§7.2 Estimating a Mean ← Read

Get Spencer Stats Programs from Math Lab

Find a Confidence Interval for a **Mean** $= \mu$

1. Requirements: $n > 30$ or Population is Normal
SRS
CLT

2. PE: $\bar{x} = \frac{\Sigma x}{n}$ → given or 1-VarStats
Point estimate = best guess of population parameter

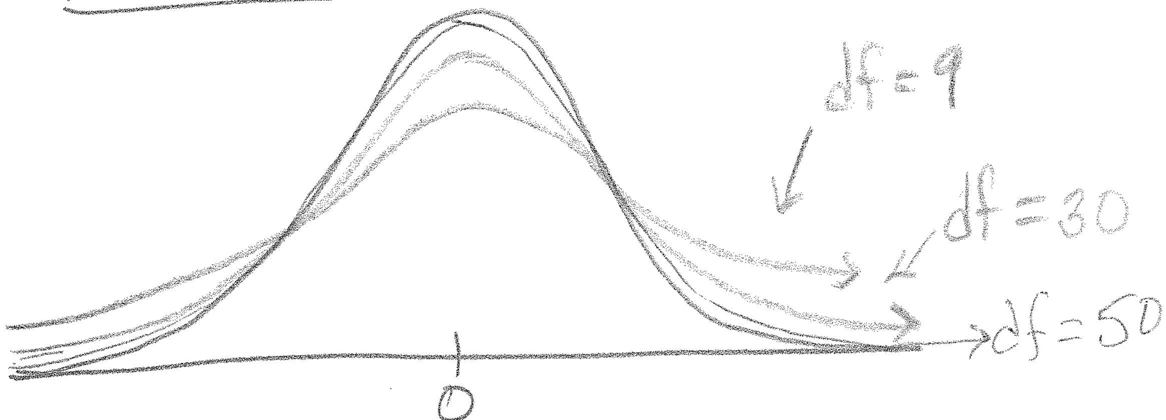
3. CV: $t_{\alpha/2} = \text{invT}(1 - \alpha/2, df)$ $df = n - 1$

4. $E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$
Standard Error $= \frac{s}{\sqrt{n}}$
= Standard deviation of distribution of sample means $\bar{x}$.

5. CI: $\bar{x} \pm E$ $\bar{x} - E < \mu < \bar{x} + E$
$(\bar{x} - E, \bar{x} + E)$

6. Sentence We are confidence level Confident that "Describe population mean" is between $\bar{x} - E$ and $\bar{x} + E$

The T-Distribution



(1) Dist of Sample means follows a t-distributio

(2) $df = n-1$ $\quad n =$ Sample Size
$= $ degrees of freedom

(3) Tails get thinner as df increases
Smaller samples have larger variability in their Sample means

(4) CV: $t_{\alpha/2} = \text{invT}(1-\alpha/2, df)$

$1-\alpha/2 =$ Rt Tail Area
$df = n-1$

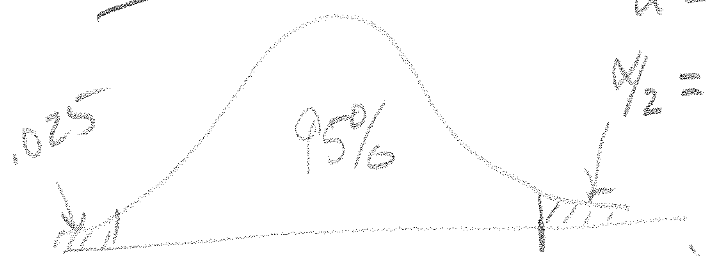
Get Spencer Stats programs

Find t - critical Values

Ex 1 If $n = 20$ and $CL = .95$ Find $t_{\alpha/2}$

$\alpha = .05$

$\alpha/2 = .025$



$t_{\alpha/2} = \text{invT}(1 - \alpha/2, df)$

$= \text{invT}(1 - .05/2, 19)$

$\boxed{t_{\alpha/2} = 2.093}$

Spencer Stats prog. Inv X 2

3 Methods

- By hand
- With TI-84 - STAT TESTS 8: TInterval
- With Statcrunch

97% = CL

Find a Confidence Interval for the mean Shower length of SR Residents.

