§4.1 Probability

Goal: Understand Vocabulary of Probability

The Probability of Event E = $P(E)$ = % The Percentage of time event occurs = Proportion of Occurences out of the Total Number of Trials = $\frac{X}{n}$

$P(E)$ = The Relative frequency frequency = $\frac{X}{n} = \frac{\#\text{ of occurences}}{\#\text{ of Trials}}$

Ex If we flip a coin 100 times and See 48 heads

$$P(H) = \frac{48}{100} = .48$$

The Law of Large Numbers Says that the Relative frequency will approach the Classical Probability as the Sample Size increases.

Classical Probability

Let S = Sample Space = Set of all possible Outcomes to a random trial.

Ex Flip 3 Coins.

a) $S = \{HHH, TTT, HTH, THT, THH, HHT, TTH, HTT\}$

"{" "The Set"

one outcome

$n(S) = 2^3 = 8$

$\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}$ ← 2 choices for each

← 3 spots

b) Find the Probability of one Tail

$E = \{HTH, THH, HHT\}$ $n(E) = 3$ = the Number of elements of E

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$$

c) At least one head? $P(F) = \frac{7}{8} = 1 - \frac{1}{8} = 1 - P(\text{No heads})$

$P(\#\text{of heads} \geq 1)$

one head, 2 heads, or 3 heads

Ex 2 Flip 7 coins

a) Sample Space = $S = \{$ HHHHHHH, THHHHHH, HTHHHHH, ... $\}$

$n(S) = 2^7 = 128$

b) $P(\text{No heads}) = \frac{1}{128} = .0078$ Not Likely, unusual

c) $P(\text{One Tail}) = \frac{7}{128} = \boxed{.0547}$ ← $0 \le P \le 1$ Round to 4 decimals

d) $P(\text{At least one head}) = \frac{127}{128} = 1 - \frac{1}{128} = .9922$ Very likely

$= 1 - P(\text{All Tails})$

$P(S) = 1 = \frac{n}{n}$ = Certain Event

Quiz 4 4.1 and 4.2 Math 15 Jones Name Key

1) What is the sample space for the experiment that we roll two dice.

$S = \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\}$

$n(S) = 6 \cdot 6 = 36 = 6^2$

$E = \{12, 21\}$ $n(E) = 2$

For each of the following, write out the subset that corresponds to the event and then find the probability of that event.

a) The sum is 3 $P(\text{Sum of 3}) = \frac{n(E)}{n(S)} = \frac{2}{36} = .0556$

b) The sum is 7 $P(\text{Sum of 7}) = \frac{6}{36} = .1667$ OR means Add

c) The sum is odd $P(\text{Sum is Odd}) = 18/36 = .5$

d) The sum is 3 or 7 $P(3 \text{ or } 7 \text{ Sum}) = \frac{8}{36} = \frac{2}{36} + \frac{6}{36} = .2222$ (a, b)

e) At least one of the die has an even number.

$P(\text{One die is even}) = 1 - P(\text{Neither is even})$ ← Complement Rule

$1 - \frac{9}{36} = .75$

f) Now suppose that you roll 5 dice, How many elements will be in the sample space.

$S = \{13312, 11111, \ldots\}$ $n(S) = 6^5$

g) What is the probability that at least one of the dice will show a 3?

Two dice

$P(\text{At least One 3}) = 1 - P(\text{No 3's})$ → Mult. principle "And Means Multiply

$= 1 - \frac{5}{6} \cdot \frac{5}{6} = \frac{11}{36}$

$= \frac{36}{36} - \frac{25}{36} = \frac{11}{36} = .3056$

We did Not get a 3 on first trial "and"
We did Not get 3 on 2nd trial

$P(\text{At least One 3 in 5 Tosses}) = 1 - P(\text{No's}) = 1 - \left(\frac{5}{6}\right)^5 = .5981$

$= 1 -$

Dice

a) Find the Probability of a 7 if two dice are rolled.

$n(S) = 6 \cdot 6 = 36$

$S = \{11, 12, 13, 14, 15, 16, 21, 22, 25, 34, 43, 46, 52, 55, 56, 61, 64, 65, 66\}$

$E = \{61, 52, 43, 34, 25, 16\}$

$P(\text{Sum}=7) = \frac{6}{36} = \frac{1}{6} = \boxed{.1667}$

b) Find the prob of a Sum of 3 or 7.

$P(F) = P(\text{Sum}=3) = \frac{2}{36} = \frac{1}{18} = .0556$

$P(F \text{ or } E) = .0556 + .1667 = \boxed{.2222}$

c) Prob of a Sum of at least 10?

