Integrated Review 7: Intervals of Numbers

Elementary Statistics Chapter 7: Estimating Parameters and Determining Sample Sizes

Objectives:

1. Find the middle value for an interval written either as an inequality or in interval notation.
2. Find the distance from the middle value of an interval to its endpoints.
3. Write and interpret three different forms of intervals (as used for confidence intervals).
4. Evaluate formulas used for confidence intervals.

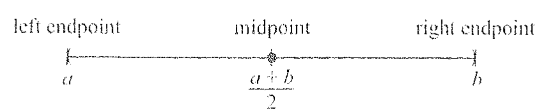
In the chapter in the text on estimating parameters and determining sample sizes, we will be utilizing intervals a lot. So, in this integrated review chapter, we will spend some time discussing several topics related to intervals.

Objective 1: Find the middle value for an interval written either as an inequality or in interval notation.

One topic that you will have to understand in the text is how to find the middle value for a given interval. The given interval can be written either as an inequality or in interval notation. We will review how to handle both of these formats.

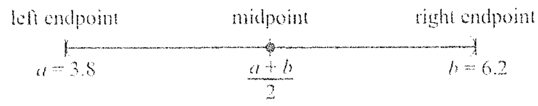
Example 1 Let $3.8 < \mu < 6.2$ represent an interval on the number line. Find the value that is in the middle of the interval.

From an algebra class, you may recall that this is the same thing as finding the midpoint for two points on a number line. In order to find the middle value, you have to add the two values of the endpoints and divide that sum by 2.



$$\text{The middle value of } a < \mu < b \text{ is } \frac{a+b}{2}.$$

We have $3.8 < \mu < 6.2$. So, substitute 3.8 for a and 6.2 for b. Then, the middle value of the given interval of numbers is found by following the order of operations. (Recall that the entire numerator can be thought of as being within parentheses and is therefore simplified first.)



$$\frac{a+b}{2} = \frac{3.8+6.2}{2} = \frac{10}{2} = 5$$

Answer The value of 5 is the middle value of the interval $3.8 < \mu < 6.2$.

My Turn!

Let $105.67 < \mu < 118.59$ represent an interval on the number line. Find the value that is in the middle of the interval.

Example 2 If $(0.124, 0.498)$ is an interval on the number line, find the value that is in the middle of the interval.

This problem is completed in the same way as Example 1. It simply differs in how the given interval is written. The interval $(0.124, 0.498)$ could be rewritten as $0.124 < \mu < 0.498$. In order to find the middle value, you have to add the two values of the endpoints and divide that sum by 2. That is,

$$\text{the middle value of } (a,b) = \frac{a+b}{2}$$

We have $(0.124, 0.498) = (a,b)$. So, substitute 0.124 for a and 0.498 for b and simplify.

$$\frac{a+b}{2} = \frac{0.124+0.498}{2} = \frac{0.622}{2} = 0.311$$

Answer 0.311 is the middle value of the interval $(0.124, 0.498)$.

My Turn!

If $(6.48, 15.84)$ is an interval on the number line, find the value that is in the middle of the interval.

Objective 2: Find the distance from the middle value of an interval to its endpoints.

Example 3 Let $(0.368, 0.549)$ represent an interval on the number line. Find the distance from the middle of the interval to either endpoint.

Let E represent the distance from the midpoint to an endpoint.

There are two ways to solve this problem:

Method 1: Find the middle value of the interval first and then find the distance from the midpoint to the endpoint.

Step 1: Follow the process for Objective 1 to find the middle value of the interval.

The middle value of $(0.368, 0.549)$ is

$$\frac{0.368+0.549}{2}=\frac{0.917}{2}=0.4585$$

Step 2: Find the distance from the middle value that you found in step 1 to the endpoint.

You can find the distance from the middle value to an endpoint by subtracting the left endpoint from the midpoint of the interval.

$$\text{distance from middle value to endpoint} = \text{middle value} - \text{left endpoint}$$

(You can also calculate this distance by subtracting the middle value from the right endpoint.)