Given $\bar{x} = 11.8$
$s = 4.26$
$n = 38$

Always use t for Means

Round t-score to 3-decimals

Find $\boxed{CV: t_{\alpha/2} = invT(1-\alpha/2, df_{n-1})}$
$= invT(1 - .03/2, 37) = 2.257$

Find $\boxed{E = t_{\alpha/2} \frac{s}{\sqrt{n}}} = 2.257 \cdot \frac{4.26}{\sqrt{38}} = 1.56$

Find CI $\bar{x} \pm E = 11.8 \pm 1.56$ $\boxed{10.24 < \mu < 13.36}$

(10.24, 13.36)

Sentence We are 97% confident that the True Mean shower length of SR Residents is Between 10.24 minutes and 13.36 minutes

↑ units

How long is your shower?

Put these in your Calculator

8 mins
8 mins
10 mins
9 min
6 min
6 mins
15 mins
7 mins
15 min
15 mins
9 mins
10 mins
10 min
8 min
3 min
15 mins
7 mins
8 mins
15 mins
15 mins
24 min
17 min
15 mins
7 min
9 min
15 mins
15 mins
15 mins

$\mu = 11.3$ $n = 28$
med = 10
range = 21

$S = 4.62$

Statcrunch
Applets Resampling
Bootstrap a statistic

$\sigma_{\bar{x}} = .85$

$\frac{S}{\sqrt{28}} = \frac{4.62}{\sqrt{28}} = .873$

Ex2 If mean Subway fair CI is

$\$1.25 < \mu < \$1.75$.

Find the point estimate and the Margin of error.

Point Estimate $= \bar{x}$

$1.25 = \bar{x} - E = LB$

$1.75 = \bar{x} + E = UB$

PE: $\boxed{\bar{x} = \frac{LB + UB}{2}} = \frac{1.25 + 1.75}{2} = 1.50$

$E = .25 = \boxed{\frac{UB - LB}{2}} = \frac{1.75 - 1.25}{2} = \frac{.50}{2} = .25$

Ex In a random sample of 105 light bulbs we find that the mean life is 441 hrs and a standard deviation of 40 hrs. Construct a 90% CI for the mean life μ, of all light bulbs of this Type.

note: $\boxed{s=40}$

① Requirements $\boxed{n=105} > 30$ & SRS ✓

$CL = .90$

$\alpha = 1-.90 = .10$

$df = n-1 = 105-1$

$df = 104$

② PE: $\boxed{\bar{x} = 441}$

③ CV: $t^* = \text{invT}(1-.10/2, 104) = \boxed{1.660} = t_{\alpha/2}$

④ $E = t_{\alpha/2} \cdot s/\sqrt{n} = 1.660 \cdot 40/\sqrt{105} = 6.48$ hrs

$\boxed{6.5 = E}$

$\frac{s}{\sqrt{n}}$ = Standard Error

⑤ CI → $\bar{x} \pm E = 441 \pm 6.5$

$441 - 6.5 < \mu < 441 + 6.5$

$\boxed{434.5 < \mu < 447.5}$

⑥ Interperet

We are 90% confedent that the True mean life of all such light bulbs is between 434.5 hrs and 447.5 hrs.

