## Integrated Review 3: Getting Ready to Evaluate Statistical Formulas

## *Elementary Statistics* Chapter 3: Statistics for Describing, Exploring, and Comparing Data

Objectives:

1. Apply the order of operations.
2. Evaluate square roots.
3. Evaluate expressions and formulas.
4. Apply summation notation.

In this chapter of the Triola text, you will begin working with some basic statistical formulas that will be used to help describe data. In this integrated review chapter, we will review the concepts that will help you evaluate these statistical formulas.

**Objective 1: Apply the order of operations.**

We use the order of operations to simplify a numerical expression. You will have many numerical expressions that you will have to simplify in statistics, especially if you are evaluating formulas.

It is important that we do the operations in the correct order to arrive at a correct answer. It is critical that each step is completed from left to right.

Below you will find a listing of the order of operations. Always calculate the operations from left to right. This is especially important to remember for steps 3 and 4. This means that if there is a division to the left of multiplication in an expression, you would perform the division first.

1. **P**arentheses

2. **E**xponents

3. **M**ultiply or **D**ivide

4. **A**dd or **S**ubtract

**Example 1** Perform the indicated operations: $5 \cdot 8^3$.

| | |
|---|---|
| First we must calculate the exponent, since exponents should be done before multiplication according to the order of operations. (We reviewed calculating exponents in *Integrated Review* Chapter 1.) | $5 \cdot \boxed{8^3} = 5 \cdot \boxed{512}$ |
| Next, we multiply. | $5\boxed{\cdot}8^3 = 5\boxed{\cdot}512 = 2560$ |

**Answer** $5 \cdot 8^3 = 2560$

**My Turn!**

Perform the indicated operations: $6^4 \div 4$.

**Example 2** Perform the indicated operations: $15 \div 3 \cdot 2 + 9$.

| | |
|---|---|
| First, we must multiply or divide from left to right. Since the division is to the left of the multiplication, we must do that first. It is a common misconception to think that you always multiply before dividing. This is not the case; you complete the operations of multiplication and division as they appear from left to right. | $15 \boxed{\div} 3 \cdot 2 + 9 = 5 \cdot 2 + 9$ |
| Now, we multiply. | $5 \boxed{\cdot} 2 + 9 = 10 + 9$ |
| Finally, we perform addition. | $10 \boxed{+} 9 = 19$ |

**Answer** $15 \div 3 \cdot 2 + 9 = 19$

**My Turn!**

Perform the indicated operations.

$5 + 2 \cdot 8$

**Example 3**: Perform the indicated operations: $\frac{(5-3)^2+(7-3)^2+(9-3)^2}{7}$

| | |
|---|---|
| First, we must simplify within each set of parentheses. | $\frac{(5-3)^2+(7-3)^2+(9-3)^2}{7}=\frac{2^2+4^2+6^2}{7}$ |
| Now, calculate the exponents. | $=\frac{4+16+36}{7}$ |
| You can think of the entire numerator as being within an invisible set of parentheses. So, you would simplify that next. | $=\frac{56}{7}$ |
| Finally, you can perform the division (or reduce the fraction). | $=8$ |

**Answer** $\frac{(5-3)^2+(7-3)^2+(9-3)^2}{7}=8$

**My Turn!**

Perform the indicated operations: $\frac{10}{(5+3)^2+(8+3)^2}$.

Round the answer to the nearest tenth.

**Objective 2: Evaluate square roots.**

Recall the meaning of square root.

The principal square root of a number $a$ is denoted by $\sqrt{a}$. Furthermore, $x=\sqrt{a}$ if $x^2=a$. So, $3=\sqrt{9}$, since $3^2=9$.

Note that for the order of operations a square root can be treated as an exponent.

**Example 4** Evaluate $\sqrt{64}$.

We are essentially trying to find a number such that the number times itself is 64.

$$\sqrt{64}=\sqrt{8\cdot 8}=8$$

**Answer** $\sqrt{64}=8$

**My Turn!**

Evaluate $\sqrt{25}$.

If the value under the radical symbol (i.e. the radicand) is not a perfect square, you are going to have to try a different strategy. You may recall having to simplify radicals for algebra. However, for the way that you will be using square roots for statistics, you won't have to simplify square roots as you would in an algebra course. You will simply use a calculator for radicands that are not perfect squares. However, you may want to come up with an estimate of which two integers the square root is between in order to catch careless calculator entry errors.

