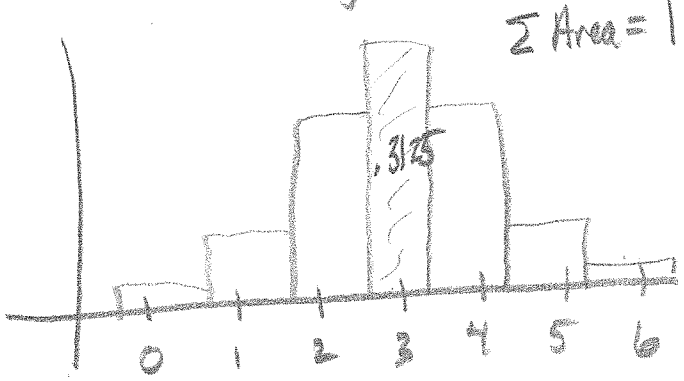
§5.1a Probability Distributions

X = Random Variable = A Numerical Outcome determined by a random experiment.

Random Variables have Probability distributions

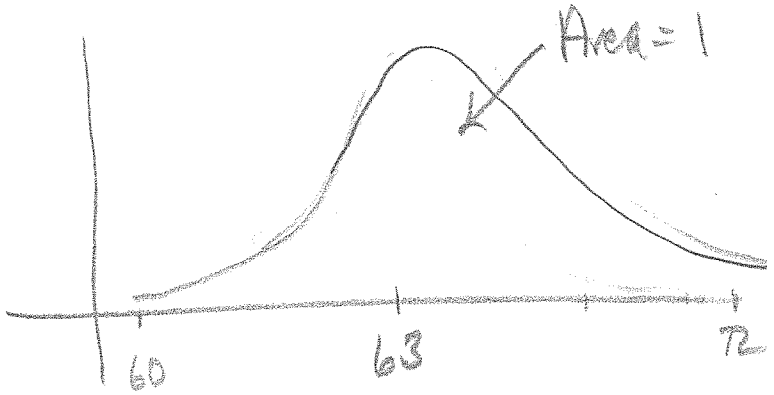
Chapter 5
Discrete
Counted

X = # of heads when 6 coins Tossed
Is Binomial



Chapter 6
Continuous
Measured

X = heights



A Probability Distribution is the graph or Table giving the Probability of an outcome.

Goals:

1. Find Probabilities
2. Find the Mean and Standard deviation of a probability Dist.
3. Applications of the expected Value of a prob. Distribution.

Ex Given the probability Distribution

X = # of Heads in 6 Tosses

X	0	1	2	3	4	5	6
$P(x)$	.0156	.09375	.2344	.3125	.2344	.09375	.0156

a) $P(X=0) = .0156$ ← Write # in Table

b) Find probability of at most 3 heads.

$P(X \leq 3) = .0156 + .09375 + .2344 + .3125$

$= .65625$

c) Show that this is a Probability Distribution.

2 Requirements (1) $\sum p(x) = .0156 + .09375 + \cdots + .0156 = 1$

(2) $0 \leq p(x) \leq 1$

Given Prob. Dist

a) Find the probability of exactly 3 heads?

$P(x=3) = .3125$

b) Find probability of at most 3 heads?

$P(X \leq 3) = .0156 + .09375 + .2344 + .3125$

$= .65625$

c) Find probability of at least 1 head?

$P(X \geq 1) = 1 - P(x=0)$

$= 1 - .0156 = \boxed{.9844}$

Ex1 Toss 6 coins, Let

x = # of heads in 6 coins = Random Variable

Make the Probability Distribution

↓ list all possible values for x

x	0	1	2	3	4	5	6
$P(x)$	.0156	.09375	.2344	.3125	.2344	.09375	.0156

a) What is the probability of No heads.