Find n — Formula for Mean

Ex 3 $\boxed{\text{Find the}}\,\boxed{\text{Sample}}\,\boxed{\text{Size}}$ Needed to estimate the $\boxed{\text{mean}}$ Salary of an SRJC Student if we want a $\boxed{99\%\ CI}$ (CL), The difference from the population mean is less than $\boxed{\$126}$ $= E$

A preliminary Study indicates $\boxed{s \approx \$534}$

$$E = t_{\alpha/2} \frac{s}{\sqrt{n}} \qquad \sqrt{n} = \frac{t_{\alpha/2} \cdot s}{E}$$

$$\boxed{n = \left(\frac{t_{\alpha/2}\, s}{E} \right)^2} \approx \left(\frac{z_{\alpha/2}\, s}{E} \right)^2$$

$\alpha = .01$

$\frac{\alpha}{2} = .005$

$$n = \left(\frac{2.576 \cdot 534}{126} \right)^2 = 119.18$$

$$\boxed{n = 120}$$

Ex 4 Find a 90% Confidence Interval For the mean Life of CF light bulbs if the Sample mean is 441 hrs with a Standard deviation of 40 hrs when 105 light bulbs are Tested.

Find CL = .90 for mean, $\overline{x} = 441$, $S = 40$, $n = 105$

① Requirements? yes $n = 105 > 30 \Rightarrow$ dist is t-dist

Round t to 3-decimals

② PE: $\overline{x} = 441$

③ CV: $t_{\alpha/2} = \text{invt}(1 - \alpha/2, df) = \text{invt}(1 - .1/2, 104) = 1.66$

$$\boxed{t_{\alpha/2} = 1.660}$$

④ $E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}} = 1.660 \cdot \frac{40}{\sqrt{105}} = 6.48$ hrs

⑤ $\overline{x} - E < \mu < \overline{x} + E$

$441 - 6.48 < \mu < 441 + 6.5$

$\boxed{434.5 < \mu < 447.5}$ ← only one more decimal than $\overline{x}$

⑥ We are 90% Confident that the mean life of all CFL Light Bulbs is between 434.5 hrs and 447.5 hrs

Ex5 Find ~~a~~ CI 93% CI for the Average age of an SRJC Statistics Student (Daytime).

$L_1 = 24 \quad 18 \quad 19. \quad 25 \quad \cdots$ ✓ $n \geq 30$

PE: $\bar{x} = 20.13 \quad S_x = 2.417 \quad n = 30$

$$t_{\alpha/2} = \text{invT}\left(1-\underbrace{\alpha}_{\alpha = 1-CL}/2, \underbrace{df}_{=n-1}\right) = \text{invT}(1-.07/2, 29)$$

$$\boxed{t_{\alpha/2} = 1.881}$$

$$E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}} = 1.881 \cdot \frac{2.417}{\sqrt{30}} = .83$$

$$\bar{x} - E < \mu < \bar{x} + E$$

$$20.13 - .83 < \mu < 20.13 + .83$$

$$19.3 < \mu < 20.96$$

We are 93% confident that the Average age of an SRJC stats student is between 19.3 & 20.9 years

Check TI T interval $\boxed{\text{Data}}$

Age

24
18
19
25
18
18
18
18
24
19
23
18
18
20
18
19
18
18
18
21
20
18
24
22
22
23
19
19
24
21

Using TI Calculator to check

STAT ▷▷ TESTS ▽ T Interval

Inpt: Data $\boxed{\text{Stats}}$ ← Enter ← If $\bar{x}$, s, n given

$\boxed{\text{Data}}$ ← Data in L_1

Stat T Stats One Sample with Data
Select Data
CI

Confidence Intervals

	§7.1 Proportions	§7.2 Means
Population Parameter	p = population proportion	μ = population mean
Sample Statistic Point estimate	(2) $\hat{p} = \frac{x}{n} = \frac{\#\text{Successes}}{\#\text{in Sample}}$	$\bar{x} = \frac{\Sigma x}{n}$, use 1-Var Stat (Need S too)
Sampling Distribution	Z or Normal	t-distribution
Requirements (1)	Binomial and $np = \#\text{Successes} = x > 5$ $nq = \#\text{failures} > 5$	$n > 30$ or original distribut of population is Normal
(3) Critical Value	CV: $Z_{\alpha/2} = \text{invnorm}(1-\alpha/2, 0, 1)$	CV: $t_{\alpha/2} = \text{invT}(1-\alpha/2, df)$
(4) Margin of Error	$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$	$E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$
(5) Confidence Interval	$\hat{p} - E < p < \hat{p} + E$	$\bar{x} - E < \mu < \bar{x} + E$
Example	$.27 < p < .35$	$\$1.25 < \mu < \$1.75$
(6) Interpret we are CL confident that	the proportion of People who drink Coffee is between .27 and .35	the mean amount spent by all coffee drinkers is between \$1.25 and \$1.75
(7) Calculator STAT ▷▷ TEST	A: 1PropZINT	8: TInterval

