§8.1b and 8.2 Hypothesis Testing Proportions

Type I and Type II <u>error</u>

	H_0 is True	H_0 is False
Reject H_0	Type I error Probability < $\alpha = .05$	Correct
fail to Reject H_0	Correct	Type II error

Describe in a Sentence what a Type I error means. Company $p = .04$ & you rejected that.

→ Your sample had an unusually high Number 15% of defects, but the companies overall defect rate really is 4%.

Describe <u>II</u>

→ The defect rate is Not 4% (H_0 is Not True)

But My Data was not statistically 5% Significant enough to Show or Support that the proportion is Not 4%

Claim In Words: prop of Blue eyes is 0.35

#30 Claim: $p = .35$

$H_0: p = .35$

$H_1: p \neq .35$

Meaning of Type I H_0 true but we Rejected H_0

The prop of people with blue eyes is .35

But our Data ~~e~~ indicated that we should

Reject that the prop. is .35.

Meaning of Type II $\underbrace{H_0 \text{ is false}}$ but we failed to Reject H_0

The proportion of people with Blue eyes is

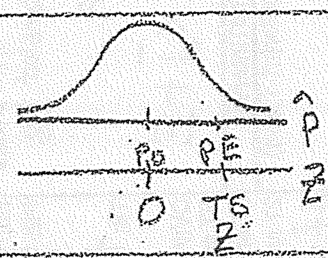
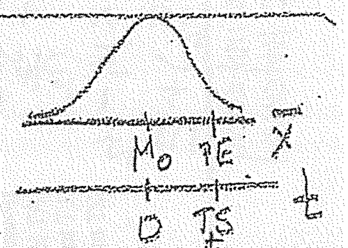
Not .35

But

there was Not sufficient data

to show that the prop. is Not .35

Steps For a hypothesis Test

	Proportions	Means
① Check Requirements SRS	$n\hat{p} > 5$ and $n\hat{q} > 5$	$n > 30$, or Population is Normal
② Write Null and Alternate Hypothesis Claim: $<, >, \neq \Rightarrow H_1$ is claim Claim: $\leq, \geq, = \Rightarrow H_1$ is opposite	$H_0: p = p_0$ $H_1: p <, >, \neq, p_0$	$H_0: \mu = \mu_1$ $H_1: \mu <, >, \neq \mu_0$
③ Point Estimate and the Summary Statistics	$\hat{p} = \frac{x}{n}$	$\bar{x}$ from 1-VarStat s, n or Given
④ Find Critical Values	CV: $Z^* =$ invnorm(Area)	CV: $t^* =$ invT(Area
⑤ Test Statistic is Score of point estimate	TS: $z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$ use p in H_0	TS: $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$ use μ in H_0 and S from 1-VarStat
⑥ Draw Sampling Distribution Vertically line up PE and TS		
⑦ P-value is the probability of being more extreme than PE	P-value Rt = normalcdf(TS, 999) Lt = ncdf(-999, TS) Two Tail = 2 · Smaller of Lt or Rt	P-value = tcdf(LB, UB, df
⑧ Make Initial Conclusion Reject H_0 or fail to Reject H_0	$H_0: \hat{p} = p_0$	$H_0: \mu = \mu_0$
⑨ Write Final Conclusion describing what the population Parameter is in words.		

A full Hypothosis Test

Ex (Asthma) A research claims that the asthma rate in her town is greater than the 11% National Average. In a random Sample of 167 people she finds that 37 of them have asthma. Test her claim at the $\alpha = .05$ level of significance.

a) What is the Sample? The 167 people tested.

Population? The entire town.

