§6.4 The Central Limit Theorem

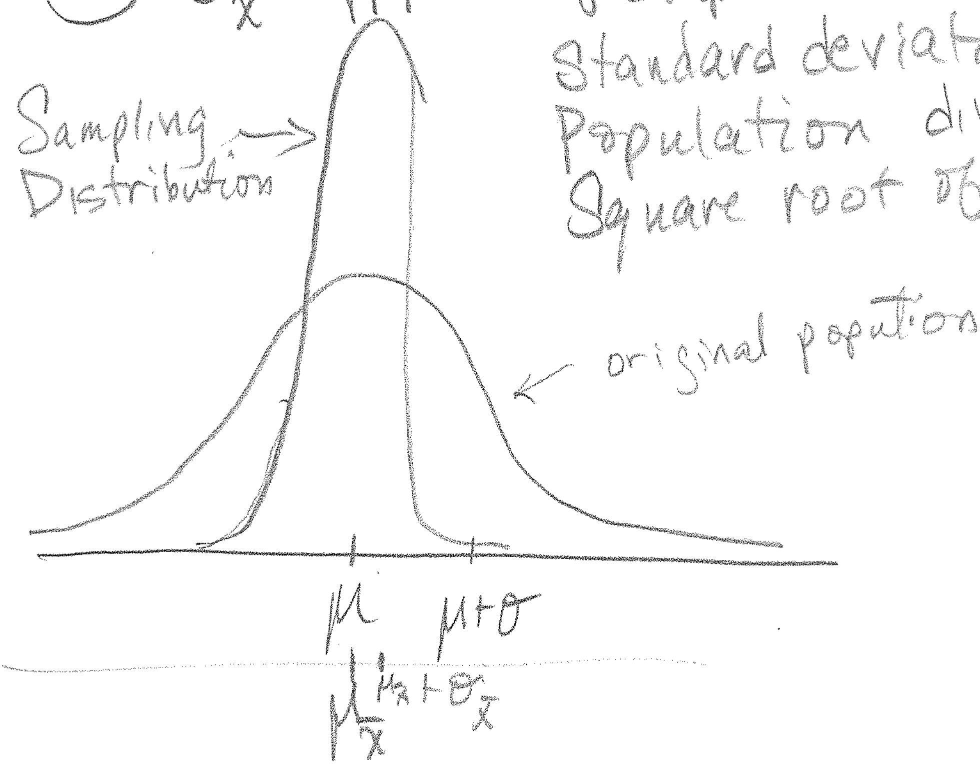
Goal: Understand and Use the Central Limit Theorem

The Central Limit Theorem for Means

① If $n > 30$ or if the original population is normal then the distribution of Sample Means is (close to) Normally Distributed.

② $\mu_{\bar{x}} = \mu$ the mean of all the Sample means is equal to the population mean.

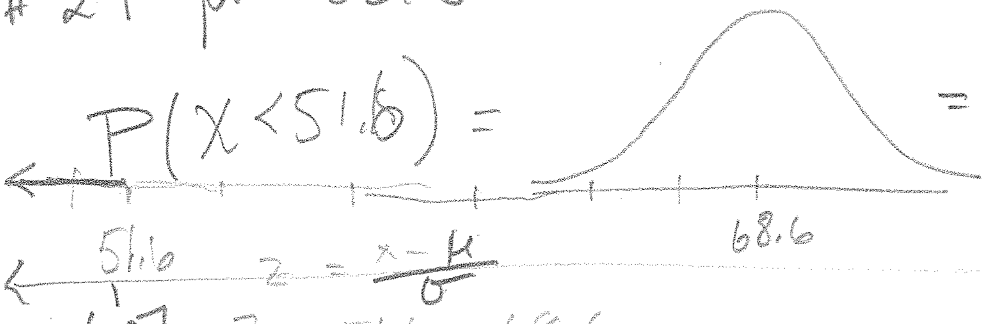
③ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ The Standard deviation of the distribution of Sample means is equal to the Standard deviation of the Original Population divided by the Square root of the Sample Size.