Ex 3 CI for a Mean age of the Class

19, 19, 21, 44, 19, 24, 19, 20, 18, 19, 19, 20

19, 19, 19, 18, 19, 18, 18, 21, 21, 17, 21

Summary Stats $\boxed{\bar{x} = 20.48}$ $\boxed{S = 5.33}$ $\boxed{n = 23}$

Make a 98% CI. So $\boxed{\alpha = .02}$

1. Requirements? NO $n < 30$ & $x = 44$ is an outlier

But we'll make the interval anyway - with & without

$\alpha = 1 - CL$ $\quad df = n-1$

2. PE: $\boxed{\bar{x} = 20.48}$

3. CV: $t_{\alpha/2} = \text{invT}(1 - .02/2, 22) = \boxed{2.508 = t_{\alpha/2}}$

4. $E = t_{\alpha/2} \dfrac{S}{\sqrt{n}} = 2.508 \cdot \dfrac{5.33}{\sqrt{23}} = \boxed{2.79 = E}$

5. CI: $\bar{x} - E < \mu < \bar{x} + E$

$20.48 - 2.79 < \mu < 20.48 + 2.79$

$\boxed{17.69 < \mu < 23.27}$

5' Check with 8: T Interval $\quad$ STAT ▸▸ TESTS ✓

6. The mean age of Stats Students is between 17.69 and 23.27

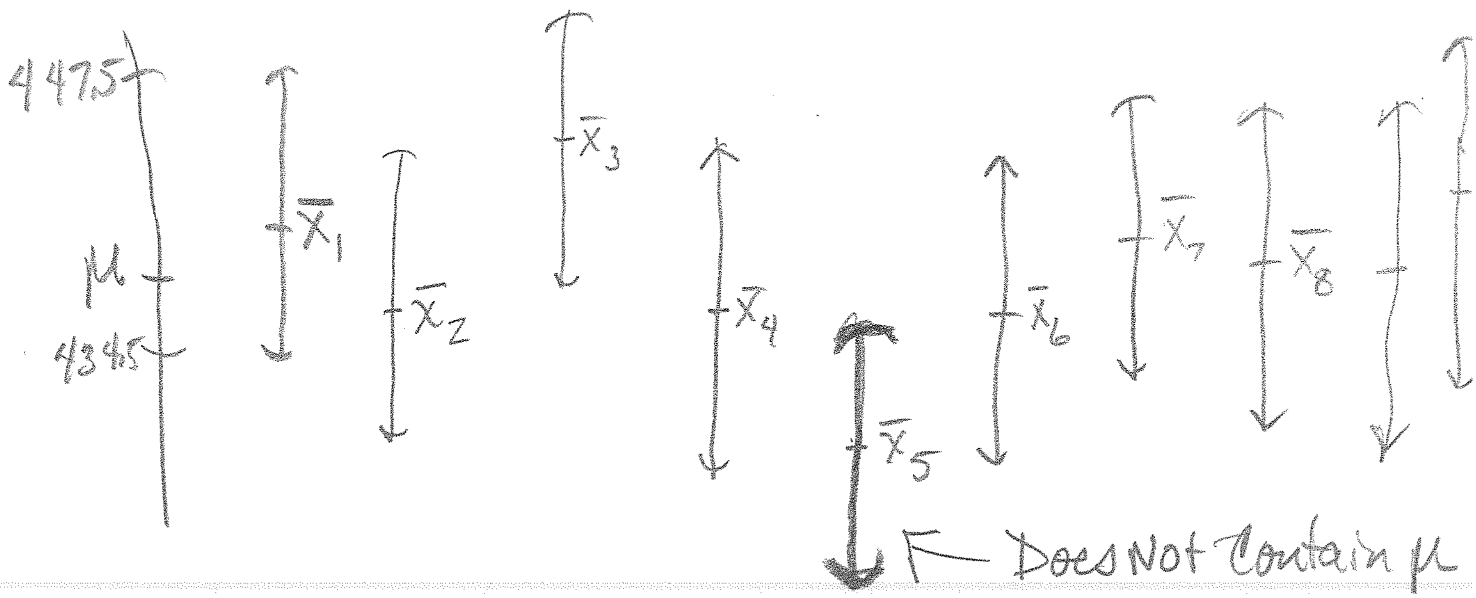
Without Outlier (18.6, 20.2)