1 b) Requirements? Yes SRS $n = 167$ $\hat{p} = \frac{37}{167}$

$n\hat{p} = 37 > 5$ $n\hat{q} = 130 > 5$

2 Claim: asthma proportion in town is greater than .11.

Claim: $p > .11$

$H_0: p = .11$

$H_1: p > .11$ ← Right Tailed

3 $\boxed{\text{PE: } \hat{p} = \frac{37}{167} = .2216}$

4 CV: $Z^* = \text{invnorm}(1-.05, 0, 1) = \boxed{1.645 = Z^*: CV}$

Rt: Area $= 1-\alpha$

Lt: Area $= \alpha$

Two: Area $= 1-\frac{\alpha}{2}$

⑤ TS: $Z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}}$

PE: $\hat{p} = \dfrac{x}{n} = \dfrac{37}{167} = .2216$

p_0 = value in claim ↑ H_0

$\boxed{H_0: p_0 = .11}$

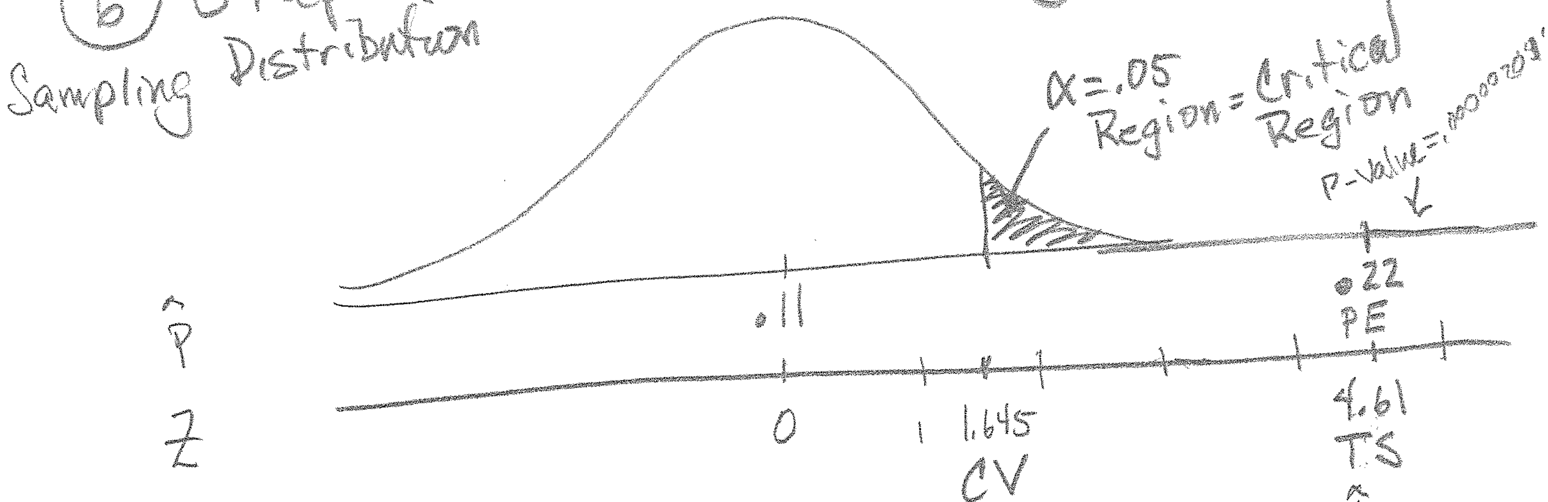
$q_0 = 1 - p_0 = .89$

$n = 167$

$Z = \dfrac{37/167 - .11}{\sqrt{\dfrac{.11 \cdot .89}{167}}} = 4.61$

$\boxed{TS: Z = 4.61}$

⑥ Graph Must have on every HT in HW



In critical Region

⇒ statistically significant

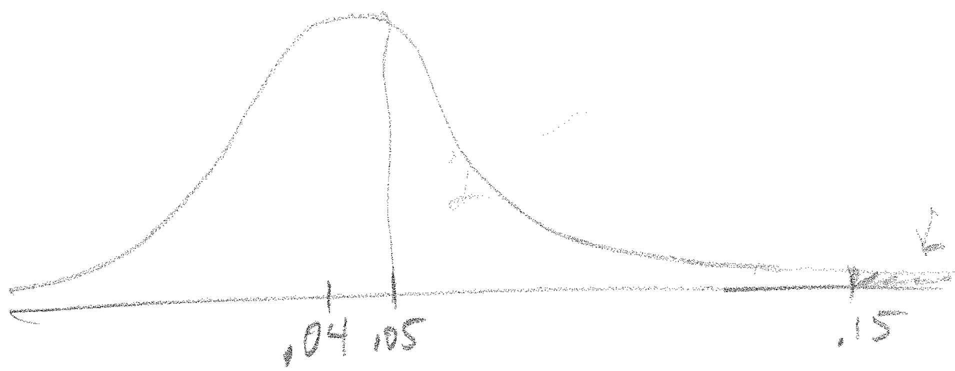
⑦ a) P-value = normal cdf (TS, ∞)

= normal cdf (4.61, 9999, 0, 1) = 2.015×10^{-6}

= .000002015 < .05

P-value

If H_0 is True then
the p-value is the probability of
Seeing Sample data as or more
extreme that the given sample.



