Integrated Review 6: The Building Blocks for Working with Normal Distributions

Elementary Statistics Chapter 6: Normal Probability Distributions

Objectives:

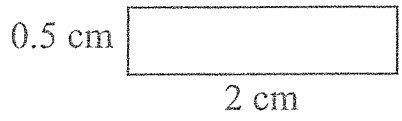
1. Find area.
2. Interpret inequality notation.
3. Evaluate formulas for normal probability distributions.

In this chapter of the Triola text, you will begin working with normal distributions (bell-shaped curves). You will be finding the area under portions of the normal curve. We will start this chapter of the integrated review by briefly looking at the area of a rectangle. Then, we will go over some of the basics for inequality notation. Finally, we will practice evaluating the formulas you will be using in the related chapter.

Objective 1: Find area.

In the chapter in the text on normal distributions, we will be finding area and associating it with probability. For the topic of uniform distributions, you will be finding area under rectangles. So, we will take a moment to review finding the area of a rectangle.

Example 1 Find the area of the rectangle shown.



The area of a rectangle is given by base×height (or length×width). We can write this as a formula.

$$A = b \cdot h$$

For the above rectangle, the base is 2 cm and the height is 0.5 cm.

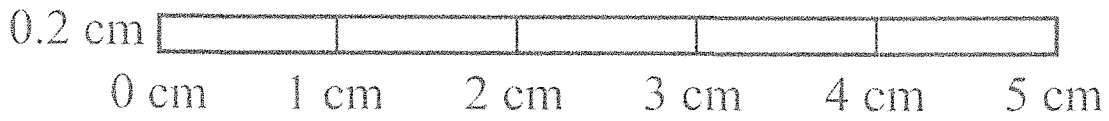
We can plug these values into the formula.

$$A = b \cdot h = 2\,\text{cm} \cdot 0.5\,\text{cm} = 1\,\text{cm}^2$$

Answer The area is 1 cm^2.

My Turn!

Find the area of the shaded portion of the rectangle.



Objective 2: Interpret inequality notation.

In the related section of the textbook, we will be dealing with translating intervals in context to notation that we can use for calculating purposes. Also, you will want to be comfortable with indicating which integers would be located within an interval. The following table includes a column for inequality notation, a column with the inequality expressed in words, and a third column that indicates which integers from 0 to 10 satisfy the inequality from the first column.

Notation	**Translation**	$X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$x < 5$	x is less than 5	0, 1, 2, 3, 4
$x > 7$	x is greater than 7	8, 9, 10
$2 < x < 6$	x is greater than 2 and less than 6	3, 4, 5
$x \leq 4$	x is less than or equal to 4	0, 1, 2, 3, 4
$x \geq 9$	x is greater than or equal to 9	9, 10

Example 2 Which integers from 0 to 10 satisfy the following inequality?

$$x < 7$$

This translates to "x is less than 7." The integers that are less than 7 include 0, 1, 2, 3, 4, 5, and 6.

Answer The integers from 0 to 10 that satisfy $x < 7$ are 0, 1, 2, 3, 4, 5, and 6.

My Turn!

Which integers from 5 to 15 satisfy the following inequality?

x is greater than or equal to 10.

There are some phrases that you may encounter as part of the text that will translate into inequalities. The following table includes a sampling of these.

Phrase	**Inequality**
At least 5 puppies	$x \geq 5$
No more than 5 puppies	$x \leq 5$
Fewer than 5 puppies	$x < 5$
More than 5 puppies	$x > 5$
Exactly 5 puppies	$x = 5$

There are other possibilities beyond the phrases used in the above table. The more problems you practice, the more comfortable you will be become with the various ways to describe inequalities.

Example 3 Translate the following phrases into inequalities.

a) A heart rate of at least 80 bpm

We can locate the phrase of *at least* in the table and see that it implies $\geq$. If a heart rate is at least 80 bpm, it means that it can be 80, 81, 82, ...bpm.
Letting h represent heart rate, we can translate the phrase into $h \geq 80$.

b) A speed of no more than 55 mph

We can locate the phrase *no more than* in the second row of the table and see that it translates to $\leq$. If the speed is no more than 55 mph, it means that it can be anywhere from 0 mph up to and including 55 mph.
If we let s represent speed, the phrase can be rewritten as $s \leq 55$.

c) A temperature below 98.6 degrees Fahrenheit

The phrase *below* is not contained within the table. However, if we think about it, a temperature below 98.6 degrees means that it can be any value less than 98.6, not including 98.6.
Therefore, using t for temperature, our translation becomes $t < 98.6$.

Answer a) $h \geq 80$ b) $s \leq 55$ c) $t < 98.6$

My Turn!

Translate the following phrases into inequalities.

a) A weight of no more than 180 pounds.
b) A height of at least 60 inches
c) Exactly 9 doors in a house

Objective 3: Evaluating formulas for normal probability distributions.

You will be learning about several formulas related to the normal distribution in the corresponding section of the textbook. Here, we will practice evaluating these formulas. You will learn the meaning of the formulas and how to apply them in context within the text section itself.

Example 4 Evaluate $z = \frac{x - \mu}{\sigma}$ if $x = 21.3$, $\mu = 23.5$, and $\sigma = 2.1$. Round the answer to the nearest hundredth.