§6.2 #32, ~~2~~4

#24 $\mu = 68.6$ $\sigma = 2.8$

$P(X < 51.6) =$ $\qquad$ LB, UB, μ, σ
$= \text{normalcdf}(-9999, 51.6, 68.6, 2.8)$



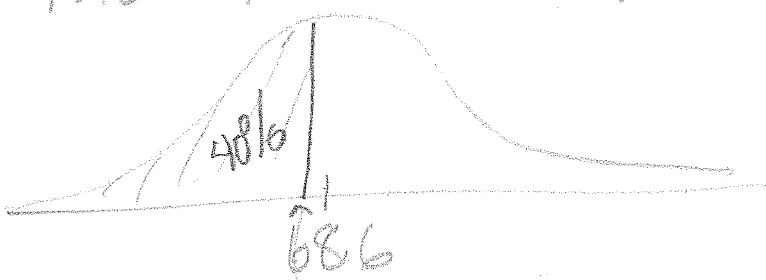
$= 6.36 \times 10^{-10}$

a) $\approx$ $\boxed{.000000000636}$

$Z = \frac{51.6 - 68.6}{2.8} = -6.07$

b) Not large enough but They Need Smaller plane to fly effiently

c) 40% of Men will fit if



$X = \text{inverseNorm}(\text{Area Left}, \mu, \sigma)$ ☆

$X = \text{InverseNorm}(.40, 68.6, 2.8)$

If Door is $\boxed{X = 67.89\text{ in}}$ then 40% men will fit.

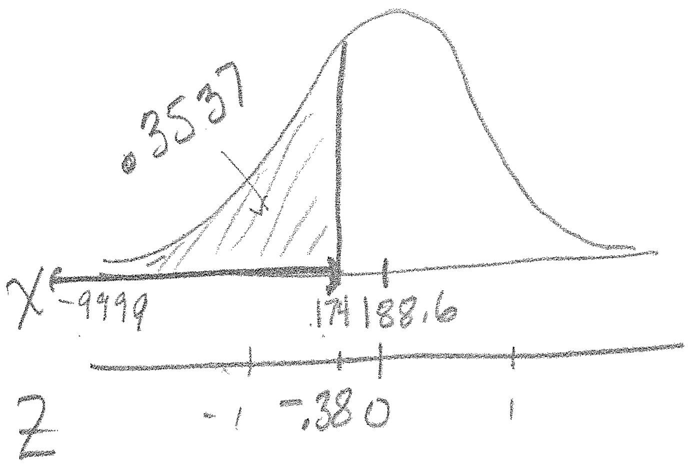
§6.2

32 a) $P(X < 174)$ $\qquad \mu = 188.6$

$= \text{ncdf}(LB, UB, \mu, \theta)$ $\qquad \theta = 38.9$

$= \text{ncdf}(-9999, 174, 188.6, 38.9) = \boxed{.3537}$

$$z = \frac{x-\mu}{\theta} = \frac{174-188.6}{38.9} = \boxed{-.38}$$



b) Load = 3500 lb, How many men @ 140 lb?

n = # men $\qquad n \cdot 140 = 3500$

$$n = \frac{3500}{140} = 25 \text{ men}$$

c ~~b~~) load = 3500 lb but men weigh 188.6 lb

$$n = \frac{3500}{188.6} = 18.5 \rightsquigarrow \text{limit to 18 men}$$

d) Because the mean weight changes

The elevator Question

$\mu = 188.6$ lb $\quad \sigma = 38.91$ lb for Men

An Elevator List Weight Limit of 3125 lb
and passenger limit of 16.
if full?

a) Find Avg. weight per passenger

$$\frac{3125 \text{ lb}}{16 \text{ men}} = 195.31 \text{ lb/man}$$

b) (6.2 Quest) Find proportion of all men who weigh above 195.31 lbs.

$P(X > 195.31) = \text{ncdf}(LB, UB, \mu, \sigma)$

$= \text{ncdf}(195.31, 9999, 188.6, 38.91)$

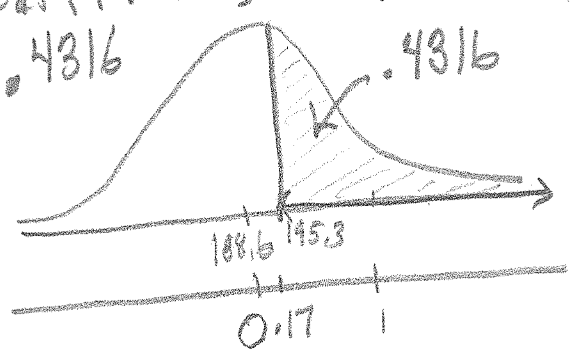
$= .4316$

$$Z = \frac{x - \mu}{\sigma} = \frac{195.31 - 188.6}{38.91}$$

$Z = .17$

Original Population → X

Z



c) (6.4) Find the probability that 16 men have a mean weight above 195.31 lbs. $n = 16$