7 b) Write a Sentence explaining the Meaning of the p-value.

If H_0 is true → If the true Asthma rate in town is 11%

then p-value chance → then there is a $\boxed{.000002015}$ chance

More extreme than sample → of getting an sample asthma rate of 22% or higher.

8 Reject H_0:

- If $\hat{p}$ is statistically Significant

Traditional → - $|TS| > |CV|$

→ TS is in Critical Region

P-value Method - p-value $= .000002015 < .05 = \alpha$

p-value is close to zero

9 Final Conclusion

Reject H_0 and Support H_1

↑ Reject that Asthma rate is 11%

Support that Asthma rate is greater than 11%

⑨ Final Conclusions

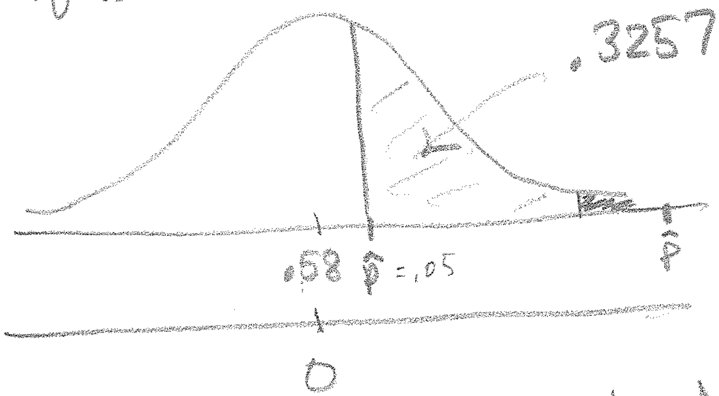
#25 Claim: more than 58% of adult would erase data.

Claim: $p > .58$

$H_0: p = .58$

Claim → $H_1: p > .58$ $\hat{p}$

P-value $= .3257 > .05$



$\hat{p}$ is not statistically significant

⑧ fail to Reject H_0 ⇒ fail to Support H_1 ← Write when claim

⑨ There is not sufficient evidence to Support that the proportion of people who would erase their data online is greater than .58.

§8.1

#3 a) $H_0: \mu = 174.1$ ← when $=, \leq, \geq$

b) $H_1: \mu \neq 174.1$ ← put opposite

c) Reject H_0 or fail to Reject H_0

Conclusion: There is sufficient evidence to Reject that the mean height is 174.1 cm.

If we fail to Reject H_0

Conclusion: there is Not Sufficient evidence to Reject that the mean height is 174.1 cm.

d) No, a HT can Never prove height <u>equals</u> 174.1cm.

#6 Claim: fewer than 95% of Adult have a cell phone

a) Claim: $p < .95$ ← No $=, \leq, \geq$ If $<, >, \neq$ then Same

$H_0: p = .95$

$H_1: p < .95$

$$\hat{p} = \frac{x}{n} = \frac{x}{1128} = .87$$

#14 TS: $$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.87 - .95}{\sqrt{\frac{.95 \cdot .05}{1128}}} = \boxed{-12.328}$$

#10 Reject H_0 ⇒ Support H_1 ⇒ there is sufficient evidence to Support that fewer than 95% of Adults have cell phones.

§8.1b&8.2 Hypothosis Testing

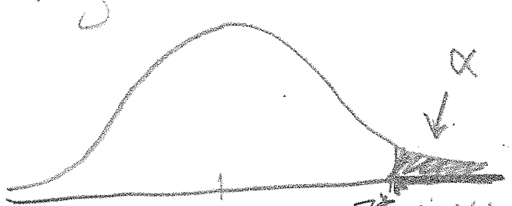
③ Finding Critical Values

The α-level is the Significance level of the Test. Either given or use .05

P_0 or μ_0 is value given in the origmal claim

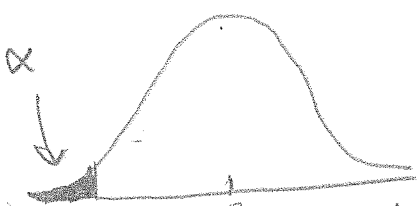
H_1 = Alternate hypothosis always contains a Strict inequality. One Prop Z Test

