

§5.3 Poisson Distribution

- ① Another example of a Discrete Prob. Dist
- ② Work with as a Table or Graph with Rectangles
- ③ $X = \# \text{ of Occurrences on an interval: Time, Dist., region}$
- ④ Defined for all $X = 0, 1, 2, 3, \dots$
- ⑤ $P(X=r) = \frac{\mu^r e^{-\mu}}{X!} = \frac{\lambda^X e^{-\lambda}}{X!} = \text{poissonpdf}(\lambda, X)$
Note: $\lambda = \mu$

$\lambda = \text{lambda lower case}$ $e = 2.718281828$

★ $\lambda = \frac{\# \text{ of Occurrences}}{\# \text{ of Intervals}}$ = Average Number expected on each interval.

- ⑥ Occurrences must be random, independent and uniformly distributed.

Ex Hurricanes hit the East Coast 336 times in the last 55 years.

a) What is the Average Number of hurricanes per year?

$$\lambda = \frac{336}{55} = 6.1$$

b) What proportion of years do we expect to get 5 hurricanes according to the Poisson distribution? Two ways

$$P(X=5) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(6.1)^5 e^{-6.1}}{5!} = .1579$$

$$= \text{Poisson Pdf}(\lambda, x) = \text{poissonpdf}(6.1, 5) = .1579 \checkmark$$

c) What does this mean? In 55 years, we expect 15.79% to have 5 hurricanes or $.1579 \cdot 55 = 8.7$ years

d) Actually there were 8 years with 5 hurricanes
 $8 \approx 8.7$ is really close, so Poisson is a good Model.

Poisson pdf (6.1, x)

X	0	1	2	3	4	5	6	7	8	9	10	11
P(x)	.0022	.0137	.0417	.0848	.1294	.1579	.160	.140	.107	.072	.044	.024
E(x)	.1				7.1	8.7						
= p(x) · 55						8						
Actual	0	1										

$$P(x) = \text{poisson}(\lambda, x) = \text{poisson}(6.1, x)$$

$$E(x) = p(x) \cdot 55 = .1579 \cdot 55 = 8.7$$

$E(x)$ = Expected Number of years that we would see 5 hurricanes