$P(\bar{x} > 195.31) = \text{ncdf}(195.31, 9999, 188.6, 38.91/\sqrt{16})$

$= \boxed{.2451}$

$\mu_{\bar{x}} = \mu = 188.6$

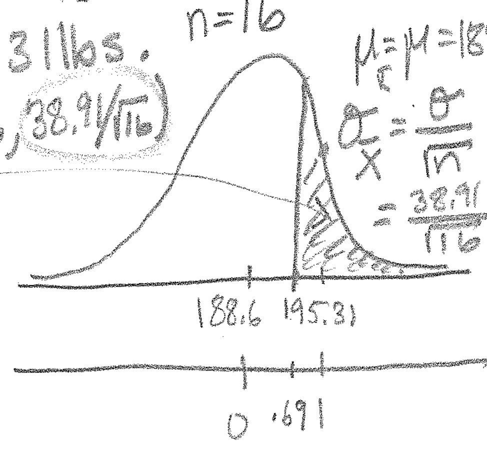
$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{38.91}{\sqrt{16}}$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{(195.31 - 188.6)}{(38.91/\sqrt{16})}$$

$Z = .69$

Sample dist → $\bar{X}$

Z



Quiz 7 6.3 Math 15 Spring 2019 Jones Name________________________

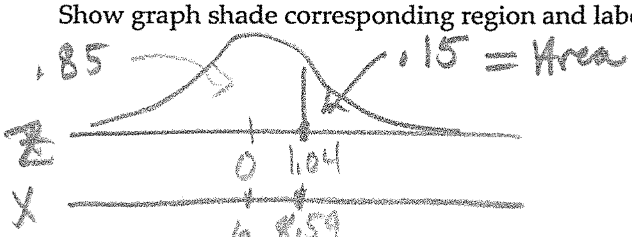
For each part draw a normal distribution with a region that has area equal to the desired probability shaded.

1) (12 Points) The distribution of weights of discarded glass for families of 4 is normally sdistributed with a mean of 6 pounds and a standard deviation of 2.5 pounds. $\mu = 6$ $\sigma = 2.5$

a) (2 Points) Graph the distribution with both an x–axes and a z–axes. Show mean and standard deviation. Calculate the z–score of a family with 8 poinds of garbage.Label both x and z on your graph.

b) (2 Points) What is the probability that a family will have at least 8 lbs? Show all work.
Use proper probability notation, calculator inputs and shade region with equal area on the graph above.
Would it be unusual to get a family with at least 8 lbs?

c) (2 Points) What weight separates the largest 15% of households?
Show graph shade corresponding region and label x and z.



invnorm (Arealeft, μ, σ)

z = invnorm (.85, 0, 1) = 1.04

x = invnorm (.85, 6, 2.5) = 8.59

d) (2 Points) On a given day the inspector samples 25 homes, and finds the sample mean. Use the Central Limit Theorem to find the mean $\mu_{\bar{x}}$ and standard deviation $\sigma_{\bar{x}}$ of the population of sample means for samples of size $n = 25$.

$\mu_{\bar{x}} = \mu = 6$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.5}{\sqrt{25}} = .5$

State CLT
For this prob.

e) (1 Points) Find the z–score of a sample mean $\bar{x} = 8$ lbs in this sampling distribution.

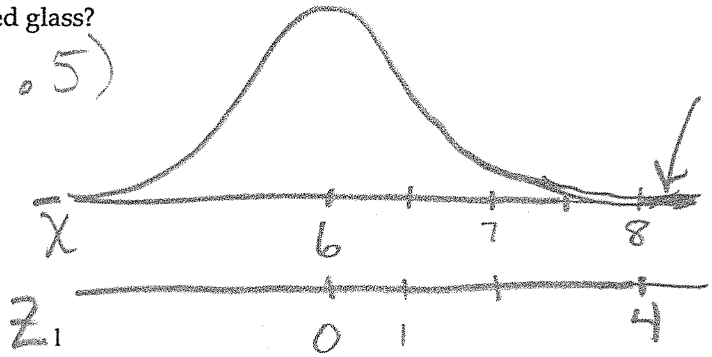
$z = (\bar{x} - \mu_{\bar{x}}) / (\sigma/\sqrt{n}) = (8-6)/.5 = 4$

f) (3 Points) For a sample of size 25, what is the probability that the sample **mean** family will beat lease 8 lbs? Graph the distribution of sample means when the sample size is 25 with both an $\bar{x}$–axes and a z–axes. Label $\bar{x}$, z and shade the region with the desired probability. Would it be unusual to get a sample of 25 homes that have a mean weight of 8 pound of discarded glass?

$P(\bar{x} > 8)$ = ncdf(8, 9999, 6, .5)

= .00000 3168

Yes, very unusual



Find the indicated critical z value.

2) (2 Points) Find the critical value $z_{0.02}$

Draw the corresponding normal distribution and label the area and z-score.