$H_1: p > p_0$
Right Tailed



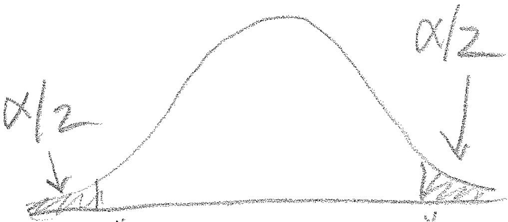
critical Region

$H_1: p < p_0$
Left Tailed



$Z^* = \text{invnorm}(\alpha)$

$H_1: p \neq p_0$
Two Tailed



$Z^* = \pm\text{invnorm}(1-\alpha/2)$

Same CV as for Confidence intervals with a $\pm$

CV: $Z^* = \text{invnorm}(1-\alpha)$

If

α	CV
.05	1.645
.01	2.33

α	Z^*
.05	-1.645
.01	-2.33

α	Z^*
.05	$\pm$ 1.96

④ and ⑤ Point Estimate and Test Statistic

$z = \frac{x - \bar{x}}{s}$

PE: $\hat{p}$ or $\bar{x}$ → Value of Sample Statistic actually observed when we collected Some data

TS: Test Statistic of data given that the Null hypothesis is True TS

$H_0: p = p_0$ TS: $z = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$ z-Score of Sample prop. on dististribution of all Sample proportions

$H_0: \mu = \mu_0$

TS: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ t-Score of Sample Mean on dist of all Sample means.

(5) Test Statistic

Determine if Proportion or Mean

#13 Claim $p = .75$ $n = 1021$ $\hat{p} = .89$

Proportions

Proportion

$$TS: z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{.89 - .75}{\sqrt{\frac{.75 \cdot .25}{1021}}} = \boxed{10.33}$$

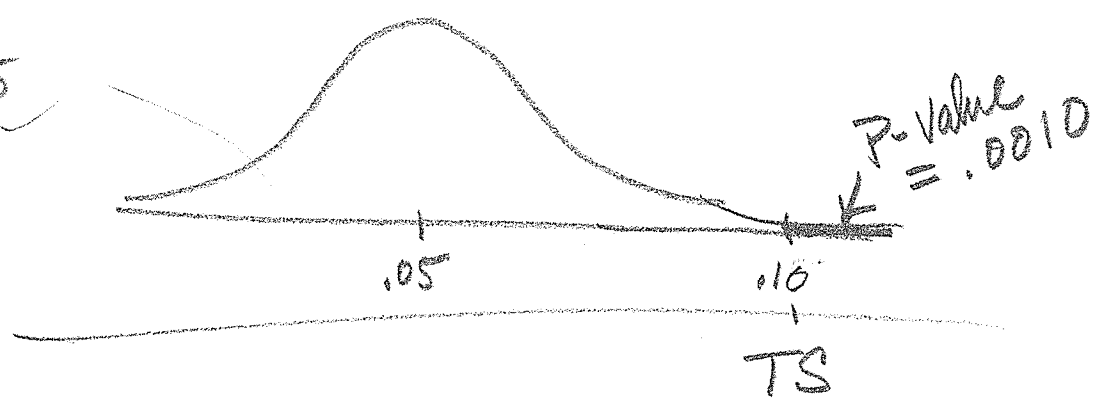
#16 Claim: $\mu = 8.00 \Rightarrow$ Population mean

Sample $\bar{x} = 7.15$ $s = 2.28$ $n = 40$

Mean

$$TS: t = \frac{(\bar{x} - \mu)}{(s/\sqrt{n})} = \frac{(7.15 - 8.00)}{(2.28/\sqrt{40})} = \boxed{-2.357}$$

