§6.3 Sampling Distributions

Distributions of Sample Statistics

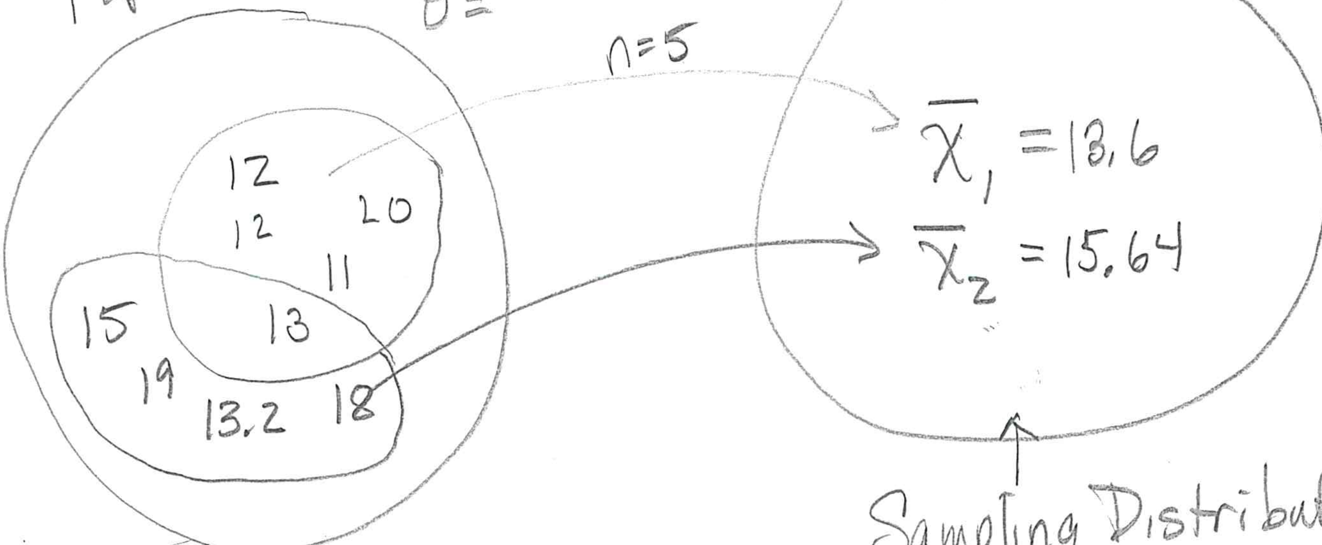
Statistics → mean, proportion, median, range,

$\bar{x}$, $\hat{p}$, s, s^2

The distribution of all sample means

Population $\mu=$

$\sigma=$



N=23

$\mu = 16.326$

$\sigma = 3.813$

Sampling Distribution

Takes every possible Sample of size n=1

Makes New Distribution

§6.3 Sampling Distributions

Normal Distributions are Very important to Statistics.