Provide an appropriate response. $\{0,1,2,3,4,5,6,7,8,9\}$

3) (6 points) Samples of size $n = 1500$ are randomly selected from the population of numbers (0 through 9) produced by a random-number generator, and the proportion of odd numbers is found for each sample. According to the Central limit theorem for proportions,

a) What is the shape of the distribution of the sample proportions? ① Normal

① $\hat{p}$ = prop of odds in the Samples

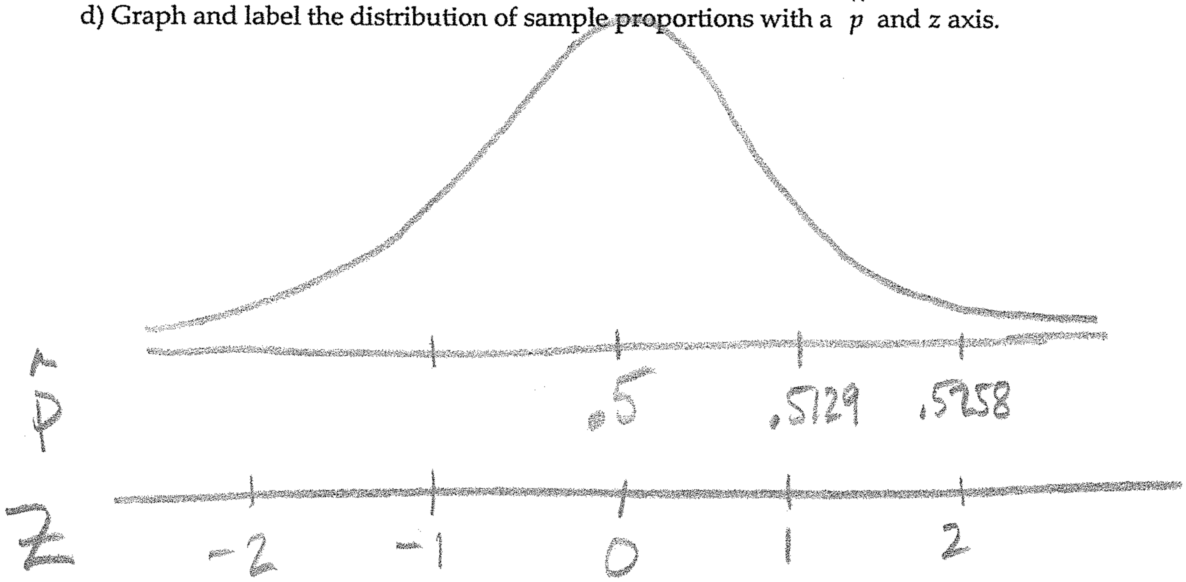
b) What is the mean of the distribution of sample proportions?

② $\mu_{\hat{p}} = .5 = p$

c) What is the standard deviation of the distribution of sample proportions? (Give the correct symbolic variable.)

③ $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.5 \cdot .5}{1500}} = .0129$

d) Graph and label the distribution of sample proportions with a $\hat{p}$ and z axis.