#25

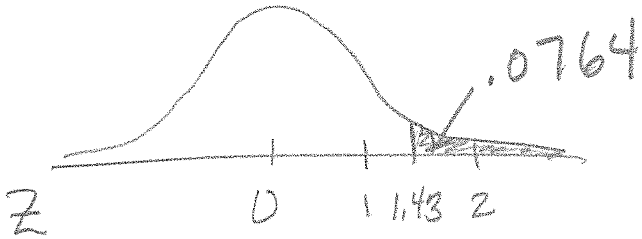


(7) Finding P-Values

Ex (1) Rt Tailed

TS: $z = 1.43$

Find p-value

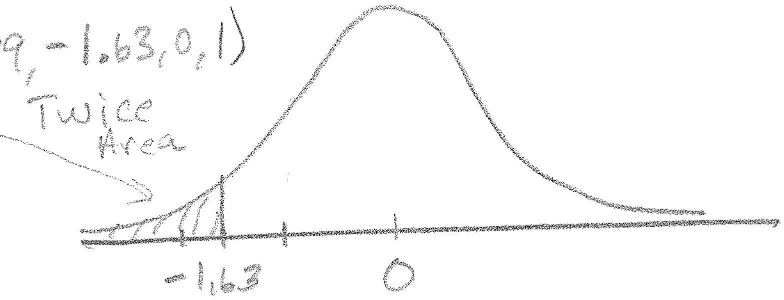


P-value = ncdf(1.43, 9999, 0, 1) = .0764

(2) Two Tailed TS: $z = -1.63$

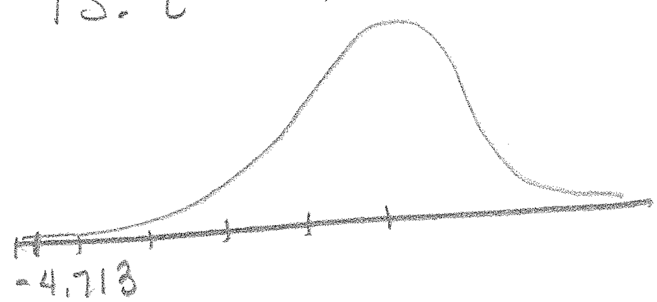
Find p-value = 2ncdf(-9999, -1.63, 0, 1)

= .1031



c) Claim: $\mu < 98.6$ and TS: $t = -4.713$

Lt Tailed $H_1: \mu < 98.6$ $n = 24$



P-Value = Tcdf(LB, UB, df)

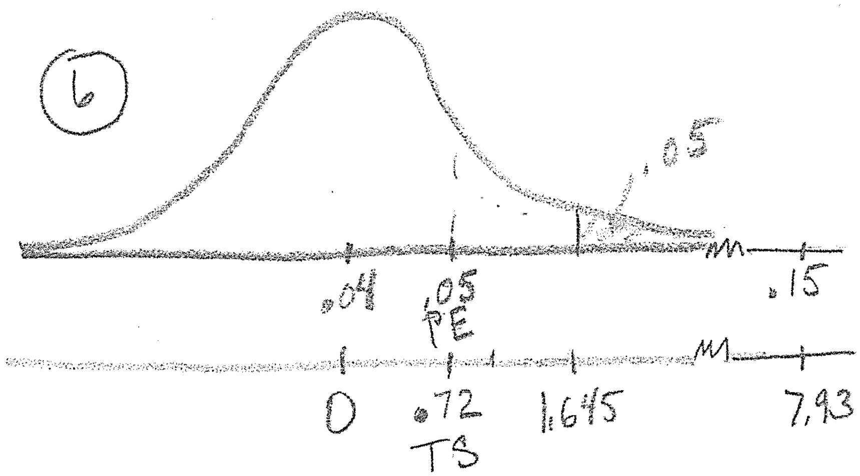
= Tcdf(-9999, -4.713, 23)

p-value = .0000476 = 4.76×10^{-5}

Example A company says that the defect rate is 4%
We recieve a shipment of 200 parts
and we get
a) 10 defective parts
b) 30 defective parts
We claim that the defect rate is above 4%.
Test at the $\alpha = .05$ level of significance.

(2) $H_0: p = .04$
$H_1: p > .04$

(6)



(3) $\hat{p}_a = \frac{10}{200} = .05$

$\hat{p}_b = \frac{30}{200} = .15$

(4) $\alpha = .05$, Rt tailed
CV: $Z^* = \text{invnorm}(1 - .05) = 1.645$

(5) TS: $Z_a = \dfrac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \dfrac{(.05 - .04)}{\sqrt{.04 \cdot .96/200}} = .72 < 1.645 = CV$

TS: $Z_b = \dfrac{(.15 - .04)}{\sqrt{.04 \cdot .96/200}} = 7.93 > 1.645 = CV$

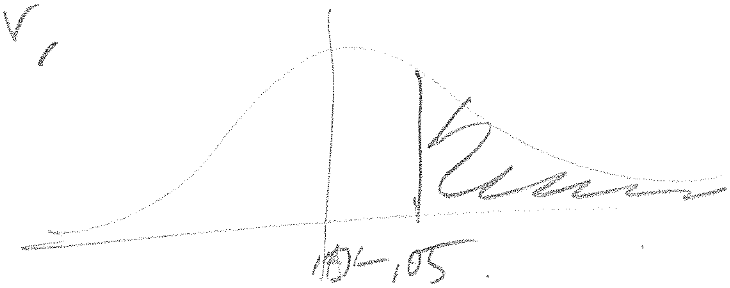
(7) P-value$_a$ = normalcdf(.72, 9999) = .2357 $> \alpha$
P-Value$_b$ = normalcdf(7.93, 9999) = .000000000000000102
$\approx 0^+$ $\qquad 1.02 \times 10^{-15} < \alpha$