First, we must substitute for the variables in the correct location. Then, we must simplify the numerator. (This is because you can envision the entire numerator within parentheses, and the order of operations dictates that parentheses must be simplified first.) Next, we will divide, and then we will round our final answer.

$$z = \frac{21.3 - 23.5}{2.1} = \frac{-2.2}{2.1} \approx -1.047 \approx -1.05$$

Answer $z \approx -1.05$

My Turn!

Evaluate $z = \frac{x - \mu}{\sigma}$ if $x = 10.3$, $\mu = 10.2$, and $\sigma = 1.3$. Round the answer to the nearest hundredth.

Example 5 Evaluate $x = \mu + z \cdot \sigma$ if $\mu = 6.7$, $z = 1.96$, and $\sigma = 0.9$. Round the answer to the nearest tenth.

First, we must substitute for the variables in the correct location. Then, we will follow the order of operations for the right-hand side. So, we will have to multiply before adding.

$$x = \mu + z \cdot \sigma = 6.7 + 1.96 \cdot 0.9 = 6.7 + 1.764 = 8.464 \approx 8.5$$

Answer $x \approx 8.5$

My Turn!

Evaluate $x = \mu + z \cdot \sigma$ if $\mu = 9.8$, $z = 2.575$, and $\sigma = 0.3$. Round the answer to the nearest tenth.

Example 6 Solve for x when $z = \dfrac{x - \mu}{\sigma}$ and $z = 2.15$, $\mu = 3.5$, and $\sigma = 0.3$. Round the answer to the nearest tenth.

We need to substitute 2.15 for z to the left of the equal sign, and 3.5 for μ and 0.3 for σ on the right-hand side.

$$2.15 = \frac{x - 3.5}{0.3}$$

Now, we need to solve for x. The first step is to multiply both sides of the equation by 0.3 to eliminate the fraction.

$$2.15(0.3) = \frac{(x - 3.5)}{0.3}(0.3)$$
$$0.645 = x - 3.5$$

Add 3.5 to both sides to isolate x.

$$0.645 + 3.5 = x - 3.5 + 3.5$$

$$4.145 = x$$

Now, we must round our final answer to the nearest tenth and $x \approx 4.1$.

Answer $x \approx 4.1$

My Turn!

Solve for x when $z = \dfrac{x - \mu}{\sigma}$ and $z = -0.34$, $\mu = 21.5$, and $\sigma = 0.8$. Round the answer to the nearest tenth.

Example 7 Evaluate

$$z = \frac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$$

if $\bar{x} = 9.8, \mu = 25.6, \sigma = 1.9$, and $n = 49$. Round your answer to the nearest hundredth.

We need to substitute 9.8 for $\bar{x}$ and 25.6 for μ in the numerator, and 1.9 for σ and 49 for n in the denominator.

$$z = \frac{9.8 - 25.6}{\frac{1.9}{\sqrt{49}}}$$

We can then simplify the right-hand side using the order of operations. You may want to visualize an invisible set of parentheses around the numerator and another invisible set of parentheses around the denominator.

$$\begin{aligned} z &= \frac{9.8 - 25.6}{\frac{1.9}{\sqrt{49}}} \\ &= \frac{(9.8 - 25.6)}{\left(\frac{1.9}{\sqrt{49}}\right)} \\ &= \frac{-15.8}{\left(\frac{1.9}{\sqrt{49}}\right)} \\ &= \frac{-15.8}{\left(\frac{1.9}{7}\right)} \\ &= \frac{-15.8}{0.27142857} \\ &= -58.2105266 \end{aligned}$$

Finally, we must round our final answer to the hundredth, which gives us $z \approx -58.21$.

Answer $z \approx -58.21$

My Turn!

Evaluate

$$z = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

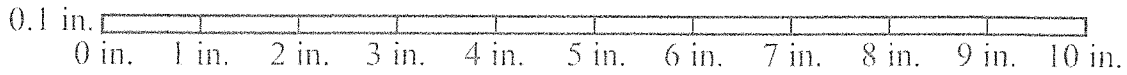
if $\overline{x} = 7.2, \mu = 5.8, \sigma = 1.2,$ and $n = 81$. Round the answer to the nearest hundredth.

Answers to My Turn!

1. 0.2 in^2
2. 10, 11, 12, 13, 14, 15
3. a) $w \leq 180$ b) $h \geq 60$ c) $d = 9$
4. $z \approx 0.08$
5. $x \approx 10.6$
6. $x \approx 21.2$
7. $z = 10.5$

Practice Problems

1. Find the area of the shaded region of the following rectangle.



2. Which integers from 10 to 20 satisfy the following inequality? $x < 12$
3. Translate the following phrases into inequalities.
   A length of more than 18 inches
   A volume of less than 60 ft^3
4. Evaluate $z = \frac{x - \mu}{\sigma}$ if $x = 67.8$, $\mu = 70.2$, and $\sigma = 2.3$. Round the answer to the nearest hundredth.
5. Evaluate $x = \mu + z \cdot \sigma$ if $\mu = 78$, $z = 1.96$ and $\sigma = 10.3$. Round the answer to the nearest tenth
6. Solve for x when $z = \frac{x - \mu}{\sigma}$ and $z = 1.47$, $\mu = 99.5$ and $\sigma = 4.8$. Round the answer to the nearest tenth.
7. Evaluate
   $z = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$ if $\overline{x} = 54.2, \mu = 52.6, \sigma = 3.2$ *and* $n = 64$. Round the answer to the nearest hundredth.