Interpreting Confidence Interval

Be careful

Not There is a 90% chance μ will be in interval

Not 90% of data values will be in confidence interval.



Does Say 9 out 10 confidence intervals will contain the population mean

§7.1

#32 To Estimate $\boxed{p \leftarrow \text{Proportion}}$

Find n, $CL = .90$, within 2% $\Rightarrow E = .02$

$E = |\hat{p} - p| =$ distance between p & $\hat{p}$

a) $\boxed{n = \dfrac{(Z_{\alpha/2})^2 \cdot .25}{E^2}} = \dfrac{(1.645)^2 \cdot .25}{(.02)^2} = 1692$

$Z_{\alpha/2} = \text{invnorm}(1 - \alpha/2) = \text{invnorm}(1 - .10/2) = 1.645$

$\alpha = 1 - CL = 1 - .90 = .10$

b) If $\hat{p} \approx .95$

$\boxed{n = \dfrac{(Z_{\alpha/2})^2 \cdot \hat{p}\,\hat{q}}{E^2}} = \dfrac{(1.645)^2 \cdot .95 \cdot .05}{(.02)^2}$

$=$ $\boxed{\text{322 people}}$

c) Yes, we need a lot less people

① PE

#24 $70\% = \boxed{\hat{p} = .70}$ say they voted

$n = 1002$

a) 98% CI $\Rightarrow$ $\alpha = .02$ $\quad Z_{\alpha/2} = \text{invnorm}(1 - .02/2, 0, 1)$

$Z_{\alpha/2} = 2.33$

②

$$E = Z_{\alpha/2}\sqrt{\frac{\hat{p}\,\hat{q}}{n}} = 2.33 \cdot \sqrt{\frac{.7 \cdot .3}{1002}} = \boxed{.0337 = E}$$

$$\hat{p} - E < p < \hat{p} + E$$

$$.7 - .0337 < p < .7 + .0337$$

③ $\boxed{.6663 < p < .7337}$ ← what they say

we are 98% confident that the prop who will say they voted is between .6663 and .7337

b) Inconsistent, More people are saying that they voted that actually voting.

§7.2 #28, 35

#28 a) $\hat{p} = \frac{x}{n} \Rightarrow x = n \cdot \hat{p} = 3005 \cdot .817 = 2455$

Round Normally

b) Find a 90% CI

$\alpha = .10 \Rightarrow \alpha/2 = .05 \Rightarrow 1 - \alpha/2 = .95$

$z_{\alpha/2} = \text{invnorm}(1-\alpha/2, 0, 1) = \boxed{1.645}$

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\,\hat{q}}{n}} = 1.645\sqrt{\frac{.817 \cdot .183}{3005}} = .0116$$

$\hat{p} - E < p < \hat{p} + E$

$.817 - .0116 < p < .817 + .0116$

$.8054 < p < .8286$ or $(.8057, .8286)$

We 90% Confident that the true population proportion of (57 to 87 year olds who use perscription drugs) is between .8054 and .8286