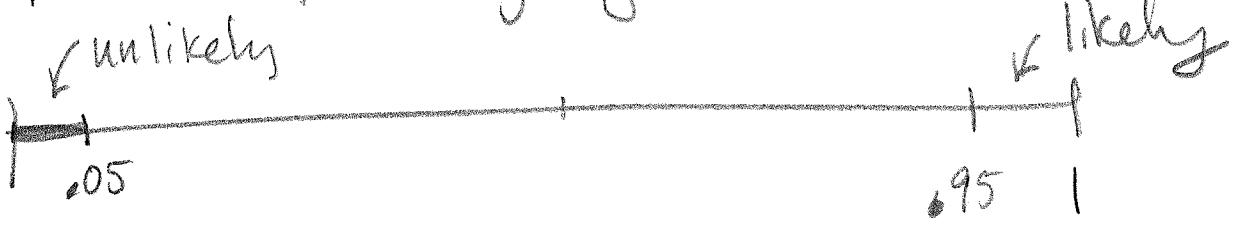
$P(\text{Sum} \geq 10) = \frac{6}{36} = \boxed{.1667} = P(G)$

d) Prob of a Sum less than 10?

Complement of an Event = G does Not occur = G^c

$P(G^c) = 1 - P(G) = 1 - \frac{6}{36} = \frac{30}{36} = \boxed{.8333}$

Probability Language



$0 \leq P(E) \leq 1$

$P(E) = 0$ → Never, impossible

$P(E) = 1$ → Always, Certain

$P(E) < .05$ → unlikely

$P(E) < .001$ → Really unlikely
Almost impossible

$P(E) > .95$ → Likely

The law of large Numbers

"The casino always wins"

As an experiment or procedure is repeated the relative frequence of an observed event approaches the classical probability

10 times	100	1000	P(H)
$\frac{4}{10}$	$\frac{47}{100}$	$\frac{493}{1000}$	
.4	.47	.493	→ .5

Sample Space for 10 flips?

$S = \{$ HHHHHHHHHH, TH...H, HTH...H, ... $\}$ (10)

$n(S)$ = the size of $S = 2^{10}$ Multiplication principle of counting 4.3

$n(S) = 1024$

$P(\text{one head}) = \frac{10}{1024} = .00977$

Def An Event is Statistically Significant if the probability of the event is small, use $(P(E) < .05)$

$\alpha = .05$ Alpha level of Test

Ex If a family has 4 children is having 4 boys Statistically Significant?

Find $P(4\text{Boys}) = \frac{n(E)}{n(S)} = \frac{\#\text{ of ways to get 4 boys}}{\#\text{ of family birth orders when 4 children}}$

$n(S) = 2^4 = 16$

$S = \{bbbb, gbbb, bgbg, \ldots\}$

$E = \{bbbb\}$ $\quad n(E) = 1$

$P(4\text{Boys}) = \frac{1}{16} = \boxed{.0625} > .05$ Not Statistically Significant!

We don't Always Use .05

Ex If 35 out of 40 busses were ontime Recently. a) What is the probability it will be ontime today?

Use Relative Freq. Definition

$$P(E) = \frac{x}{n} = \frac{\#\text{ Successes}}{\#\text{ Trials}} = \frac{35}{40} = \boxed{.875}$$

b) What is probability that bus is late?

$$P(\text{Late}) = P(\overline{E}) = \frac{5}{40} = 1 - .875 = \boxed{.125}$$

$\overline{E}$ ↑ the complement of E = Everything Not in E

Ex A class has 46 women and 81 Men What is probability of randomly selecting a women?

$n = \text{class} = 46 + 81 = 127$

$$P(W) = \frac{46}{127} = \boxed{.3622}$$

Math 15/215 Plan for Test 2

Date	Cover	Collect
Mon 9/16	4.1	Nothing
Tue 9/17	4.2,4.3	Lesson 2: Struggle, Integrated review 4
Wed 9/18	4.4	4.1 and 4.2
Mon 9/23	5.1	4.3 and 4.4
Tue 9/24	5.2	Test Corrections, Integrated Review 5
Wed 9/25	5.3	5.1
Mon 9/30	6.1	5.2 and 5.3
Tue 10/1	6.2	Lesson 3: 100 Rectangles, Integrated Review 6
Wed 10/2	6.3	6.1 and 6.2
Mon 10/7	7.1	6.3
Tue 10/8	Review	Practice Test 2
Wed 10/9	Test 2	

Math 215 Grade Test Corrections

→ Separate Paper

→ Problem Number
Score on that problem ?/10
Complete Solution

- Due Tues. 9/24

- You can Get Help
Do your own work