Integrated Review 8: Formulas for Hypothesis Testing

Elementary Statistics Chapter 8: Hypothesis Testing

Objective:

1. Evaluate formulas for hypothesis testing.

You will be learning about several formulas for hypothesis testing in the related section of the textbook. Here, we will practice evaluating these formulas. You will learn the meaning of the formulas and how to apply them in context within the text section itself.

Objective 1: Evaluate formulas for hypothesis testing.

Example 1 Evaluate the formula

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$

when $\hat{p} = \frac{x}{n}$, $x = 80$, $n = 100$, $p = 0.75$, and $q = 1 - p$. Round your answer to the nearest hundredth.

First we need to figure out the value for $\hat{p}$ so that we can substitute in that value. Since, $\hat{p} = \frac{x}{n}$, with $x = 80$ and $n = 100$, we get

$$\hat{p} = \frac{x}{n} = \frac{80}{100} = 0.8$$

We may also want to find the value of q. Since $q = 1 - p$ and $p = 0.75$ (Be really careful with p and $\hat{p}$, as they are indeed different), we get
$q = 1 - p = 1 - 0.75 = 0.25$

Now, we can substitute in values for all of the variables and solve for z.

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}} = \frac{0.8 - 0.75}{\sqrt{\frac{0.75 \cdot 0.25}{100}}} = \frac{0.05}{\sqrt{\frac{0.1875}{100}}} = \frac{0.05}{\sqrt{0.001875}} \approx \frac{0.05}{0.0433} \approx 1.1547$$

Now, we will round our value for z to the nearest hundredth.

$$z \approx 1.15$$

Answer $z \approx 1.15$

My Turn!

Evaluate the formula

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$

when $\hat{p} = \frac{x}{n}$, $x = 90$, $n = 120$, $p = 0.35$ and $q = 1 - p$. Round your answer to the nearest hundredth.

Example 2 Evaluate the formula

$$t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$$

when $\mu = 99.2$, $\overline{x} = 99.5$, $n = 100$, and $s = 1.3$. Round your answer to the nearest hundredth.

Begin by substituting the given values for the variables in the formula. Then, follow the order of operations to simplify the right hand side.

$$t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{99.5 - 99.2}{\frac{1.3}{\sqrt{100}}} = \frac{0.3}{\frac{1.3}{\sqrt{100}}} = \frac{0.3}{\frac{1.3}{10}} = \frac{0.3}{0.13} \approx 2.3077 \approx 2.31$$

Note that if 100 were not a perfect square, you would want to use your calculator to find the value of the square root.

Answer $t \approx 2.31$

My Turn!

Evaluate the formula

$$t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$$

when $\mu = 16$, $\overline{x} = 17.5$, $n = 80$, and $s = 2.4$. Round your answer to the nearest hundredth.

Example 3 Evaluate the formula

$$\chi^2 = \frac{(n-1) \cdot s^2}{\sigma^2}$$

when $\sigma = 3.9$, $n = 50$, and $s = 3.67$.

Round your answer to the nearest tenth.

Note that χ^2 (chi-squared) is considered as an entity. You do not approach this problem trying to solve for χ. You strive to get a value for χ^2. That is, $\chi^2 =$.

We can go straight to substituting the given values in for the variables and simplifying.

$$\begin{aligned}\chi^2 &= \frac{(n-1)\cdot s^2}{\sigma^2} \\ &= \frac{(50-1)\cdot 3.67^2}{3.9^2} \\ &= \frac{(49)\cdot 3.67^2}{3.9^2} \\ &= \frac{(49)\cdot 13.4689}{15.21} \\ &= \frac{659.9761}{15.21} \\ &\approx 43.3909 \\ &\approx 43.4\end{aligned}$$

Answer $\chi^2 \approx 43.4$

My Turn!

Evaluate the formula

$$\chi^2 = \frac{(n-1)\cdot s^2}{\sigma^2}$$

when $\sigma = 4.5$, $n = 60$, and $s = 4.6$. Round your answer to the nearest tenth.

Answers to My Turn!

1. $z \approx 9.19$
2. $t \approx 5.59$
3. $\chi^2 \approx 61.7$

Practice Problems

1. Evaluate the formula

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$

when $\hat{p} = \frac{x}{n}$, $x = 90$, $n = 150$, $p = 0.82$, and $q = 1 - p$. Round your answer to the nearest hundredth.

2. Evaluate the formula

$$t = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}}$$

when $\mu = 44$, $\overline{x} = 45.2$, $n = 80$, and $s = 3.1$. Round your answer to the nearest hundredth.

3. Evaluate the formula

$$\chi^2 = \frac{(n-1) \cdot s^2}{\sigma^2}$$

when $\sigma = 11.3$, $n = 160$, and $s = 11.4$. Round your answer to the nearest tenth.