#10
d) A sample statistic is unbiased estimator if $\mu_{\hat{p}} = p$

Mean of the sample proportion is equal to the population proportion

$\{4, 5, 9\}$ = The population

p = poportion of odd in population $\Rightarrow p = \frac{2}{3}$

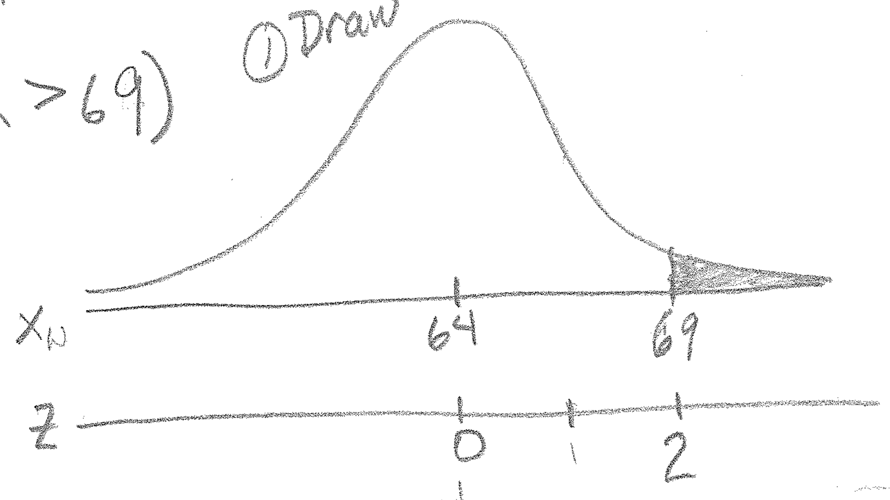
§6.4 Applications of the CLT

__Example__ Suppose women have a mean height of 64 inches and a standard deviation of 2.5.

a) Find the prob an individual woman has a height above 69 inches

$\mu = 64$ $\quad P(X > 69)$ $\quad$ ① Draw

$\sigma = 2.5$



② $Z = \dfrac{X - \mu}{\sigma}$

$Z = \dfrac{69 - 64}{2.5}$

$Z = 2$

③ Put Z-score on Z-axis
Directly above put $X = 69$

④ Shade region cooresponding to prob. you want to find

LB, UB, μ, σ

⑤ $P(X > 69) = \text{normalcdf}(69, 9999, 64, 2.5)$

$= \boxed{.0228} \rightarrow$ or 2.28%

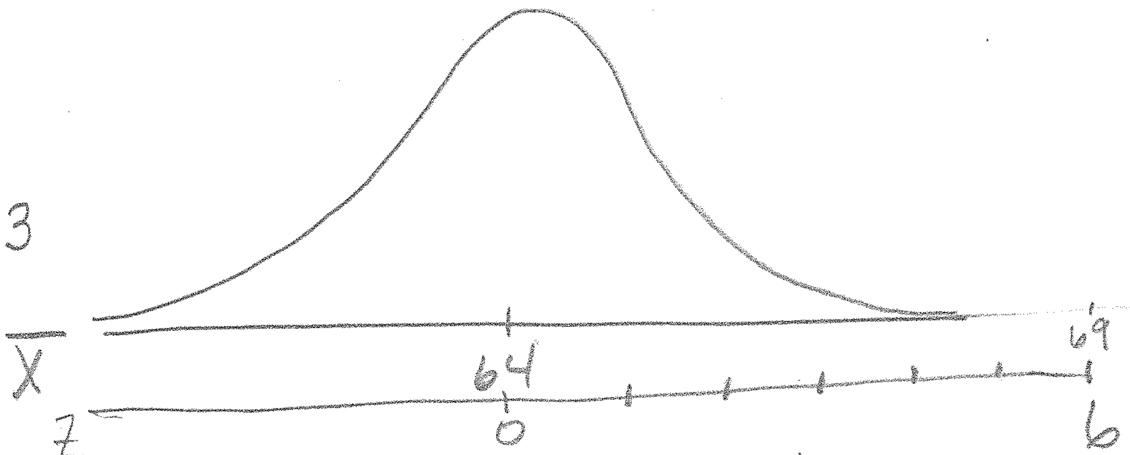
b) If I have a group of 9 women What is the probability that their $\boxed{\text{Mean}}$ height will be above 69 inches?

We have to consider the Sampling distribution of Sample mean when $n=9$

the CLT

$\mu_{\bar{x}} = 64$

$\sigma_{\bar{x}} = \frac{2.5}{\sqrt{9}} = .83$



$$Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma/\sqrt{n}} = \frac{69 - 64}{.83} = 6$$

$$P(\bar{X} > 69) = \text{normalcdf}(69, 9999, 64, .833)$$

$$= 9.88 \times 10^{-10} = .000000000988$$

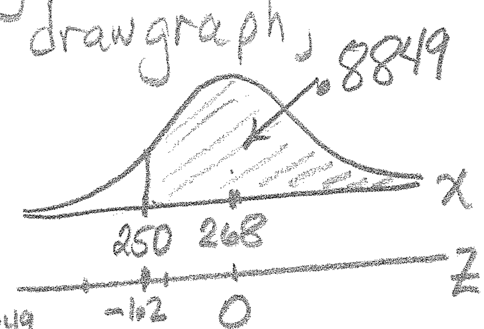
$$< .0001$$

It is very unlikely to get a random group of 9 women who have a mean height that is above 69 inches.

Class Try

Lengths of pregnancies are Normally Distributed With a Mean of 268 days and a Standard deviation of 15 days.

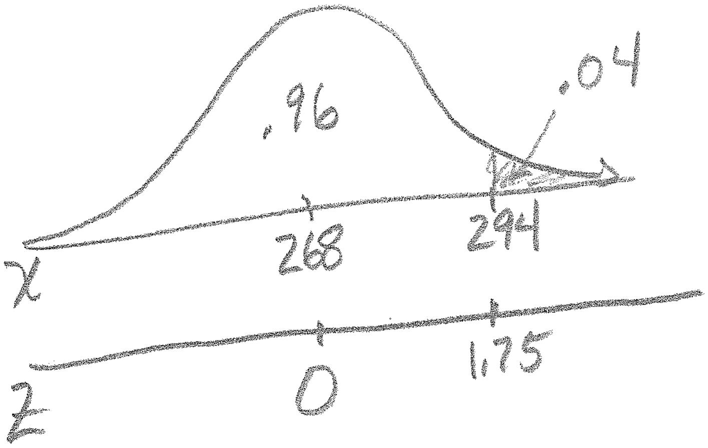
a) Find prop. of pregnancies lasting more than 250 days. Find z, draw graph, label. x, z, μ, and Area.