**Example 5** Which two integers is $\sqrt{88}$ between?

You need figure out which two perfect squares 88 is between.

$$81 < 88 < 100$$

$$9^2 < 88 < 10^2$$

So,

$$9 < \sqrt{88} < 10$$

**Answer**: $\sqrt{88}$ is between 9 and 10.

**My Turn!**

Which two integers is $\sqrt{65}$ between?

**Example 6** Approximate $\sqrt{88}$ to the nearest hundredth.

Use a calculator to evaluate this, since 88 is not a perfect square.

For the TI-83/84 calculators, the keystrokes are 2nd $\sqrt{\ }$ 88 enter. Note that the $\sqrt{\ }$ is the second function for the $x^2$ button. If you need to find other roots, press MATH.

Your calculator output is approximately 9.3808. This rounds to 9.38, which agrees with our findings from Example 5, as it is between 9 and 10.

**Answer** $\sqrt{88} \approx 9.38$. The symbol $\approx$ means "is approximately equal to." We use this symbol to indicate that our value is not an exact answer, since it is a rounded value.

**My Turn!**

Approximate $\sqrt{130}$ to the nearest hundredth.

**Objective 3: Evaluate expressions and formulas.**

**Example 7** Perform the indicated operations:

$$\frac{18-11}{\frac{3}{\sqrt{9}}}$$

In order to simplify this expression, we will apply the order of operations to an expression that contains a square root. We treat a radical symbol as an exponent in the order of operations. We can do this because radicals can be rewritten as rational exponents (e.g. $\sqrt{9}=9^{\frac{1}{2}}$ ).

| | |
|---|---|
| We can begin by viewing the numerator 18 – 11 as being contained within invisible parentheses and the denominator $\frac{3}{\sqrt{9}}$ as being contained within another set of parentheses. We can work on simplifying each of these. The simplification of the numerator is shown first. | $\frac{(18-11)}{\left(\frac{3}{\sqrt{9}}\right)}=\frac{7}{\left(\frac{3}{\sqrt{9}}\right)}$ |
| To simplify the denominator $\frac{3}{\sqrt{9}}$, we must find $\sqrt{9}$. | $\frac{7}{\left(\frac{3}{\sqrt{9}}\right)}=\frac{7}{\left(\frac{3}{3}\right)}=\frac{7}{1}$ |
| We find our final answer by dividing (or simplifying the fraction). | $\frac{7}{1}=7$ |

**Answer** $\frac{18-11}{\frac{3}{\sqrt{9}}}=7$

**My Turn!**

Perform the indicated operations:

$$\frac{6.8-6.1}{\frac{1.1}{\sqrt{16}}}$$

Round the answer to the nearest hundredth.

A formula is a special type of equation that expresses the relationship between several variables. On one side of the equation (usually the left side), you have the dependent variable. On the other side you have the independent variables. To evaluate a formula, we substitute known values for the dependent variables and follow the order of operations to simplify. We will include one example here, and you will see numerous examples in the chapters that follow.

**Example 8** Evaluate the formula

$$E = z \cdot \frac{\sigma}{\sqrt{n}}$$

for $z = 1.96$, $\sigma = 2.1$, and $n = 100$. Round the answer to the nearest thousandth.

First, we must substitute the given values for the independent variables.

$$E = z \cdot \frac{\sigma}{\sqrt{n}} = 1.96 \cdot \frac{2.1}{\sqrt{100}}$$

In order to simplify the right-hand side, we will apply the order of operations to an expression that contains a square root.

| | |
|---|---|
| We treat a radical symbol as an exponent in the order of operations. So, we will begin there. The square root of 100 is 10. | $E = 1.96 \cdot \frac{2.1}{\sqrt{100}} = 1.96 \cdot \frac{2.1}{10}$ |
| We can simplify the fraction by dividing by 10. | $E = 1.96 \cdot \frac{2.1}{10} = 1.96 \cdot 0.21$ |
| We find our value for the dependent variable, *E*, by multiplying. | $E = 1.96 \cdot 0.21 = 0.4116$ |
| Now, we round our final answer to the nearest thousandth. Since the value in the ten thousandth place is a 6, we round to 0.412. | $E \approx 0.412$ |

**Answer** $E \approx 0.412$ when $E = z \cdot \frac{\sigma}{\sqrt{n}}$ and $z = 1.96$, $\sigma = 2.1$, and $n = 100$.

**My Turn!**

Evaluate the formula

$$n = \frac{z^2 \cdot \hat{p} \cdot \hat{q}}{E^2}$$

For $z = 2.575$, $\hat{p} = 0.3$, $\hat{q} = 0.7$, and $E = 0.05$. Round the answer to the nearest whole number.