6.4 The central limit theorem

① the distribution of Sample Means or proportions is Normal

6.3 Goal: Understand what a distribution Of Sample means our proportions is.

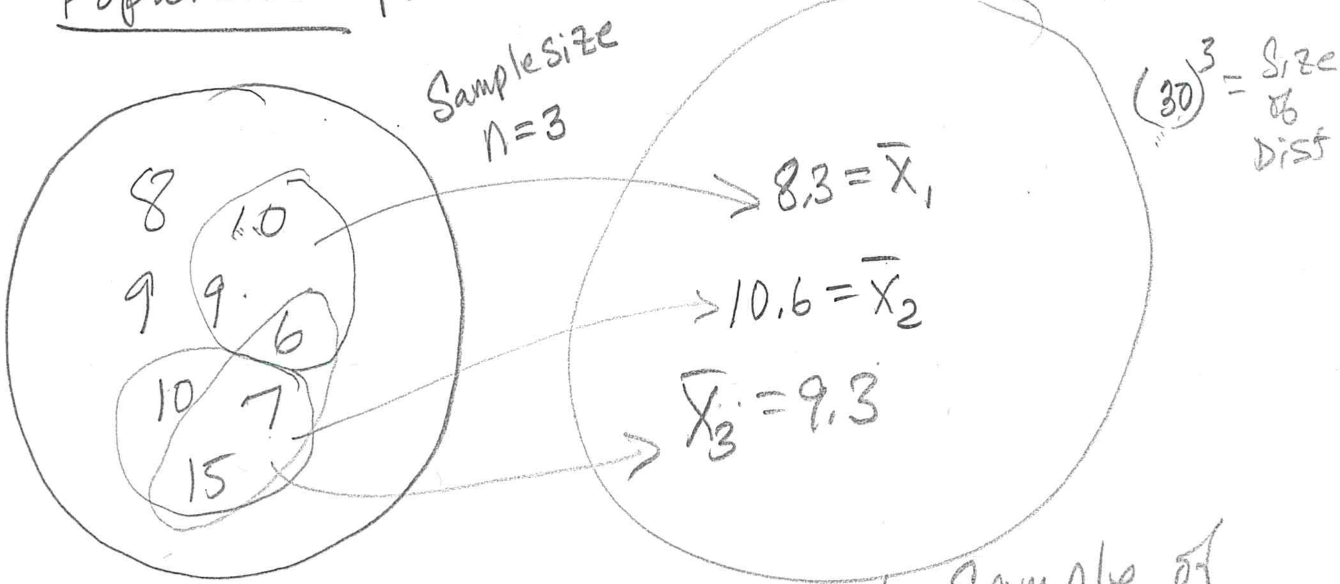
Sampling Distribution of Means

Time for a shower

$\mu =$

Population $\mu = 30$

Distribution of Sample Means



Find the mean of every possible sample of size 3.

Central limit theorem

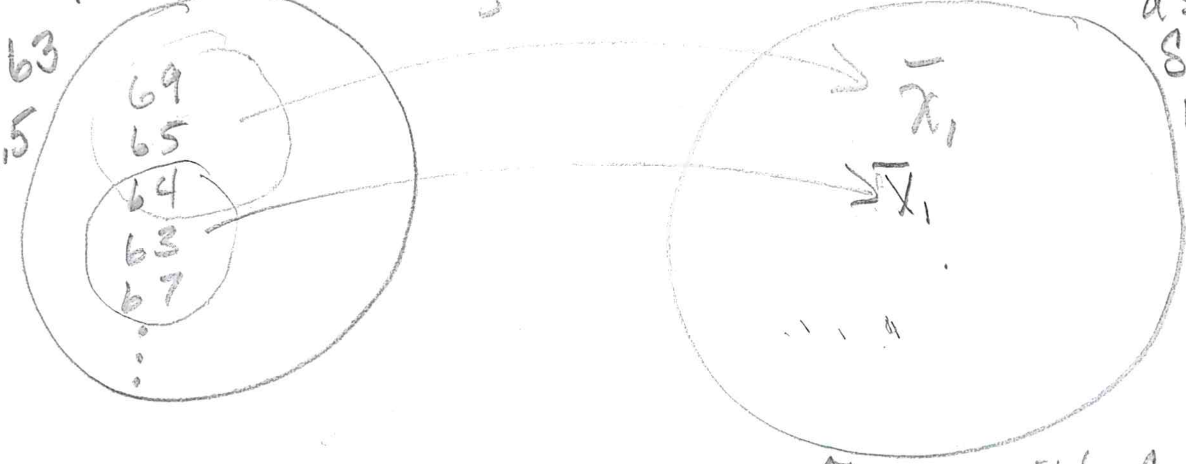
1. $\mu_{\bar{x}} = \mu$
2. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
3. Dist is Normal

Sampling Distribution of Means

Population = All womens Heights

$\mu = 63$

$\sigma = 2.5$



Sampling Dist

a set of Sample means All of size n

$\mu_{\bar{x}} =$

Mean of every possible Sample of size n.