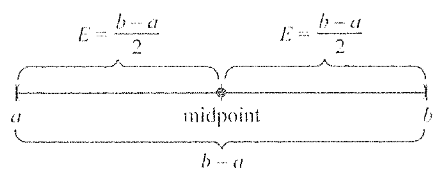
For this example, the distance from the middle value to either endpoint is

$$E = 0.4585 - 0.368 = 0.0905$$

Method 2: Find the distance directly without actually finding the midpoint.

The distance from the middle value to the endpoint of an interval (a, b) can be given by

$$E = \frac{b-a}{2}$$



So, the distance from the middle value to the endpoint of the interval $(0.368, 0.549)$ can be found as follows:

$$E = \frac{0.549 - 0.368}{2} = \frac{0.181}{2} = 0.0905$$

You can feel free to use the method that is more intuitive for you. Sometimes you may need to know the middle value anyway. In that case, method 1 is definitely the way to go. If you don't need to know the midpoint, then you may want to use method 2.

Answer The distance is 0.0905.

My Turn!

Let $(26.8, 32.4)$ represent an interval on the number line. Find the distance from the middle of the interval to either endpoint.

Objective 3: Write and interpret three different forms of intervals (as used for confidence intervals).

As you may have gathered from the examples above, there are several different forms of intervals. We will review the various forms you will see in the textbook. Often, technology will give you the output in a form that is different from how you may want to write it. So, it is important to understand how to translate from one format to another.

The key point that we have to remember is that $a < \mu < b$ and (a, b) represent the same interval for μ.

Example 4: Express $2.5 < \mu < 5.3$ in interval notation.

The endpoints of the interval are 2.5 and 5.3. So, we can rewrite $2.5 < \mu < 5.3$ as $(2.5, 5.3)$.

Answer $(2.5, 5.3)$

My Turn!

Express $-3.5 < \mu < 1.9$ in interval notation.

Example 5 Express $(0.88, 0.96)$ in inequality notation, assuming that the variable under study is μ.

The endpoints of the interval are 0.88 and 0.96. So, we can rewrite $(0.88, 0.96)$ as $0.88 < \mu < 0.96$.

Answer $0.88 < \mu < 0.96$

My Turn!

Express $(0.63, 0.92)$ in inequality notation, assuming that the variable under study is p.

Sometimes, you will want to write your interval in the following form:

$$\text{middle value}(m) \pm \text{distance from middle value to endpoint}(E) = m \pm E$$

The notation $\pm$ translates to *plus or minus*.

Example 6 Let $4.0 < \mu < 4.8$ represent an interval on the number line. Write the given interval in the format $m \pm E$.

First, we must find the value that is in the middle of the interval and let the variable m represent that.

$$m = (4.0, 4.8) = \frac{4.0 + 4.8}{2} = \frac{8.8}{2} = 4.4$$

Next, we must find the distance from the middle of the interval to either endpoint and let E represent that.

$$E = 4.4 - 4.0 = 0.4$$

Finally, we must write the given interval in the desired format $m \pm E$.

So, $4.0 < \mu < 4.8$ can be rewritten in the desired form of 4.4 ± 0.4.

Answer 4.4 ± 0.4

My Turn!

Let $0.18 < \mu < 0.36$ represent an interval on the number line. Write the given interval in the format $m \pm E$.

Objective 4: Evaluate formulas used for confidence intervals.

You will be learning about several formulas in the related section of the textbook on confidence intervals. Here, we will practice evaluating these formulas. You will learn the meaning of the formulas and how to apply them in context within the text section itself.

Example 7 Evaluate the formula

$$E = z \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

when $z = 1.96$, $\hat{p} = 0.516$, and $n = 1000$. Round the answer to the thousandths.