(8) Initial conclusion
a) fail to Reject H_0 $\qquad$ b) Reject H_0

Meaning of P-value

$\hat{p} = .05$ p-value $= .2357$ $H_0: p = .04$

H_0 If the true defect rate is 4% then there is a 23.57% chance of getting a sample with a 5% defect rate or higher.



Type I Error $\hat{p} = .16$

The data ~~rejec~~ supports the defect rate is greater than .04 when actually it is Not

Type II Error $\hat{p} = .05$

The defect rate $p > .04$ but we failed to show $p > .04$

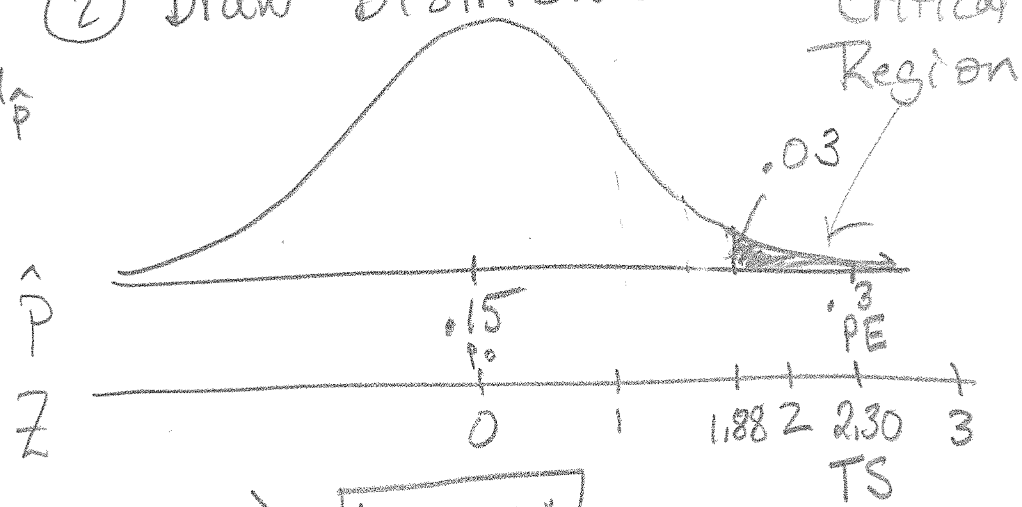
§8.2 Review Class Try

Ex LED lights. Of 30 people, 9 own at least One LED light. Claim is that SRans own LED lights at a rate above the National proportion of .15. @ $\alpha = .0$:

① Claim: $p > .15$

$$\boxed{\begin{array}{l} H_0: p = .15 \\ H_1: p > .15 \end{array}} = \mu_{\hat{p}}$$

② Draw Distribution



③ PE: $\boxed{\hat{p} = \frac{x}{n} = \frac{9}{30} = .3}$

④ CV: $Z^* = \text{invnorm}(1 - .03) = \boxed{1.88 = Z^*_{CV}}$

⑤ TS: $$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.3 - .15}{\sqrt{\frac{.15 \cdot .85}{30}}} = \boxed{2.30 = Z_{TS}}$$

⑥ Traditional: If TS lies in the Critical Region we Reject H_0: Sample data is Statistically Significant.

⑦ Final Conclusion:

We can Support that the proportion of Santa Rosens who use LED's is greater than 15% or .15. (Never write the claim)

How to use Calculator to get TS & p-value

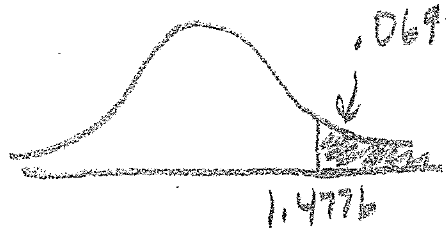
1-Prop Z Test

STAT ▷▷ TEST 5: 1-Prop Z Test

P_0: .8 = Value in H_0
X: 170 = # Successes
n: 202 = Sample size
Prop > .8 is the Alternate Hypothosis

Output
H_1: p > .8
TS: Z = 1.48 use more decimals
p = .0697 = p-value
$\hat{p}$ = .841584 = PE
n = 202

Draw



Not critical Region

there is a 6.98% chance of seeing a sample with 84% having significant weight loss (samp) or greater Given That the true prop in whole population is 80%. (H_0)

Quiz 9 Math 15 Fall 2016 Jones Name____________________

Identify the null hypothesis, alternative hypothesis, test statistic, P–value, conclusion about the null hypothesis, and final conclusion that addresses the original claim.