If % 80.54% to 82.86%

c) Nothing about college Students

#35 Find ~~#~~ n when CL = 90% and

E = .05

a) $\hat{p}$ is unknown

$$n = \frac{Z_{\alpha/2}^{2} \cdot .25}{E^{2}} = \frac{1.645^{2} \cdot .25}{(.05)^{2}} = 270.60$$

$\boxed{n = 271}$

b) $\hat{p} = .85$ is know

$$n = \frac{Z_{\alpha/2}^{2} \cdot \hat{p} \cdot \hat{q}}{E^{2}} = \frac{1.645^{2} \cdot .85 \cdot .15}{(.05)^{2}}$$

$\boxed{n = 138}$

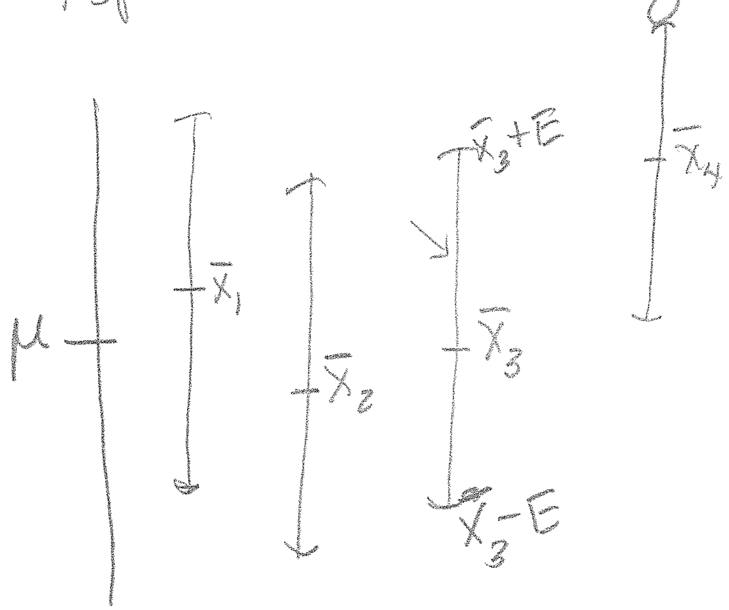
c) No, We Need a random Sample of all Adults

Confidence Intervals

	Proportion	Mean	Standard Deviation
Population Parameter	p	μ	σ
Sample Statistic	$\hat{p} = \frac{x}{n}$	$\bar{x} = \frac{\Sigma x}{n}$ or use 1-varstat	s use 1-Var stat
Sampling Distribution	z	t	χ^2
Margin of Error	$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$	$E = t_{\alpha/2}\frac{s}{\sqrt{n}}$	$\sqrt{\frac{s^2(n-1)}{\chi^2_R}} < \sigma < \sqrt{\frac{s^2(n-1)}{\chi^2_L}}$
Critical Value	$z_{\alpha/2} = \text{invnorm}(1-\alpha/2, 0, 1)$	Program $t_{\alpha/2} = \text{invT}(1-\alpha/2, 0, 1, df)$	InvX2 Deg Freedom = n- 4: Conf. Int Conf Level = .95
Confidence Interval	$\hat{p} - E < p < \hat{p} + E$	$\bar{x} - E < \mu < \bar{x} + E$ $65 < \mu < 67$	See Above 1.5 inches $< \sigma < 3.7$
Example We are 95% Confident	$.52 < p < .57$ The proportion in favor of ... is between .52 an .57	The mean height of all SRJC students is between 65 inches and 67 inches	The Standard Deviation of all heights is be 1.5 and 3.7 inches

(5) Interpreting the Meaning

Population Mean is fixed



90% CI means that 9 out of 10 Time the CI will contain the True Population mean.