Integrated Review 9: Formulas for Two-Sample Hypothesis Testing

Elementary Statistics Chapter 9: Inferences from Two-Samples

Objective:

1. Evaluate formulas for two-sample hypothesis testing.

You will be learning more formulas for hypothesis testing in the related section of the textbook. Once again, we will practice evaluating these formulas here. You will learn the meaning of the formulas and how to apply them in context within the text section itself.

Objective 1: Evaluate formulas for two-sample hypothesis testing.

Example 1 Evaluate the formula

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

for $x_1 = 15$, $x_2 = 18$, $n_1 = 35$, and $n_2 = 25$. Round your answer to the nearest ten thousandth.

First, we substitute the given values in for the variables in the formula. Notice that the variables with x and n have subscripts. The variables x_1 and x_2 represent two different unknowns. So, it is critical that we substitute the appropriate values into the variables with the correct subscripts.

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{15 + 18}{35 + 25}$$

Once we have completed substituting, we apply the order of operations to the right-hand side. We can group the numerator and the denominator in two sets of parentheses to remind us to individually simplify each of them first by adding. Then, we divide the numerator by the denominator to change the fraction to a decimal. No rounding of the final answer is necessary in this problem.

$$\bar{p} = \frac{(15 + 18)}{(35 + 25)} = \frac{33}{60} = 0.55$$

Answer $\bar{p} = 0.55$

My Turn!

Evaluate the formula

$$\overline{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

for $x_1 = 22$, $x_2 = 18$, $n_1 = 40$, and $n_2 = 30$. Round your answer to the nearest ten thousandth.

Example 2 Evaluate the formula

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\overline{p} \cdot \overline{q}}{n_1} + \frac{\overline{p} \cdot \overline{q}}{n_2}}}$$

if $p_1 - p_2 = 0$, $x_1 = 40$, $x_2 = 60, n_1 = 80, n_2 = 100$, $\hat{p}_1 = \frac{x_1}{n_1}$, $\hat{p}_2 = \frac{x_2}{n_2}$, $\overline{p} = \frac{x_1 + x_2}{n_1 + n_2}$, and $\overline{q} = 1 - \overline{p}$.

Round your answer to the nearest hundredth.

We have a lot of variables to substitute for in the formulas. We even have formulas to find out the values for some of these variables! Let's begin by using the formulas for the variables that we need to plug into the formula for z. We will find numerical values for $\hat{p}_1$, $\hat{p}_2$, $\overline{p}$, and $\overline{q} = 1 - \overline{p}$.

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{40}{80} = 0.5$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{60}{100} = 0.6$$

$$\overline{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{40 + 60}{80 + 100} = \frac{100}{180} = 0.5556$$

$$\overline{q} = 1 - \overline{p} = 1 - 0.5556 = 0.4444$$

As you can see, we round $\overline{p}$ and $\overline{q}$ to the nearest ten thousandth even though we are rounding our final answer to the nearest hundredth. We keep a few extra places in intermediate steps to minimize rounding error. Ideally, we would try to complete all the calculations at one time using technology so that there is no intermediate rounding. However, when that is not possible, an extra two places should generally be sufficient.

Now, we are ready to substitute into the formula for z and simplify. Note that entire expression under the radical symbol can be thought of as being inside parentheses. This means we cannot take a square root until we have simplified the entire expression under the $\sqrt{\ }$.

$$
\begin{aligned}
z &= \frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{\overline{p}\cdot\overline{q}}{n_1}+\frac{\overline{p}\cdot\overline{q}}{n_2}}} \\
&= \frac{(0.5-0.6)-0}{\sqrt{\frac{0.5556\cdot 0.4444}{80}+\frac{0.5556\cdot 0.4444}{100}}} \\
&= \frac{-0.1}{\sqrt{\frac{0.5556\cdot 0.4444}{80}+\frac{0.5556\cdot 0.4444}{100}}} \\
&\approx \frac{-0.1}{\sqrt{\frac{0.2469}{80}+\frac{0.2469}{100}}} \\
&\approx \frac{-0.1}{\sqrt{0.0031+0.0025}} \\
&= \frac{-0.1}{\sqrt{0.0056}} \\
&\approx \frac{-0.1}{0.0748} \\
&\approx -1.3369 \\
&\approx -1.34
\end{aligned}
$$

Answer $z \approx -1.34$

My Turn!

Evaluate the formula

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\overline{p} \cdot \overline{q}}{n_1} + \frac{\overline{p} \cdot \overline{q}}{n_2}}}$$

if $p_1 - p_2 = 0$, $x_1 = 40$, $x_2 = 30, n_1 = 60, n_2 = 80,$ $\hat{p}_1 = \frac{x_1}{n_1}$, $\hat{p}_2 = \frac{x_2}{n_2}$, $\overline{p} = \frac{x_1 + x_2}{n_1 + n_2}$, and $\overline{q} = 1 - \overline{p}$.