CLT for Means

(1) $\mu_{\bar{x}}$ = mean of the Sample Means $= \mu$

(2) Distribution of Sample means is Normal

if (1) Original Population is Normal

"OR"

(2) $n > 30$

(3) $\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$ = Standard deviation of Distribution of Sample Means

$N=3$

<u>Ex</u> Let $\{5, 7, 14\}$ be the population

Let $n = 2$ = Sample Size, Sample with replacement

$n=2$ Samples	$\bar{X}$	$\hat{p}$ = odd	med.	range
5,5	5	1		
5,7	6	1		
5,14	9.5	.5		
7,5	6	1		
7,7	7	1		
7,14	10.5	.5		
14,5	9.5	.5		
14,7	10.5	.5		
14,14	14	0		

Distribution of Sample proportions

This is the Sampling Distribution of all Sample means.

Now Check Central limit theorem

a) Write Sampling Distribution as a Probability Distribution

List each possible out come and it's probability

$L_1 = x$	5	6	9.5	7	10.5	14
$L_2 = P(x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

Find $\mu_{\bar{x}} = E(x) = \sum x \cdot p(x) = 8.67 = \mu_{\bar{x}}$

1-VarStats $\sigma_{\bar{x}} = 2.72845$

Check CLT

$$\mu_{\bar{x}} = \mu = \{5, 7, 14\} = \frac{5+7+14}{3} = 8.67$$ ✓

$$\sigma_{\bar{x}} = 2.728 = \frac{\sigma}{\sqrt{n}} = \frac{3.8586}{\sqrt{2}}$$

Pick 4 Gas Tank Volumes $\{15, 19, 18, 21\}$

= The population $\mu = \frac{(15+19+18+21)}{4} = 18.25$

All possible Subsets of Size 2 with replacement

① Samples | ② $\overline{X}$ | $\hat{p}$

Samples	$\overline{X}$	$\hat{p}$
{15, 15}	15	1
15 19	17	1
15 18	16.5	.5
15 21	18	1
19 15	17	1
19 19	19	1
19 18	18.5	.5
19 21	20	1
18 15	16.5	.5
18 19	18.5	.5
18 18	18	0
18 21	19.5	.5
21 15	18	1
21 19	20	1
21 18	19.5	.5
21 21	21	1

↑ Sampling dist of mean

↑ Size of Sample Space is $4 \cdot 4 = 16$

③ Probability Distribution for $\overline{x}$

$\overline{X}$	$p(\overline{x})$ ④
15	1/16
16.5	2/16
17	2/16
18	3/16
18.5	2/16
19	1/16
19.5	2/16
20	2/16
21	$\frac{1}{16}$

Find Mean of Prob. Dist

$\overline{x} = \boxed{18.25 = \mu_{\overline{x}}}$

mean of Sampling Distribution equals population Mean $\mu = 18.25$

$\hat{p}$ = proportion of odd Numbers

Prob. Dist. for the $\hat{p}$

X	0	.5	1
P(X)	$\frac{1}{16}$	$\frac{6}{16}$	$\frac{9}{16}$

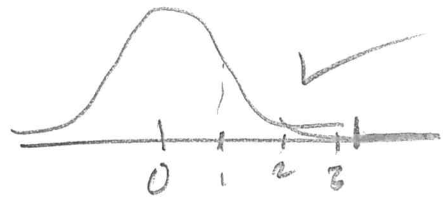
$\mu_{\hat{p}} = .75 = p$

How many gallons of gas does your cars gas tank hold?