$Z = \frac{x - \mu}{\sigma} = \frac{250 - 268}{15} = \frac{-18}{15} = -1.2$

$P(x > 250) = \text{normalcdf}(250, 9999, 268, 15) = .8849$

b) How many days will the longest 4% of Pregnancies last?



$Z = \text{invnorm}(.96, 0, 1)$
$= 1.75$

$X = \text{invnorm}(.96, 268, 15)$
$= 294$

To get from z to Value = x

$\sigma \cdot Z = \frac{x - \mu}{\sigma} \cdot \sigma$

$Z\sigma = X - \mu$

$+\mu \quad + \mu$

$X = Z\sigma + \mu$

$\boxed{X = \mu + \sigma Z}$

$X = 268 + 15 \cdot 1.75 = 294$

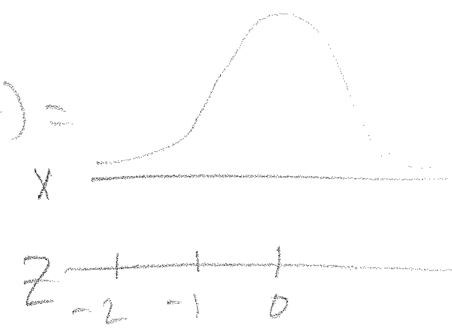
Class try

Manufacturer fills Soda bottles to a Mean of $\mu = 32.3$ oz with $\sigma = 1.2$ oz

a) Find prop of Soda bottles with less than 32 oz.

b) If you buy 12 bottles find the proportion of the time that the MEAN weight is less than 32 oz

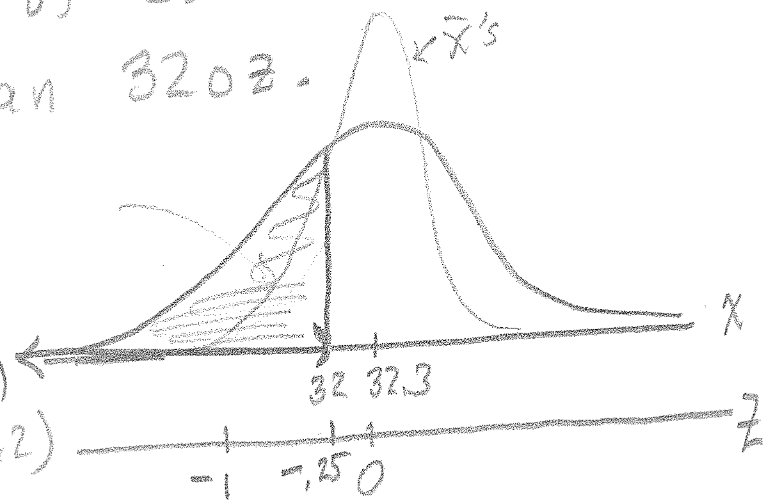
a) $Z = \frac{X - \mu}{\sigma}$ $\quad P(X < 32) =$ ncdf(LB, UB, μ, σ) =



Class Try

Soda Bottles have $\mu = 32.3$ oz and $\sigma = 1.2$ oz.

a) Find the poportion of Soda bottles that have less than 32 oz.

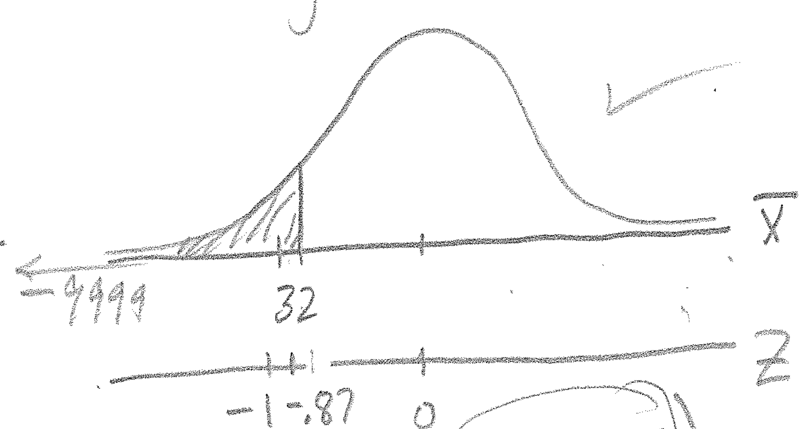


$$Z = \frac{X - \mu}{\sigma} = \frac{32 - 32.3}{1.2} = -.25$$

(LB, UB, μ, σ)