<u>Wrong</u> μ is fixed so

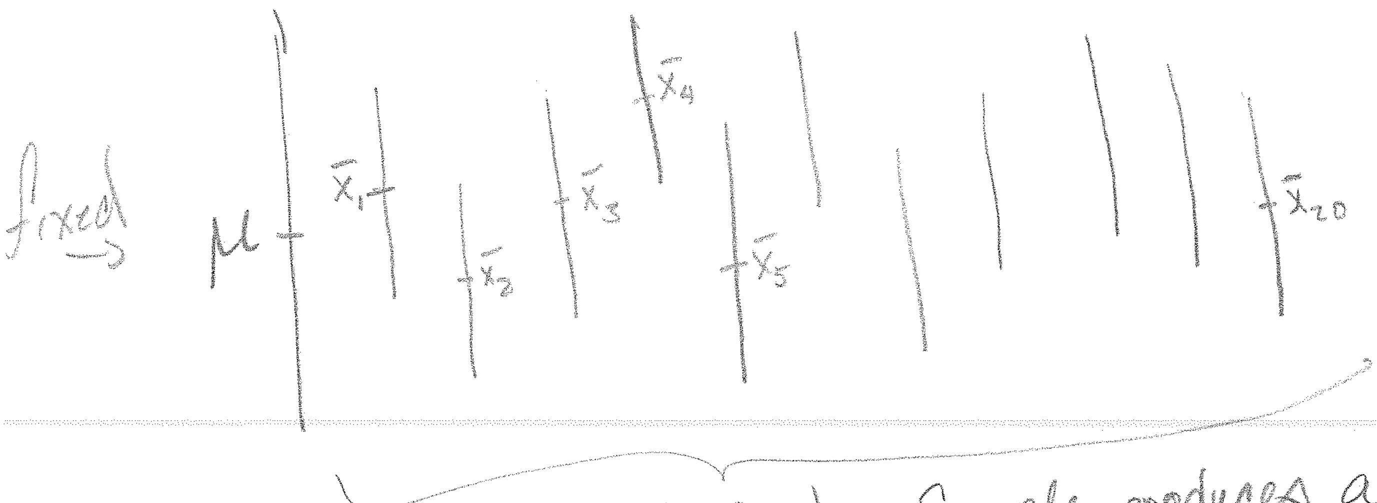
— 95% Chance that μ will fall between

or

— 95% of all values are between

CI only tells us where we think the <u>mean</u> will lie

— 95% chance that the mean will lie betwee — and —



Each Random Sample produces a different confidence interval.

In 95 out of 100 contain the mean

or 95% these CI's contain the mean

(1) How to pick n

How large a Sample is Needed

To approximate μ = mean amount of Salary for an SRJC Stat Student. Want our margin of Error to be less than \$ 126 and want CL = 99%.

$$E = t_{\alpha/2} \cdot \frac{S}{\sqrt{n}}$$

$$E \cdot \sqrt{n} = t_{\alpha/2} \cdot S$$

$$\sqrt{n} = \frac{t_{\alpha/2} \cdot S}{E}$$

$$n = \left(\frac{t_{\alpha/2} S}{E} \right)^2$$

$Z_{\alpha/2}$ = invNorm(1 − .01/2) =

To use we Need $S \approx 534$, use $Z_{\alpha/2}$ because n, and df are unknown.

$$n = \left(\frac{2.576 \cdot 534}{126} \right)^2 = 119.18$$

We need $\boxed{120 \text{ Salaries}}$

Quiz 8

4) $n = \frac{z_{\alpha/2}^2 \cdot \hat{p}\ \hat{q}}{E^2}$ = Sample Size

CL = 98% $E = .03$ $\hat{p} = .73$ $\hat{q} = 1 - \hat{p} = .27$

$Z_{.01} = Z_{\alpha/2} = \text{invnorm}(1 - .01, 0, 1) = 2.33$

CL = .98, $\alpha = 1 - .98 = .02$, $\frac{\alpha}{2} = .01$

$n = \frac{2.33^2 \cdot .73 \cdot .27}{(.03)^2} = 1186$ Big

Just survey the whole population
of N = 700 Students

$CL = .9 \quad \alpha = .1$ invNorm(1-.05,0,1)