Gal
12
12
~~13.5~~ 20
11
13
22
15
19
13.2
18
16.6
10
14.5
22.5
17
18
13.2
21.1
21.5
16
13.5
15

$N = 23$ = population size

6.1 MML #1 $P(Z \geq 3.25) =$



$= \text{normalcdf}(LB, UB, \mu, \sigma)$

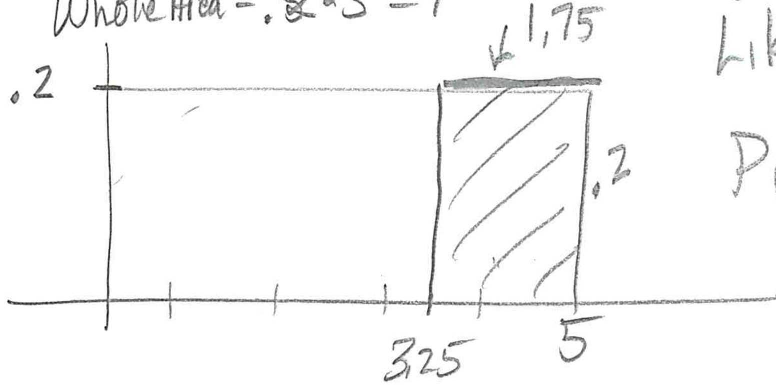
$= \text{normalcdf}(3.25, 9999, 0, 1) = .000577$

$.0006$

6×10^{-4}

MML #7 Uniform Distribution = every outcome between 0 & 5 is equally likely

Whole Area $= .2 \cdot 5 = 1$

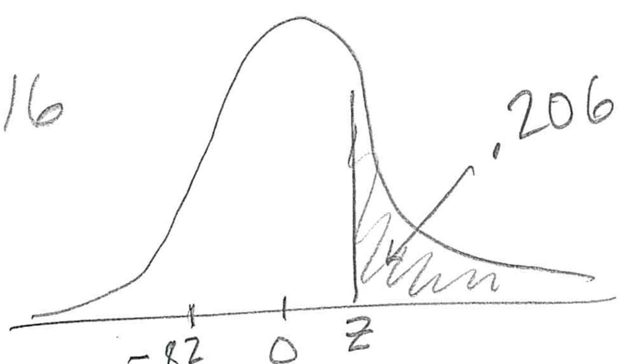


$P(3.25 < X < 5) =$ Area of rectangle

$= 1.75 \cdot .2$

$= \boxed{.35}$

6.1 #16



$z = \text{invnorm}(\text{Area Left}, \mu, \sigma)$

$z = \text{invnorm}(1 - .2061, 0, 1) = \boxed{.82}$

z $\xrightarrow{\text{normalcdf}}$ Area

Area $\xrightarrow{\text{Inv norm}}$ z

#42 $Z_{.02}$ is the Z-score with an area of .02 to the right

Definition of a Critical Value

Always Area Left

$Z_{.02} = \boxed{\text{invnorm}(1-.02, 0, 1)}$

$\boxed{Z_{.02} = 2.05}$

#13 regions = 576 Area = .25 km^2

535 bombs

$$\mu = \frac{535 \text{ bombs}}{576 \text{ regions}} = 0.929$$

a) $P(X=2) = \frac{\mu^x e^{-\mu}}{x!} = \frac{.929^2 e^{-.929}}{2!} = .170$

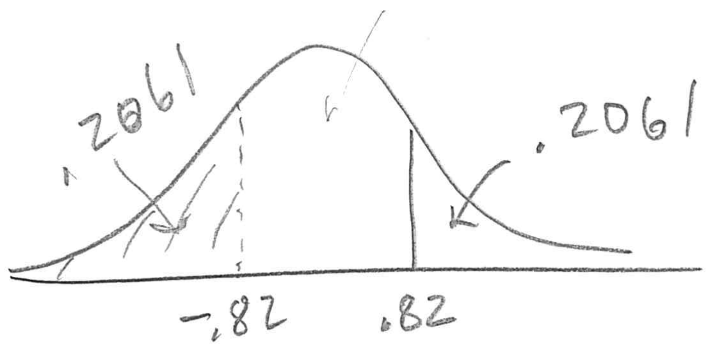
x = 2 Times

b) $E(x)$ = 17% of the 576 regions will be hit Twice

$= .17 \cdot 576 \approx 98.2$

c) Actual = 93 this is close.

6.1 #16



invnorm(.2061, 0, 1) = −.82

$\uparrow$ Area left

invnorm(1 − .2061, 0, 1)

(.7939, 0, 1) = .82