$P(HHHHHH) = \frac{1}{2^6} = \frac{n(f)}{n(S)} = \frac{n(\text{All heads})}{n(S)} = \left(\frac{1}{2}\right)^6 = .015625$

$P(X=6)$

$S = \{$ All possible Outcomes $\}$ $F =$

$F = \{ HHHHHH, \ldots$ n

b) What is prob. one head

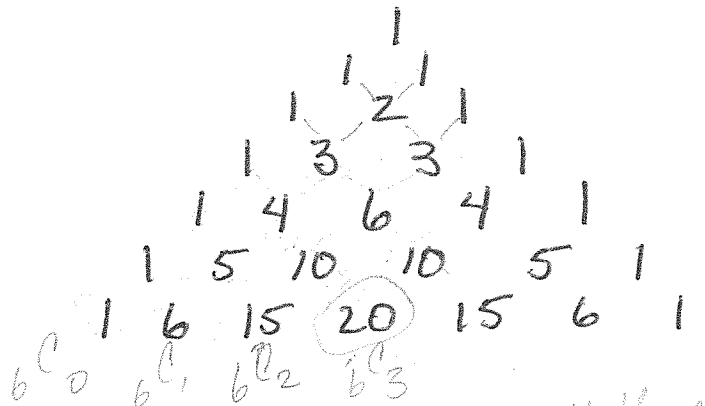
$E = \{ HTTTTTT, THTTTTT$

$P(\text{one head}) = P(X=1) = \frac{6}{2^6} = .09375$

$P(X=1) = 6 \cdot \left(\frac{1}{2}\right)^6$

$P(X=2) = 15 \cdot \left(\frac{1}{2}\right)^6 = {}_6C_2 \cdot \left(\frac{1}{2}\right)^6$

HHHTT



Can't Repeat Number 1,1 Not OK Not Multiple

Pick which 2 spots the head will occur at. order doesn't matter Not different so Combinations

$$_6C_2 = \frac{6!}{4!2!} = \frac{6 \cdot 5}{2} = 15$$

$A = \{6 \text{ tosses that contain 2 heads}\}$

$n(A) = 15$

H	T	H	T	T	T
1	2	3	4	5	6

6 spots

Choose 2 of 6 spots where the heads will appear

6

T T T T T T

H T T T T T
T H T T T T
T T H T T T
T T T H T T
T T T T H T
T T T T T H

15

H H T T T T
H T H T T T
H T T H T T
H T T T H T
H T T T T H
T H H T T T
T H T H T T
T H T T H T
T H T T T H
T T H H T T
T T H T H T
T T H T T H
T T T H H T
T H T H T H
T T T T H H

1,2 ← 2,1
1,3
1,4

2,3 ← 3,2
2,4
2,5
2,6
3,4
3,5
3,6
4,5
4,6
5,6

Same Toss $\{1,3 \quad 3,1$

So order doesn't Matter So there are

3,5

A table like this is a probability Distribution if

① Sum of Prob $= \sum P(x) = 1$

② $0 \le P(x) \le 1$

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a) Is this a probability distribution?

yes because $\Sigma p = .0156 + .0938 + \cdots + .0156 = 1$

and $0 \le p(x) \le 1$ for each x.

b) What is the probability of getting 5 heads?

Prob Notation

$P(X=5) = .0938$

Not unusual because $P(X=5) = .0938$

d) Probability of at least 5 heads?

$P(X \ge 5) = P(X=5) + P(X=6$

$= .0938 + .0156$

$= .1095$

d) Find probability of <u>at most</u> 2

$P(X \le 2) = .0156 + .0938 + .2344 = .3438$

A distribution is a Probability Distribution if

① $\sum P(x) = 1$

② $0 \le p(x) \le 1$

Read a probability Distribution

Ex a) $P(x=4) = .2344$

↑ You write Book writes What is the probability of getting 4 heads

b) $P(\text{No heads}) = P(x=0) = .0156$

c) $P(\text{At least one head}) = 1 - P(\text{No heads})$

$P(x \ge 1) = 1 - P(x=0) = 1 - .0156$

$= .9844$

d) $P(x \ge 5) = P(\underline{\text{At least}}\ 5 \text{ heads})$

↑

$= P(x=5) + P(x=6)$

$= .09375 + .0156$

$= .10935$

Translate Inequalities

- x less than 5 $x < 5$
- x more than 5 $x > 5$
- x At least 5 $x \geq 5$ ★
- x At most 5 $x \leq 5$

The Mean and Standard Deviation

$\mu = \sum x \cdot p(x)$ → weighted Average

Each outcome • the probability of that outcome

$\mu = \sum x \cdot p(x) = 0 \cdot .0156 + 1 \cdot .09375 + 2 \cdot .2344 + \dots + 6 \cdot .0156 = \boxed{3}$

$$\sigma = \sqrt{\sum x^2 p(x) - \mu^2}$$

$$\sigma = \sqrt{0^2 \cdot .0156 + 1 \cdot .09375 + 4 \cdot .2344 + 9 \cdot .3125 + 16 \cdot .2344 + 25 \cdot .09375 + 36 \cdot .0156 - 9}$$

$\sigma = 1.2246$

But! there is an easier way!