First, we must substitute for the variables in the correct location. Then, we will follow the order of operations for the right-hand side.

$$\begin{aligned}
E &= z \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\
&= 1.96 \cdot \sqrt{\frac{0.516(1-0.516)}{1000}} \\
&= 1.96 \cdot \sqrt{\frac{0.516(0.484)}{1000}} \\
&= 1.96 \cdot \sqrt{\frac{0.249744}{1000}} \\
&= 1.96 \cdot \sqrt{0.000249744} \\
&\approx 1.96 \cdot 0.0158032908 \\
&= 0.03097445 \\
&\approx 0.0310
\end{aligned}$$

Answer $E \approx 0.0310$

My Turn!

Evaluate the formula $E = z \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ when $z = 1.645$, $\hat{p} = 0.874$, and $n = 500$. Round the answer to the thousandths.

Example 8 Evaluate the formula

$$E = t \cdot \frac{s}{\sqrt{n}}$$

when $t = 2.626$, $s = 0.93$, and $n = 200$. Round your answer to the nearest hundredth.

By now, you probably have the idea that we need to substitute and simplify!

$$E = t \cdot \frac{s}{\sqrt{n}} = 2.626 \cdot \frac{0.93}{\sqrt{200}} \approx 2.626 \cdot \frac{0.93}{14.14213562} \approx 0.1726882039 \approx 0.17$$

Answer $E \approx 0.17$

My Turn!

Evaluate the formula

$$E = t \cdot \frac{s}{\sqrt{n}}$$

when $t = 2.756$, $s = 0.64$, and $n = 30$. Round your answer to the nearest hundredth.

Example 9 Evaluate the formula

$$n = \frac{z^2 \cdot p(1-p)}{E^2}$$

when $z = 1.645$, $p = 0.38$, and $E = 0.04$. Round the answer up to the next whole number.

Once again, substitute and simplify. If you are doing this on your calculator, be really careful with the order of operations.

$$\begin{aligned} n &= \frac{z^2 \cdot p(1-p)}{E^2}. \\ &= \frac{1.645^2 \cdot 0.38(1-0.38)}{0.04^2} \\ &= \frac{1.645^2 \cdot 0.38(0.62)}{0.04^2} \\ &= \frac{2.706025 \cdot 0.38 \cdot 0.62}{0.0016} \\ &= \frac{0.63753949}{0.0016} \\ &\approx 398.4621813 \end{aligned}$$

Note the special rounding instructions for this problem. We are not following traditional rounding rules. If you don't get an exact whole number, you want to automatically go up to the next higher whole number. (You will find out why we do this when you learn about this formula in the textbook.)

So, for this problem we would round up to 399.

Answer $n = 399$

My Turn!

Evaluate the formula

$$n = \frac{z^2 \cdot p(1-p)}{E^2}$$

when $z = 1.96, p = 0.24$, and $E = 0.09$. Round the answer up to the next whole number.

Answers to My Turn!

1. 112.13
2. 11.16
3. 2.8
4. $(-3.5, 1.9)$
5. $0.63 < p < 0.92$
6. 0.27 ± 0.09
7. 0.024
8. 0.32
9. 87

Practice Problems

1. Let $0.45 < p < 0.85$ represent an interval on the number line. Find the value that is in the middle of the interval.
2. If $(2.57, 17.64)$ is an interval on the number line, find the value that is in the middle of the interval.
3. Let $(36.9, 42.4)$ represent an interval on the number line. Find the distance from the middle of the interval to either endpoint.
4. Express $3.5 < \mu < 11.9$ in interval notation.
5. Express $(0.63, 0.92)$ in inequality notation, assuming that the variable under study is μ.
6. Let $24.1 < \mu < 26.3$ represent an interval on the number line. Write the given interval in the format $m \pm E$.
7. Evaluate the formula

$$E = z \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

when $z = 1.96$, $\hat{p} = 0.526$, and $n = 300$. Round the answer to the thousandths.

8. Evaluate the formula

$$E = t \cdot \frac{s}{\sqrt{n}}$$

when $t = 2.678$, $s = 0.64$, and $n = 51$. Round your answer to the nearest hundredth.

9. Evaluate the formula

$$n = \frac{z^2 \cdot p(1-p)}{E^2}$$

when $z = 1.96, p = 0.37$, and $E = 0.08$. Round the answer up to the next whole number.