1) In a clinical study of the effects of eliminating sugary drinks from the diet, 170 of the 202 subjects reported experiencing significant weight loss. At the 0.01 significance level, test the claim that more than 80% of all those who eliminated sugary drinks and juice from their diet experienced significant weight loss.

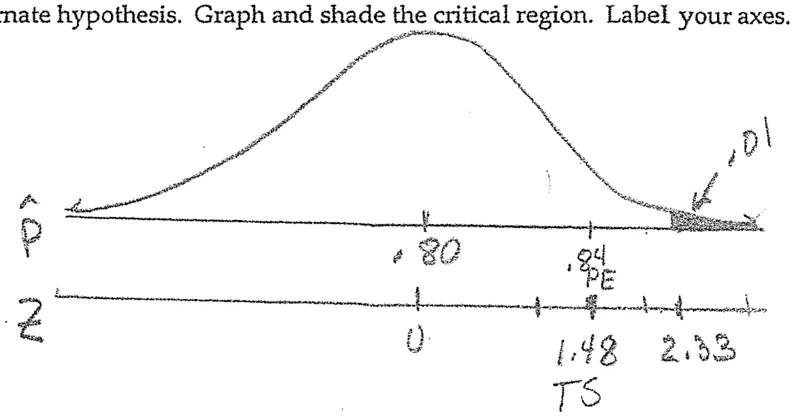
(4 Points) State the claim, null, and alternate hypothesis. Graph and shade the critical region. Label your axes. Find the critical value.

Claim: $p > .80$

H_0: $p = .80$

H_1: $p > .80$

$\alpha = .01$



CV: $Z^* = \text{invnorm}(.99,0,1) = 2.33$

(4 Points) Find the point estimate of the population proportion and it's test statistic using the formula and checking your answer with your calculator. Label these on your graph above.

PE: $\hat{p} = \frac{170}{202} = .84$

TS: $Z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{.84 - .80}{\sqrt{.80 \cdot .20/202}} = 1.42$

(4 Points) Find the P–value and explain the meaning of the P–value. Shade a new graph showing the area equal to the p–value.

P-value = normalcdf$(1.42, 9999) = \boxed{.0778}$

There is a 7.78% chance of seeing 84% or higher having significant weight loss when the true proportion is 80%.

(4 Points) Find you initial conclusion. Clearly state your final conclusion.

Initial fail to Reject H_0

Final There is Not sufficient evidence to support that more than 80% lost significant weight