$L_1 = X$

$L_2 = P(x)$

1-VarStat L_1, L_2

List = L_1

freq. = L_2

$\mu = \bar{x} = 3$

$\sigma = 1.2256$

Sh11S.1 **Expected Value** of a probability Distribution is the Mean $= \mu = \overline{X}$

Applications

1. Games of chance
2. Insurance

Ex Flip 4 Coins.
Win \$10 if 4 heads → HHHH
Win \$3 for 3 heads → HHHT, ★
Lose \$2 Otherwise.

What is the **expected value** of this game?
Average Amt of win or loss per game if played many times.

1. Make a Probability Distribution X = Net winnings

X = \$	10	3	−2
$P(x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{11}{16}$

↑ Sum must be = 1

$\frac{1}{16} + \frac{4}{16} + ? = 1$

$? = 1 - \frac{5}{16}$

$? = \frac{11}{16}$

2. $E(x) = \mu = \sum x \cdot p(x)$

$= 10 \cdot \frac{1}{16} + 3 \cdot \frac{4}{16} - 2 \cdot \frac{11}{16}$

$= 0$ ↝ Game is fair

Insurance Fire insurance for $300,000 costs $130 per year. The probability that the house burns down is .0003. (Actuary) Find the **Expected Value** of this policy to the insurance company

Outcomes	Burns Down	Good
$X = \$$	$130 - 300,000$	130
$P(x)$	$.0003$	$1 - .0003 = .9997$

$E(x) = -299,870 \cdot .0003 + 130 \cdot .9997$

$E(x) = 40 \rightsquigarrow$ On Average the company Makes $40 per policy Per year

$$\$40 = \frac{130 + 130 + \cdots + -299,870 + 130}{10,000}$$

Ex2 70% of Adults have college degrees in a certain Town.
If 4 adults are randomly selected this table gives X = # with college degrees

X	P(x)
0	.0081
1	.0756
2	.2646
3	.4116
4	.2401

a) Show this is a prob. Dist
$\sum P(x) = 1$ and $0 \le P(x) \le 1$ for each x
So it is a prob. Dist.

b) $P(X=4) = .2401$

c) Find the probability of at least one with a college degree?

$P(\text{At least one}) = P(X \ge 1) = 1 - P(X=0)$ §4.5
$= 1 - .0081 =$ **.9919**

d) Find Mean and Standard deviation
Don't use formulas use 1-Varstat L_1, L_2
List = $L_1 = X$
freq = $L_2 = P(x)$

$\bar{X} = \mu = 2.8$
$\sigma_x = \sigma = .9165$

Population parameters

Games of Chance and Insurance

Ex Prizes in a Sweepstakes and their chances of winning are listed

X = Net Winnings = winning − ticket price

Outcome	P(x)	X
Win $5000	$\frac{1}{8900}$	4999
Win $2300	$\frac{1}{7000}$	2299
lose $1	Rest	−1

$$P(\text{lose}) = 1 - \frac{1}{8900} - \frac{1}{7000} = .99974478$$

Find the expected value of this Game?

Expected Value = $E(X) = \mu$ of probability Dist

$$E(x) = \mu = \sum x \cdot p(x) = 4999 \cdot \frac{1}{8900} + 2299 \cdot \frac{1}{7000} + -1 \cdot (.99974478)$$

$$= -.1096$$

On average player loses $\$-.1096$ per game while the group makes 11¢ per ticket on average.

Lottery Problem

Pick 4 problem and # from 0 to 9

$\underline{2}\ \underline{3}\ \underline{3}\ \underline{7}$

a) How many Selection? $10^4 = 10{,}000$ 0000 to 9999

b) What is prob of winning? $P(\text{win}) = \frac{1}{10{,}000}$

c) Win $500 and each ticket cost $1
Find the expected value of the game?

