200)/50 = 1.5$ 5) ____________

a) If an applicant is randomly selected, find the probability of a rating that is between 200 and 275.

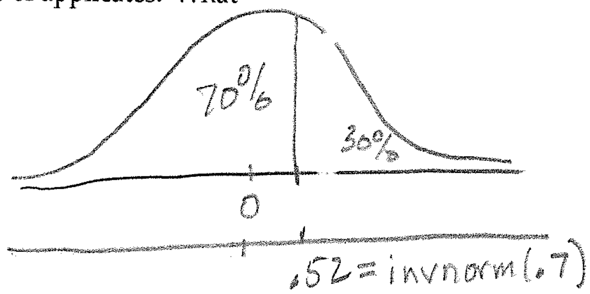


$P(200 < X < 275) = P(0 < Z < 1.5)$

$= \text{normalcdf}(200, 275, 200, 50)$ (Lower bound, upper bound, μ, σ)

$= \boxed{.4332}$

b) In todays market the loan officer is only giving loans to the top 30% of applicates. What rating will separate the top 30% of applicants from the bottom 70%.



$x = \text{invnorm}(.7, 200, 50)$

$\boxed{X = 226.2}$ ← Always use Area to the left of the desired score.

Solve the problem.

(20) 6) A bank's loan officer rates applicants for credit. The ratings are normally distributed with a mean of 200 and a standard deviation of 50. 6) ____________

If 40 different applicants are randomly selected, find the probability that their mean score is above 215. ← must use CLT, distribution of means of samples of size

$P(\bar{x} > 215)$ n=40 has $\mu_{\bar{x}} = \mu = 200$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 50/\sqrt{40}$

$= \text{normalcdf}(215, 9999, 200, 50/\sqrt{40}) = \boxed{.0289}$

(21)

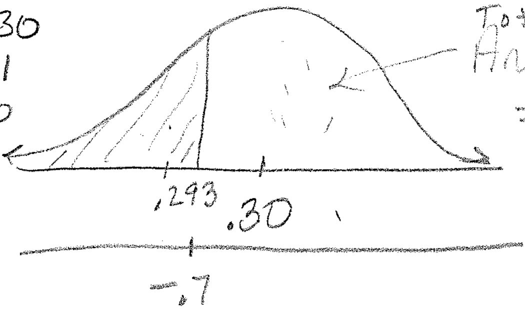
Not a CLT Problem

7) The diameters of pencils produced by a certain machine are normally distributed with a mean of 0.30 inches and a standard deviation of 0.01 inches. In a random sample of 450 pencils, approximately HOW MANY would you expect to have a diameter less than 0.293 inches? (Hint: Find a proportion first.) ← Not looking at a distribution of means 7) ____________

$\mu = .30$

$\sigma = .01$

$n = 450$



Total Area under curve = 100% of population or all = 450 pencils

Area = proportion, Probability, % of pop.

$Z = \dfrac{.293 - .30}{.01} = -.7 = \text{normalcdf}(-9999, -.7)$

$P(X < .293) = \text{normalcdf}(-9999, .293, .3, .01)$

$= .2419 \Rightarrow 24.19\%$ of $\boxed{450}$ ← whole population

# of pencils with diameter less than .293 $= \boxed{109}$

Provide an appropriate response.

22 5) Sampling without replacement involves dependent events, so this would not be considered a binomial experiment. Explain the circumstances under which sampling without replacement could be considered independent and, thus, binomial. To make calculations easier.

When $n < 5\%$ of N then the probabilities corresponding to selection with replacement are so close to the probabilities when sampling is done without replacement that we can use the independent (with replacement) values instead.

23 6) Under what conditions can we apply the results of the central limit theorem?

For means when $n > 30$ or dist. of the parent population is Normal.

For proportions when $np > 5$ and $nq > 5$.

(Note: np & $nq > 9$ is much better)

26 7) The typical computer random–number generator yields numbers in a uniform distribution between 0 and 1 with a mean of 0.500 and a standard deviation of 0.289. (a) Suppose a sample of size 50 is randomly generated. Find the probability that the mean is below 0.300. (b) Suppose a sample size of 15 is randomly generated. Find the probability that the mean is below 0.300. These two problems appear to be very similar. Only one can be solved by the central limit theorem. Which one and why?

a) $n = 50 > 30$ so dist of sample means is Normal.

b) $n = 15 < 30$ & pop = Uniform $\neq$ Normal so distribution of sample means will not be Normal (Yet) Bootstrap!