$P(X < 32) = \text{ncdf}(-9999, 32, 32.3, 1.2)$
$= \boxed{.4013}$

b) If you buy 12 bottles, Find the proportion of the time that the mean weight is less than 32 oz.



$$Z = \frac{\bar{X} - \mu_{\bar{x}}}{\sigma/\sqrt{n}} = \frac{32 - 32.3}{(1.2/\sqrt{12})} = \boxed{-.87}$$

$P(\bar{x} < 32) = \text{ncdf}(-9999, 32, 32.3, (1.2/\sqrt{12}))$
.346
only change
$= \boxed{.1932}$

§6.4 The Central limit Theorem For Proportions

1. Dist of Sample prop is Normal
   $np > 5 \quad nq > 5$

2. $\mu_{\hat{p}} = p$

3. $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$

Four our example Population $= \{5, 7, 14\}$
Population prop of odds is $p = \frac{2}{3}$

So $\mu_{\hat{p}} = \frac{2}{3} = \frac{1+1+.5+1+1+.5+.5+.5+0}{9} = \frac{2}{3}$

Write as a prob. Dist.

X	1	.5	0
P(x)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

1-varStat $\cancel{\bar{x}} = \frac{2}{3}$ $\mu = \frac{1}{3}$

$\mu_{\hat{p}} = \frac{2}{3}$

$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$

$\sigma_{\hat{p}} = \sqrt{\frac{2/3 \cdot 1/3}{2}} = \frac{1}{3}$

$p = \frac{1}{3}$ = prop. of even

Ex Let $\{5, 7, 14\}$ be a population

Note $\mu = \frac{5+7+14}{3} = 8.67$ σ = 1-Var stats L_1

$\sigma = 3.8586$

$n=2$

Samples	$\bar{x}$	$\hat{p}$ = prop of even
5, 5	5	0
5, 7	6	0
5, 14	9.5	.5
7, 5	6	0
7, 7	7	0
7, 14	10.5	.5
14, 5	9.5	.5
14, 7	10.5	.5
14, 14	14	1

Mean Probability Distribution

$L_1 = \bar{X}$	$P(\bar{x}) = L_2$
5	1/9
6	2/9
7	1/9
9.5	2/9
10.5	2/9
14	1/9

(1) $\mu_{\bar{x}} = 8.667 = \mu$ ✓

(3) $\sigma_{\bar{x}} = 2.728 = \frac{\sigma}{\sqrt{n}} = \frac{3.858}{\sqrt{2}}$

Prop. Prob Dist

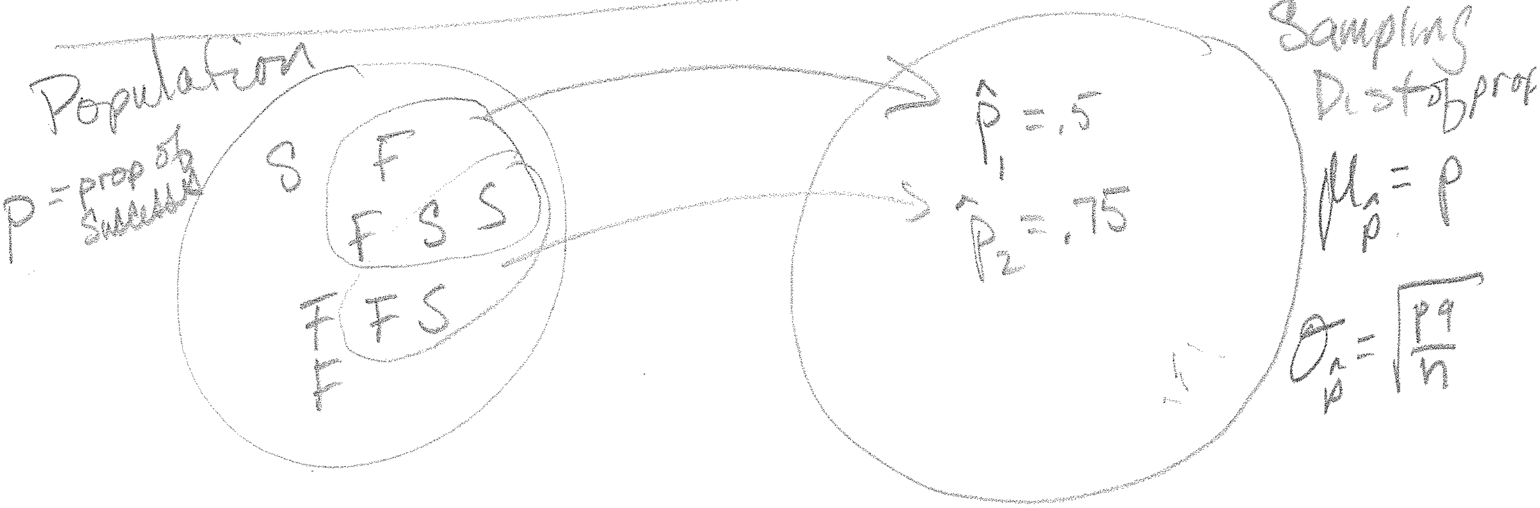
$\hat{p}$	$P(\hat{p})$
0	4/9
.5	4/9
1	1/9