Round your answer to the nearest hundredth.

Example 3 Evaluate

$$E = z \cdot \sqrt{\frac{\hat{p}_1 \cdot \hat{q}_1}{n_1} + \frac{\hat{p}_2 \cdot \hat{q}_2}{n_2}}$$

if $z = 1.96$, $x_1 = 50$, $x_2 = 60, n_1 = 70, n_2 = 100$, $\hat{p}_1 = \frac{x_1}{n_1}, \hat{p}_2 = \frac{x_2}{n_2}$, $\hat{q}_1 = 1 - \hat{p}_1$, and $\hat{q}_2 = 1 - \hat{p}_2$.

Round your answer to 3 significant digits.

Let's begin by using the formulas for the variables that we need to plug into the formula for E. Let's find numerical values for $\hat{p}_1$, $\hat{p}_2$, $\hat{q}_1$, and $\hat{q}_2$. Throughout this solution, we will round to five significant digits (two more significant digits than what is desired in our final answer is normally adequate) in intermediate steps to minimize rounding error. Recall that the leading zeros don't count as significant digits.

$$\hat{p}_1 = \frac{x_1}{n_1} = \frac{50}{70} \approx 0.71429$$

$$\hat{p}_2 = \frac{x_2}{n_2} = \frac{60}{100} = 0.6$$

$$\hat{q}_1 = 1 - \hat{p}_1 \approx 1 - 0.71429 = 0.28571$$

$$\hat{q}_2 = 1 - \hat{p}_2 = 1 - 0.6 = 0.4$$

Since we now have numbers for each of the variables on the right-hand side, we can plug them into the formula for E. Then, we follow the order of operations on the right-hand side to simplify it.

$$\begin{aligned} E &= z \cdot \sqrt{\frac{\hat{p}_1 \cdot \hat{q}_1}{n_1} + \frac{\hat{p}_2 \cdot \hat{q}_2}{n_2}} \\ &\approx 1.96\sqrt{\frac{0.71429 \cdot 0.28571}{70} + \frac{0.6 \cdot 0.4}{100}} \\ &\approx 1.96\sqrt{\frac{0.20408}{70} + \frac{0.24}{100}} \\ &\approx 1.96\sqrt{0.0029154 + 0.0024} \\ &= 1.96\sqrt{0.0053154} \\ &\approx 1.96 \cdot 0.072907 \\ &\approx 0.14290 \\ &\approx 0.143 \end{aligned}$$

Answer $E \approx 0.143$

My Turn!

Evaluate

$$E = z \cdot \sqrt{\frac{\hat{p}_1 \cdot \hat{q}_1}{n_1} + \frac{\hat{p}_2 \cdot \hat{q}_2}{n_2}}$$

if $z = 2.575$, $x_1 = 32$, $x_2 = 33, n_1 = 50, n_2 = 60,\ \hat{p}_1 = \frac{x_1}{n_1}, \hat{p}_2 = \frac{x_2}{n_2},\ \hat{q}_1 = 1 - \hat{p}_1$, and $\hat{q}_2 = 1 - \hat{p}_2$.

Round your answer to 3 significant digits.

Example 4 Evaluate the formula

$$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^{\,2}}{n_1}+\frac{s_2^{\,2}}{n_2}}}$$

for $\bar{x}_1=210, \bar{x}_2=190$, $\mu_1-\mu_2=0$, $s_1=11$, $s_2=7$, $n_1=45$, and $n_2=60$. Round your answer to the nearest hundredth.

We can begin this problem by immediately substituting the given values for the variables. Once again, we use the order of operations to guide us in simplifying the right-hand side. Be careful not to evaluate the square root until you have simplified the entire expression under the $\sqrt{\ }$.

$$\begin{aligned}
t&=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^{\,2}}{n_1}+\frac{s_2^{\,2}}{n_2}}}\\
&=\frac{(210-190)-(0)}{\sqrt{\frac{11^2}{45}+\frac{7^2}{60}}}\\
&=\frac{20}{\sqrt{\frac{11^2}{45}+\frac{7^2}{60}}}\\
&=\frac{20}{\sqrt{\frac{121}{45}+\frac{49}{60}}}\\
&\approx\frac{20}{\sqrt{2.6889+0.8167}}\\
&=\frac{20}{\sqrt{3.5056}}\\
&\approx\frac{20}{1.8723}\\
&\approx 10.6820\\
&\approx 10.68
\end{aligned}$$

Answer $t\approx 10.68$

My Turn!