HW §7.1

a) $n = \dfrac{Z_{\alpha/2}^2 \cdot .25}{E^2} = \dfrac{(1.645)^2 \cdot .25}{(.02)^2} = 1692$

b) $n = \dfrac{Z_{\alpha/2}^2 \, \hat{p} \cdot \hat{q}}{E^2} = \dfrac{(1.645)^2 \cdot .95 \cdot .05}{(.02)^2} = 322$

c) Yes it has a big impact when $\hat{p}$ is close to zero or 1.

#24 $n = 1002$, $\hat{p} = .70$ $p = .61$

a) $CL = .98$ Find a Confidence Interval

2pt ① Req. SRS $\hat{p}n > 5$ $\hat{q}n > 5$

② PE = $\boxed{\hat{p} = .70}$

2 ③ $\hat{q} = 1 - \hat{p} = .3$ $\alpha = .02$ $\boxed{Z_{\alpha/2} = \text{invnorm}(1 - \frac{\alpha}{2}, 0, 1)}$

$= \text{invnorm}(1 - .02/2, 0, 1)$

$Z_{\alpha/2} = 2.33$

2 ④ $\boxed{E = Z_{\alpha/2}\sqrt{\dfrac{\hat{p}\,\hat{q}}{n}}} = 2.33\sqrt{\dfrac{.7 \cdot .3}{1002}} = .0337$

2 ⑤ $\hat{p} \pm E = .7 \pm .0337$ or $.6663 < p < .7337$

$(.6663, .7337)$

Check with TI Note: $\hat{p} = \frac{x}{n}$ $x = \hat{p} \cdot n = .7 \cdot 1002 = 701$

Integer

5 ⑥

(6) We are $\underset{CL}{\underline{98\%}}$ Confident that the $\underset{\text{Description of Pop parameter}}{\underline{\qquad\qquad}}$

% or
Proportion of people who say ~~they voted~~ in election is between $\underset{CI}{\underline{.666 \text{ and } .734}}$

b) Consistent with Turn out of $61\% = p$

No people lied

Bring Tuesday

Project Part 1 Statistics Jones Math 15 Name____________________________

For Part 1 on the class project everyone will answer these questions individually. This portion of the project is worth 15 points. Do not start collecting data yet. This is Due on Thursday, March 28.

1) Make up a question that you could ask for which the summary statistic will be a proportion, for example
   - A. Do you use a reusable coffee cup?
   - B. Do you print two sided?
   - C. How do you use sustainable transportation like walking, riding, or taking the bus to get to school?
   - D. Is the car in the parking lot an SUV?
   - E. Do you wear a smart watch?
   - F. Your Question?

2) Determine a question that you could ask for which the summary statistic will be a mean, for example

	level	Discrete/Cts
a. How many miles is it from your home to school?	Ratio	Cts
b. How many straws have you used in the last week?	Ratio	Discrete
c. How many ounces of coffee do you drink per day?	Ratio	Cts
d. What is the gas mileage of your car in miles per gallon?	Ratio	Cts
e. How many steps did you take yesterday?	Ratio	Discrete
f. How many minutes did you exercise last week?	Ratio	Cts
g. Your Question?		

3) For your quantitative question and the examples in 2) determine the level of measurement and if the values are discrete or continuous.

You can use these Populati

4) Match these populations to the questions in 1) and 2)
   SRJC faculty and students, A, B, C, E, a, b c
   All men and women, b c
   cars in Safeway parking lot and Grocery Outlet parking lot, D
   people who exercise regularly and do not exercise regularly,
   Gym members and non-gym members
5) Determine two populations to compare for your questions.

6) For both of your questions explain how you could collect your sample data. Refer to the sampling methods in chapter 1. Your method must be something that you can do. (Write on back of page.)
7) When you make inference to your population, explain how your sampling method may introduce bias and how it avoids bias. (Write on back of page.)