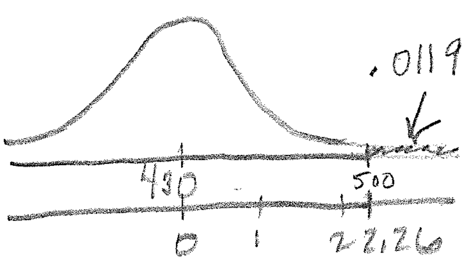
27 8) SAT verbal scores are normally distributed with a mean of 430 and a standard deviation of 120 (based on the data from the College Board ATP). If a sample of 15 students is selected randomly, find the probability that the sample mean is above 500. Does the central limit theorem apply for this problem?

$\mu = 430$ $\sigma = 120$ $n = 15$, Normal yes CLT Applys $\bar{x} = 500$

$$Z_{\bar{x}=500} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma/\sqrt{n}} = \frac{(500-430)}{(120/\sqrt{15})} = 2.26$$

$$.0119 = P(\bar{x} > 500) = \text{ncdf}(500, 9999, 430, 120/\sqrt{15})$$

$$= \boxed{.0119}$$

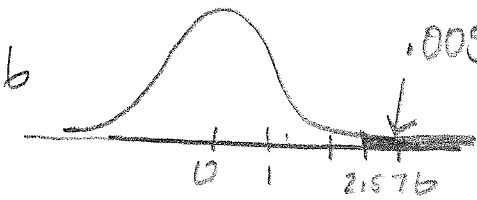


24 9) Which of the following notations represents the standard deviation of the population consisting of all sample means?

A) $\sigma_{\bar{x}}$ B) s C) $\sqrt{npq}$ D) μ

Find the indicated value.

25 10) $z_{0.005}$ = invnorm$(1-.005, 0, 1) = 2.576$



Provide an appropriate response.

28

~~11~~) A poll of 1100 randomly selected students in grades 6 through 8 was conducted and found that 54% enjoy playing sports.

Is the 54% result a statistic or a parameter? Explain. Statistic, $.54 \cdot 1100 = 594 = X = \#$ in Sample who enjoy sports.

Does the 54% refer to sample mean or a sample proportion?

$$54\% = .54 = \frac{594}{1100} = \frac{X}{n} = \hat{p} = \text{a Sample Proportion.}$$

30

~~12~~) Tell whether the following statistic is a biased or unbiased estimator of a population parameter:

Sample proportion used to estimate a population proportion. unbiased

Sample mean used to estimate a population mean. unbiased

Sample standard deviation used to estimate a population standard deviation. biased (But used)

Sample varience used to estimate a population varience. unbiased

Median - Biased

29

~~13~~) Apply the Central Limit Theorem. Samples of size n = 800 are randomly selected from the population of numbers (0 through 9) produced by a random–number generator.

a) If the proportion of odd numbers is found for each sample what type of distribution is the distribution of the sample proportions? What is it's mean and what is it's standard deviation?

a) $\boxed{\text{Normal}}$

b) $\boxed{\mu_{\hat{p}} = .5} = P = \frac{5}{10} = \frac{\#\text{ odd}}{\#\text{ of values}}$

c) $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.5 \cdot .5}{800}} = \boxed{.017 = \sigma_{\hat{p}}}$

} CLT for Proportions

b) If the mean of the 800 values is found for each of the samples what type of distribution is the distribution of sample mean? What is the mean and what is the standard deviation of the distribution of sample means? (please use correct notation.)

