Integrated Review 8: Formulas for Hypothesis Testing

Elementary Statistics Chapter 8: Hypothesis Testing

Objective:

1. Evaluate formulas for hypothesis testing.

You will be learning about several formulas for hypothesis testing in the related section of the textbook. Here, we will practice evaluating these formulas. You will learn the meaning of the formulas and how to apply them in context within the text section itself.

Objective 1: Evaluate formulas for hypothesis testing.

Example 1 Evaluate the formula

$$z = \frac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}}$$

when $\hat{p} = \frac{x}{n}$, $x = 80$, $n = 100$, $p = 0.75$, and $q = 1 - p$. Round your answer to the nearest hundredth.

First we need to figure out the value for $\hat{p}$ so that we can substitute in that value. Since, $\hat{p} = \frac{x}{n}$, with $x = 80$ and $n = 100$, we get

$$\hat{p} = \frac{x}{n} = \frac{80}{100} = 0.8$$

We may also want to find the value of q. Since $q = 1 - p$ and $p = 0.75$ (Be really careful with p and $\hat{p}$, as they are indeed different), we get
$q = 1 - p = 1 - 0.75 = 0.25$

Now, we can substitute in values for all of the variables and solve for z.

$$z = \frac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}} = \frac{0.8 - 0.75}{\sqrt{\dfrac{0.75 \cdot 0.25}{100}}} = \frac{0.05}{\sqrt{\dfrac{0.1875}{100}}} = \frac{0.05}{\sqrt{0.001875}} \approx \frac{0.05}{0.0433} \approx 1.1547$$

Now, we will round our value for z to the nearest hundredth.

$$z \approx 1.15$$

Answer $z \approx 1.15$

My Turn!

Evaluate the formula

$$z = \frac{\hat{p} - p}{\sqrt{\dfrac{p \cdot q}{n}}}$$

when $\hat{p} = \frac{x}{n}$, $x = 90$, $n = 120$, $p = 0.35$ and $q = 1 - p$. Round your answer to the nearest hundredth.

Example 2 Evaluate the formula

$$t = \frac{\overline{x} - \mu}{\dfrac{s}{\sqrt{n}}}$$

when $\mu = 99.2$, $\overline{x} = 99.5$, $n = 100$, and $s = 1.3$. Round your answer to the nearest hundredth.

Begin by substituting the given values for the variables in the formula. Then, follow the order of operations to simplify the right hand side.

$$t = \frac{\overline{x} - \mu}{\dfrac{s}{\sqrt{n}}} = \frac{99.5 - 99.2}{\dfrac{1.3}{\sqrt{100}}} = \frac{0.3}{\dfrac{1.3}{\sqrt{100}}} = \frac{0.3}{\dfrac{1.3}{10}} = \frac{0.3}{0.13} \approx 2.3077 \approx 2.31$$

Note that if 100 were not a perfect square, you would want to use your calculator to find the value of the square root.

Answer $t \approx 2.31$

My Turn!

Evaluate the formula

$$t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$$

when $\mu = 16$, $\overline{x} = 17.5$, $n = 80$, and $s = 2.4$. Round your answer to the nearest hundredth.

Example 3 Evaluate the formula

$$\chi^2 = \frac{(n-1) \cdot s^2}{\sigma^2}$$

when $\sigma = 3.9$, $n = 50$, and $s = 3.67$.

Round your answer to the nearest tenth.

Note that χ^2 (chi-squared) is considered as an entity. You do not approach this problem trying to solve for χ. You strive to get a value for χ^2. That is, $\chi^2 = \square$.

We can go straight to substituting the given values in for the variables and simplifying.

$$\begin{aligned}\chi^2 &= \frac{(n-1)\cdot s^2}{\sigma^2} \\ &= \frac{(50-1)\cdot 3.67^2}{3.9^2} \\ &= \frac{(49)\cdot 3.67^2}{3.9^2} \\ &= \frac{(49)\cdot 13.4689}{15.21} \\ &= \frac{659.9761}{15.21} \\ &\approx 43.3909 \\ &\approx 43.4\end{aligned}$$

Answer $\chi^2 \approx 43.4$

My Turn!

Evaluate the formula

$$\chi^2 = \frac{(n-1)\cdot s^2}{\sigma^2}$$

when $\sigma = 4.5$, $n = 60$, and $s = 4.6$. Round your answer to the nearest tenth.

Answers to My Turn!

1. $z \approx 9.19$
2. $t \approx 5.59$
3. $\chi^2 \approx 61.7$

Practice Problems

1. Evaluate the formula

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$

when $\hat{p} = \frac{x}{n}$, $x = 90$, $n = 150$, $p = 0.82$, and $q = 1 - p$. Round your answer to the nearest hundredth.

2. Evaluate the formula

$$t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$$

when $\mu = 44$, $\overline{x} = 45.2$, $n = 80$, and $s = 3.1$. Round your answer to the nearest hundredth.

3. Evaluate the formula

$$\chi^2 = \frac{(n-1) \cdot s^2}{\sigma^2}$$

when $\sigma = 11.3$, $n = 160$, and $s = 11.4$. Round your answer to the nearest tenth.