**Objective 4: Apply summation notation.**

You will often see the summation symbol, $\Sigma$, in statistical formulas. It is the capital Greek letter "sigma." Whenever you see this symbol in a formula, it is telling you to add up values.

For instance, $\bar{x} = \dfrac{\Sigma x}{n}$ is the formula for finding the mean (average), $\bar{x}$, where $x$ represents the data values and $n$ is the number of data values.

**Example 9** Find $\bar{x} = \dfrac{\Sigma x}{n}$ for the values in the accompanying table.

| $x$ |
|---|
| 34 |
| 48 |
| 25 |
| 76 |
| 22 |
| 29 |

There are 6 values for $x$, so $n = 6$.

The formula is telling us to add up the values of $x$ and divide by $n$.

$$\bar{x} = \frac{\Sigma x}{n} = \frac{34+48+25+76+22+29}{6} = \frac{234}{6} = 39$$

Note that the above formula could also be written as

$$\sum_{i=1}^{6} \frac{x_i}{n}$$

**Answer** $\bar{x} = 39$

**My Turn!**

Find $\bar{x} = \dfrac{\Sigma x}{n}$ for the values in the accompanying table, where $n$ is the number of given values. Round to the nearest tenth, as needed.

| $x$ |
|---|
| 5 |
| 4 |
| 8 |
| 7 |
| 2 |

**Example 10** Find

$$\frac{\Sigma(x-39)^2}{n-1}$$

for the accompanying table and $n = 6$.

| $x$ |
|---|
| 34 |
| 48 |
| 25 |
| 76 |
| 22 |
| 29 |

This notation is telling us to subtract 39 from each $x$ value first, since we work out parentheses first in the order of operations. Next we would calculate exponents per the order of operations. Then, we sum up these squares of these differences. Once we have this sum, then we can perform division by the denominator.

$$\frac{\Sigma(x-39)^2}{n-1} = \frac{(34-39)^2+(48-39)^2+(25-39)^2+(76-39)^2+(22-39)^2+(29-39)^2}{6-1}$$

$$= \frac{(-5)^2+(9)^2+(-14)^2+(37)^2+(-17)^2+(-10)^2}{6-1}$$

$$= \frac{25+81+196+1369+289+100}{5}$$

$$= \frac{2060}{5}$$

$$= 412$$

**Answer** $\frac{\Sigma(x-39)^2}{n-1} = 412$ for the given table of values.

**My Turn!**

Find

$\frac{\Sigma(x-2)^2}{n-1}$ for the accompanying table and $n = 4$. Round to the nearest tenth, as needed.

| $x$ |
|---|
| 4 |
| 3 |
| 2 |
| 1 |

**Answer to My Turn!**

1. 324
2. 21
3. 0.1
4. 5
5. 8 and 9
6. 11.4
7. 2.55
8. $n \approx 557$
9. $\bar{x} = 5.2$
10. 2

**Practice Problems**

1. Perform the indicated operations: $17-3\cdot4$.
2. Perform the indicated operations: $5+3^4\div9$.
3. Perform the indicated operations: $\dfrac{4^2}{(9-2)^2+(8-2)^2}$. Round the answer to the nearest hundredth.
4. Evaluate $\sqrt{36}+\sqrt{100}$.
5. Which two integers is $\sqrt{101}$ between?
6. Approximate $\sqrt{255}$ to the nearest hundredth.
7. Perform the indicated operations. Round the answer to the nearest hundredth.
   $$\frac{0.9-0.8}{\sqrt{\frac{0.8\cdot0.2}{50}}}$$
8. Evaluate the formula $\sigma=\sqrt{n\cdot p\cdot(1-p)}$ for $n=60$ and $p=0.25$. Round the answer to the nearest tenth.
9. Find $\dfrac{\Sigma x}{n}$ for the values in the given table, where $n$ is the number of given values. Round to answer to the nearest tenth.
10. Find $\dfrac{\Sigma(x-2)^2}{n-1}$ for the values in the given table and $n=4$. Round the answer to the nearest tenth.

| $x$ |
|---|
| 8 |
| 7 |
| 2 |
| 9 |