$\mu_{\hat{p}} = \sum x \cdot p(x) = 0 \cdot \frac{4}{9} + .5\,\frac{4}{9} + 1 \cdot \frac{1}{9}$
$= .333$

$\sigma_{\hat{p}}$ = 1-Var stats L_4, L_5 $\mu_{\hat{p}} = \frac{1}{3} = .333$

$\sigma_{\hat{p}} = \sqrt{\frac{p\,q}{n}} = \sqrt{\frac{1/3 \cdot 2/3}{2}} = \frac{1}{3}$

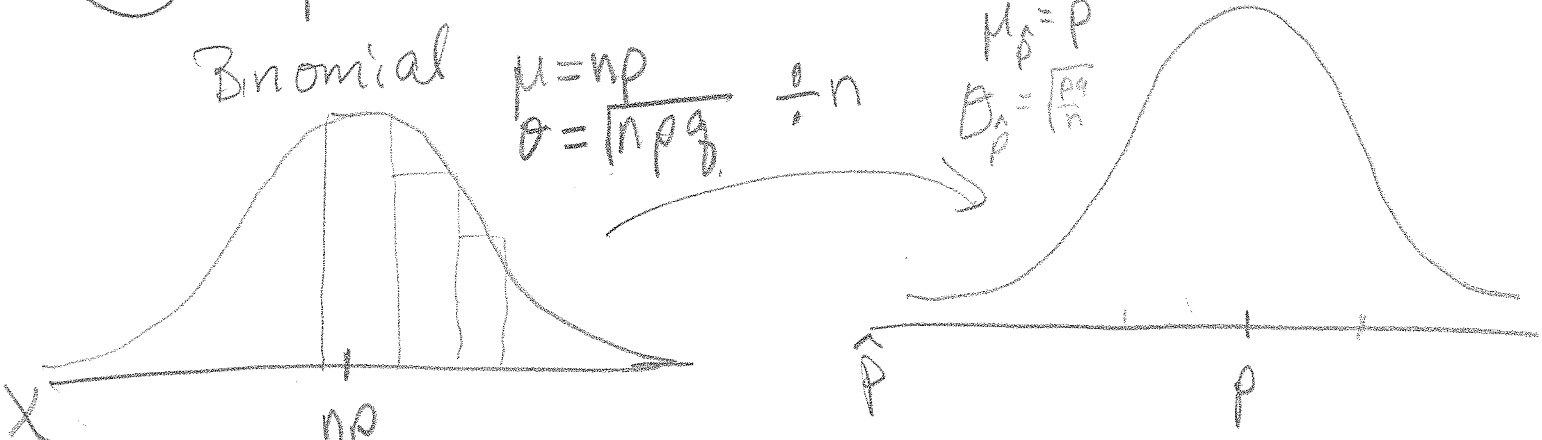
Central Limit Theorem for Proportions



① $\mu_{\hat{p}} = p$

② If $np > 5$ and $nq > 5$ then
Dist of Sample proportions is Normal

③ $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$



Which Sample Statistics Target their population parameters?

Means $\rightarrow$ $\mu_{\bar{x}} = \mu$ Mean of Sample Means is equal to pop Mean

Prop. $-$ $\mu_{\hat{p}} = p$ Mean of Sample prop is equal to pop. prop.

If $n > 30$ or $np > 5$ & $nq > 5$ then Dist are Normal

Variances $\mu_{\sigma^2} = \sigma^2$

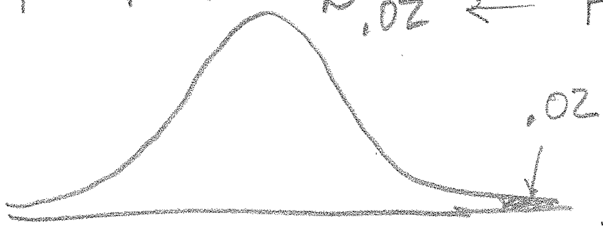
Which Don't Work?

Meadian $\mu_{med} \neq$ Median of population meadian
of Sample medians

$\mu_{range} = range$

$\mu_{\sigma} = \sigma$

#42 find $z_{.02}$ ← Area in Right Tail



$z = \text{invnorm}(\text{Arealeft}, 0, 1)$

$= \text{invnorm}(1 - .02$

$z = 2.05$