Key words "Expected Value"

Make a prob. dist. X = Net out

	X	P(x)
Win	499	$\frac{1}{10000} = .0001$
Lose	−1	.9999

$E(X) = \mu = \Sigma X \cdot P(x)$

$= 499 \cdot .0001 + -1 \cdot .9999$

$= -.05$

§4.3 #6, 3 Q2

§4.4 #28

4.3 #22 $P(\text{fail}) = .22$ P first and 2nd and 3rd fail

Find $P(\text{At least one of 3 is working}) = 1 - P(\text{None are working})$

$= 1 - P(\text{All 3 fail})$

$= 1 - P(\text{1st fail}) \cdot P(\text{2nd fail}) \, P(\text{3rd fails})$

$= 1 - (.22)^3 = .9894$

⇒ Might Not be good enough

4.4 #28 a) ${}_{16}P_{14} = \frac{16!}{2!} = \frac{16 \cdot 15 \cdot 14 \cdots 3}{2} = 1.046 \times 10^{13}$

10,461,394,944,000

b) ${}_{16}C_{14} = 120$

c) $\frac{1}{120} = P(\text{14 youngest})$

4.4

#28 Select & Arrange 14 from 16 is

a) $_{16}P_{14} = 1.046 \times 10^{13} = 10461394940000$

b) How many groups of 14 from 16

$_{16}C_{14} = 120 = 16 \cdot 15$

c) Prob of 14 youngest?

$$P(14 \text{ youngest}) = \frac{1}{120} = \boxed{.00833}$$

4.3 #20 $P(\text{No hepatitis} \mid \text{Neg. Result})$

$$P(\bar{H} \mid N) = \frac{1153}{1163} = \boxed{.9914}$$

	P	N
H	355	10
$\bar{H}$	2	1153
		1163

↑ only look at subjects with a Neg. Result

Effective at showing people that don't have hepatitus that they are Not infected

$$P(H \mid N) = \frac{10}{1163} = .0086$$

§4.4 8, 23, 28 R10

#8 a) Choose 3 players from 11 $\rightsquigarrow {}_{11}C_3 = \boxed{165}$ ✓

b) Arrange 3 players 1st 2nd 3rd (After picking 3) $\rightsquigarrow {}_3P_3 = 3! = \boxed{6}$ ✓

#23 a) Arrange 4 officers from 10 Candidates $= {}_{10}P_4 = \boxed{5040}$ ✓

b) Committee of 4 (How many ways) from 10 ${}_{10}C_4 = \boxed{210}$ ✓

c) Prob. of 4 youngest $= \frac{\#\text{ groups with 4 youngest}}{\#\text{ of groups}} = \boxed{\frac{1}{210}}$ ✓

#28 a) Sequences (# of) of 14 from 16 $= {}_{16}P_{14} = 1.05 \times 10^{13}$ ✓

$= 10,461,394,944,000$

b) Select 14 from 16 $= {}_{16}C_{14} = 120$ ✓

c) $\boxed{\frac{1}{120}}$ ✓

4.3

Prob Notation frac Decimal

#20 $P(H \mid \underset{N}{\ominus}) = \frac{10}{1163} = \boxed{.0086}$

#22 $P(\text{At least one works}) = 1 - P(\text{All don't work})$

$= 1 - (.22)^3 = \boxed{.9894}$

$P(\text{All 3 work}) = (.78)^3$

#10 Mult choice test

$P(\text{one Right}) = \frac{1}{5}$ $P(\text{one Wrong}) = \frac{4}{5}$

$P(\text{At least one correct}) = 1 - P(\text{All 10 are wrong})$

$= 1 - \left(\frac{4}{5}\right)^{10} = \boxed{.8926}$

4.2 #29 47,637 Helicopter = X

111,874 by ground

159,511 = n

✓ $P(H) = \frac{47,637}{159511} = .299$

✓ $P(\text{5 by helicopter}) = (.299)^5 = .0024$

✓ It is ok to use the With replacement Probability because 5 is much less than 5% of $159511 \approx 8000$ by Numberson Calc. Rule.