	Requirements	Sample size	Confidence Interval		Hypothesis Test	
	Formula page Requirements	Statistics	Estimate Population Parameter with Confidence Interval		Testing a claim with a Hypothesis Test	
	n=Size of population	Find Sample Size	Confidence intervals One sample	Confidnece Interval Two samples	Test Statistic One Sample	Test Statistic Two sample
PROPORTION	Proportion	approx. p known	7.1 1-PropZInt	9.1 2-PropZInt	8.2 1-PropZTest	9.1 2PropZTest
	requirements np>5, nq>5 Use Normal Distribution $\hat{p}$=x/n $\hat{q}$=1-$\hat{p}$	7.1 $n=\frac{z_{\alpha/2}^2\cdot\hat{p}\cdot\hat{q}}{E^2}$ $\hat{p}$ = given approx. approx. p Unknown 7.2 $n=\frac{z_{\alpha/2}^2\cdot .25}{E^2}$	PE: p = x/n $E= z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$ CV: $z_{\alpha/2}$ = invN(1-α/2, 0, 1) $\hat{p}-E<p<\hat{p}+E$ (LB, UB) = ($\hat{p}$-E, $\hat{p}$+E) $E=\frac{UB-LB}{2}$, $\hat{p}=\frac{UB+LB}{2}$	PE: $\hat{p}_1-\hat{p}_2$ $E= z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}$ $(\hat{p}_1-\hat{p}_2)-E<p_1-p_2<(\hat{p}_1-\hat{p}_2)+E$	Ho: $p=p_0$ PE: $\hat{p}$=x/n TS: $z=\frac{\hat{p}-p_0}{\sqrt{pq/n}}$ CV: z* = invnorm(area) P-value= normalcdf(TS,999) RT normalcdf(-999,TS) LT mult. by 2 for 2tail	Ho: $p_1=p_2$, $p_1-p_2=0$ PE: $\hat{p}_1-\hat{p}_2=\frac{x_1}{n_1}-\frac{x_2}{n_2}$ TS: $z=\frac{\hat{p}_1-\hat{p}_2}{\sqrt{\frac{\bar{p}\bar{q}}{n_1}+\frac{\bar{p}\bar{q}}{n_2}}}$ CV: z* = invnorm(area) P-value=ncdf(LB,UB) $\bar{p}=(x_1+x_2)/(n_1+n_2)$
MEANS	Mean = μ σ Known σ = Pop. SD n>30 or normal	7.2 $n=\left(\frac{z_{\alpha/2}\cdot\sigma}{E}\right)^2$	Skip ZInterval $E=z_{\alpha/2}\cdot\sigma/\sqrt{n}$ $\bar{x}-E<\mu<\bar{x}+E$	Always Assume that the Population Standard deviation is Unknown for two sample means.	8.4 Skip ZTest Ho: $\mu=\mu_0$ PE: $\bar{x}$ TS: $z=(\bar{x}-\mu_0)/(\sigma/\sqrt{n})$ CV: z* = invNorm(area)	Always Assume that the Population Standard deviation is Unknown for two sample means.
	Mean = μ σ Unknown Use t-distribution σ = Pop. SD unknown n>30 or normal parent population If n < 30 Check normal Histogram QQPlot	7.2 $n=\left(\frac{t_{\alpha/2}\cdot s}{E}\right)^2$ df = n-1 and s from preliminary Sample	7.2 TInterval PE: $\bar{x}$ α = 1-CL $E=t_{\alpha/2}\cdot s/\sqrt{n}$ For Data use 1-VarStat(L1) to find $\bar{x}$ and s $\bar{x}-E<\mu<\bar{x}+E$ $\bar{x}=\frac{UB+LB}{2}$ CV: $t_{\alpha/2}$ = invT(1-α/2, df)	9.2 TInterval on L3=L1-L2 Matched Pairs, df = n - 1, n = # pairs PE: $\bar{d}=\bar{x}$ 1-VarStat L3 $E=t_{\alpha/2}\cdot s_d/\sqrt{n}$ $\bar{d}-E<\mu_d<\bar{d}+E$ 9.2 2-SampTInterval Independent df = smaller(n1-1,n2-1) PE: $\bar{x}_1-\bar{x}_2$ $E=t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$ $(\bar{x}_1-\bar{x}_2)-E<\mu_1-\mu_2<(\bar{x}_1-\bar{x}_2)+E$	8.3 TTest Ho: $\mu=\mu_0$ PE: $\bar{x}$ TS: $t=\frac{\bar{x}-\mu_0}{s/\sqrt{n}}$ CV: t* = InvT(area,df) RT area = 1-α LT area = α 2T area = 1-α/2 df = n-1 P-value= RT tcdf(TS,9999, df) LT tcdf(-9999,TS,df) 2T multiply by 2	9.3 TTest on L3=L1-L2 Matched Pairs, df = n - 1, n = # pairs Ho: $\mu_d=0$ 1-VarStat L3 PE: $\bar{d}$ $\bar{x}=\bar{d}$ $s_x=s_d$ TS: $t=\frac{\bar{d}-\mu_d}{s_d/\sqrt{n}}$ CV: t* = InvT(Area, df) P-value=tcdf(LB,UB) df 9.3 2-SampTTest Independent df = smaller(n1-1,n2-1) Ho: $\mu_1=\mu_2$ or $\mu_1-\mu_2=0$ PE: $\bar{x}_1-\bar{x}_2$ TS: $t=(\bar{x}_1-\bar{x}_2)/\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$ CV: t* = InvT(Area, df) P-value=tcdf(LB,UB, df)
Standard Deviation	Standard Deviation	Table	7.3 S2INT(s², df, CL) $\sqrt{\frac{(n-1)s^2}{\chi_R^2}}<\sigma<\sqrt{\frac{(n-1)s^2}{\chi_L^2}}$			

UB = upper bound of confidence interval
LB = Lower bound of confidence interval

RT = right Tail
LT = Left Tail