Evaluate the formula

$$t = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

for $\overline{x}_1 = 68, \overline{x}_2 = 64$, $\mu_1 - \mu_2 = 0$, $s_1 = 4$, $s_2 = 3$, $n_1 = 80$, and $n_2 = 90$.

Round your answer to the nearest hundredth.

Example 5 Evaluate

$$E = t \cdot \sqrt{\frac{s_1^{\ 2}}{n_1} + \frac{s_2^{\ 2}}{n_2}}$$

if $t = 2.449, s_1 = 1.5, s_2 = 1.8, n_1 = 50, \text{and } n_2 = 40$. Round your answer to the nearest tenth.

By now, you know the routine; substitute and simplify!

$$\begin{aligned}
E &= t \cdot \sqrt{\frac{s_1^{\ 2}}{n_1} + \frac{s_2^{\ 2}}{n_2}} \\
&= 2.449 \cdot \sqrt{\frac{1.5^2}{50} + \frac{1.8^2}{40}} \\
&= 2.449 \cdot \sqrt{\frac{2.25}{50} + \frac{3.24}{40}} \\
&= 2.449 \cdot \sqrt{0.045 + 0.081} \\
&= 2.449 \cdot \sqrt{0.126} \\
&\approx 2.449 \cdot 0.3550 \\
&\approx 0.8694 \\
&\approx 0.9
\end{aligned}$$

Answer $E \approx 0.9$

My Turn!

Evaluate $E = t \cdot \sqrt{\frac{s_1^{\ 2}}{n_1} + \frac{s_2^{\ 2}}{n_2}}$ if $t = 1.99, s_1 = 0.6, s_2 = 0.8, n_1 = 30, \text{and } n_2 = 50$. Round your answer to the nearest tenth.

Example 6 Evaluate

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

for $\bar{d} = 1.2, \mu_d = 0, s_d = 0.14$, and $n = 49$. Round your answer to the nearest hundredth.

Let's substitute the four given values in for the appropriate variables. Be careful that you follow the order of operations carefully, especially for the denominator of the complex fraction. If you are using your calculator, you may have to insert parentheses.

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{1.2 - 0}{\frac{0.14}{\sqrt{49}}} = \frac{1.2}{\frac{0.14}{\sqrt{49}}} = \frac{1.2}{\frac{0.14}{7}} = \frac{1.2}{0.02} = 60$$

Answer $t = 60$

My Turn!

Evaluate

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

for $\bar{d} = 0.9, \mu_d = 0, s_d = 0.4$, and $n = 64$. Round your answer to the nearest hundredth.

Answers to My Turn!

1. $\bar{p} \approx 0.5714$
2. $z \approx 3.42$
3. $E \approx 0.241$
4. $t \approx 7.30$
5. $E \approx 0.3$
6. $t = 18$

Practice Problems

1. Evaluate the formula

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

for $x_1 = 46$, $x_2 = 37$, $n_1 = 80$, and $n_2 = 70$. Round your answer to the nearest ten thousandth.

2. Evaluate the formula

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p} \cdot \bar{q}}{n_1} + \frac{\bar{p} \cdot \bar{q}}{n_2}}}$$

if

$p_1 - p_2 = 0, x_1 = 96, x_2 = 83, n_1 = 100, n_2 = 100, \hat{p}_1 = \frac{x_1}{n_1}, \hat{p}_2 = \frac{x_2}{n_2}, \ \bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$, and $\bar{q} = 1 - \bar{p}$.

Round your answer to the nearest hundredth.

3. Evaluate

$$E = z \cdot \sqrt{\frac{\hat{p}_1 \cdot \hat{q}_1}{n_1} + \frac{\hat{p}_2 \cdot \hat{q}_2}{n_2}}$$

if $z = 1.96$, $x_1 = 45$, $x_2 = 76, n_1 = 50, n_2 = 80$, $\hat{p}_1 = \frac{x_1}{n_1}$, $\hat{p}_2 = \frac{x_2}{n_2}$, $\hat{q}_1 = 1 - \hat{p}_1$, and

$\hat{q}_2 = 1 - \hat{p}_2$.

Round your answer to 3 significant digits.

4. Evaluate the formula

$$t=\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$$

for $\bar{x}_1=230, \bar{x}_2=242.5, \mu_1-\mu_2=0, s_1=4.5, s_2=3.9, n_1=40,$ and $n_2=45$. Round your answer to the nearest hundredth.

5. Evaluate

$$E=t\cdot\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

if $t=1.664, s_1=0.9, s_2=0.7, n_1=80,$ and $n_2=70$. Round your answer to the nearest tenth.

6. Evaluate

$$t=\frac{\bar{d}-\mu_d}{\frac{s_d}{\sqrt{n}}}$$

for $\bar{d}=1.2, \mu_d=0,\ s_d=0.2,$ and $n=100$. Round your answer to the nearest hundredth.