Population = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} = L_1

1-Var stats $\mu = 4.5$ $\sigma = 2.87$

a) Normal

b) $\mu_{\bar{x}} = 4.5$

c) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.87}{\sqrt{800}} = .1015$

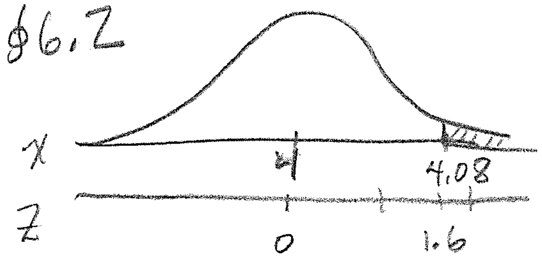
} CLT for Means

n = 800

31) 3) (21 Points) A machine in a saw mill cuts pieces of lumber to an average width of $\mu = 4$ inches with a standard deviation $\sigma = .05$ inches

a) (3 Points) Graph the distribution with both an x–axes and a z–axes. Show mean and standard deviation. Calculate the z–score of a 4.08 width for a piece of lumber and label on your graph.

66.2



$Z = \frac{X - \mu}{\sigma} = \frac{4.08 - 4}{.05} = 1.6$

$\mu = 4$

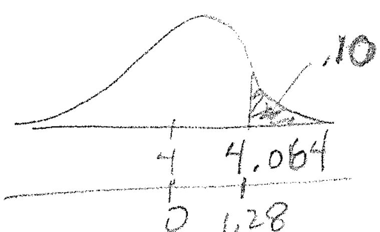
$\sigma = .05$

3

b) (3 Points) What is the probability that width is at least 4.08 for a piece of lumber? Show all work. Use proper probability notation, calculator inputs and shade region with equal area on the graph above.

$P(X \geq 4.08) = \text{ncdf}(4.08, 9999, 4, .05) = .0548$ 3

c) (3 Points) What width separates the widest 10% of cuts? Show on a new graph.



$Z = \text{invNorm}(.9, 0, 1) = 1.28$ 3

$X = \text{invNorm}(.9, 4, .05) = \boxed{4.064 \text{ inches}}$

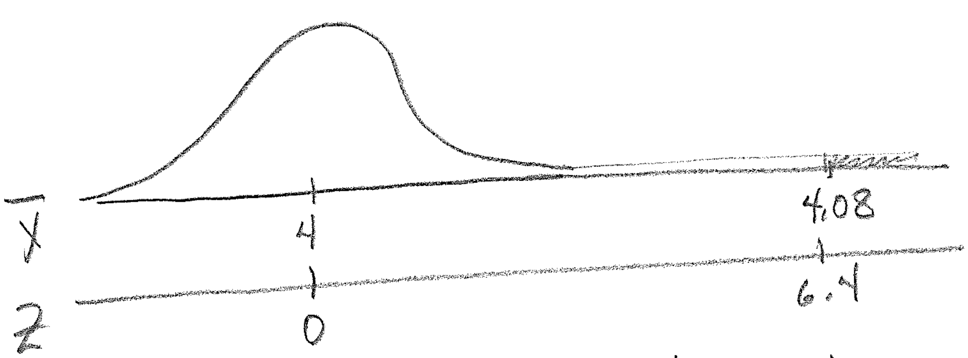
d) (3 Points) On a given day the inspector samples 16 boards, and finds the sample mean. Find the mean $\mu_{\bar{x}}$ and standard deviation $\sigma_{\bar{x}}$ of the population of sample means for samples of size n = 16.

$\mu_{\bar{x}} = 4 \qquad \sigma_{\bar{x}} = \frac{.05}{\sqrt{16}} = .0125$ 3

e) (3 Points) Find the z–score of a sample mean that is at least $\bar{x}$ =4.08 inches in the distribution of sample means.

$Z_{\bar{x}} = \frac{\bar{X} - \mu_{\bar{x}}}{\sigma/\sqrt{n}} = \frac{4.08 - 4}{.0125} = 6.4$ 3

f) (6 Points) For a sample of size 16, what is the probability that the **mean** is at least $\bar{x}$ =4.08 inches in the distribution of sample means? Graph the distribution of sample means when the sample size is 16 with both an $\bar{x}$–axes and a z–axes. Does the data indicate that the machine is working properly? No, 6



the $P(\bar{X} > 4.08) \approx$ it would be very unusual to see this mean width if the machine is adjusted properly

$P(\bar{x} > 4.08) = \text{ncdf}(4.08, 9999, 4, .0125)$

$= 7.8 \times 10^{-11}$

21