§4.4 Counting

Def $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{\#\text{ elements of } E}{\#\text{ elements in Sample Space}}$

$$P(2\text{ girls}) = \frac{3}{8}$$

$E = \{GBG, GGB, BGG\}$

$S = \{GGG, GBG$

(1) the fundamental principle of counting

Multiply → # of ways to do each task

7 Births $\underset{\uparrow 2}{G}\ \underset{\uparrow 2}{G}\ G\ B\ B\ G\ \underset{\uparrow 2}{G}$

$n(S) = 2^7 = 128$ Birth orders

$$P(\text{one girl}) = \frac{7}{128} = \boxed{.0547} > .05$$

Not statistically Significant.

§4.4 Counting

Def $P(E) = \frac{n(E)}{n(S)} = \frac{\#\text{ of elements of } E}{\#\text{ of elements of } S}$

Goal: Count the Number of elements in a Sample Space without writing the whole Space.

The Fundamental Principle of Counting.

If n ways for first Task
m ways for 2nd Task

then mn ways for Both Tasks

Ex How many license plates? $\frac{5}{N}\ \frac{M}{L}\ \frac{L}{L}\ \frac{K}{L}\ \frac{9}{N}\ \frac{3}{N}\ \frac{5}{N}$

$10 \cdot 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10$

$175,760,000$

Letters & Numbers can Repeat

Permutations (Arrangements) of n objects taken r at a time. Order Matters ADG ≠ AGD

Ex 8 Bikes but only have room to display 3 in the window

8 7 6

$${}_8P_3 = 8 \cdot 7 \cdot 6 = 336 \text{ window displays}$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{8!}{5!} =$$

$$nP_r = \frac{n!}{(n-r)!} = \frac{8!}{5!} =$$

MATH ◁ PRB 2: nPr (8, 3)

Combinations of n objects taken r at a time. Order does not Matter. A B C D E F G H

$$ADG = AGD$$

How many ways can we write the 3 letters ADG? ($\uparrow$ r things)

ADG
AGD
DAG
DGA
GDA
GAD

$3 \cdot 2 \cdot 1 = 6$

Have r! permutations

Each Combination has $3! = 6$ Permutations. there are

$${}_8P_3 = 336 \text{ permutations}$$

so there are

$${}_8C_3 = \frac{{}_8P_3}{6} = 56$$

or $6 \cdot 56 = 336$

$$56 = \frac{336}{6}$$

Each combination of 3 objects can be written as 6 permutations

ABC
ACB
BAC
BCA
CAB
CBA

ABD
ADB
BDA
BAD
DAB
DBA

ABE
AEB
BAE
BEA
EAB
EBA

ACD
ADC
CAD
CDA
DAC
DCA

ACE
AEC
CAE
CEA
EAC
ECA

ADE
AED
DAE
DEA
EDA
EAD

BCD
BDC
DBC
DCB
CDB
CBD

BDE
BED
DBE
DEB
EDB
EBD

BCE
BEC
CEB
CBE
ECB
EBC

CDE
CED
EDC
ECD
DEC
DCE

$_5P_3 = 60 =$ # permutations of 5 objects taken 3 at a time.

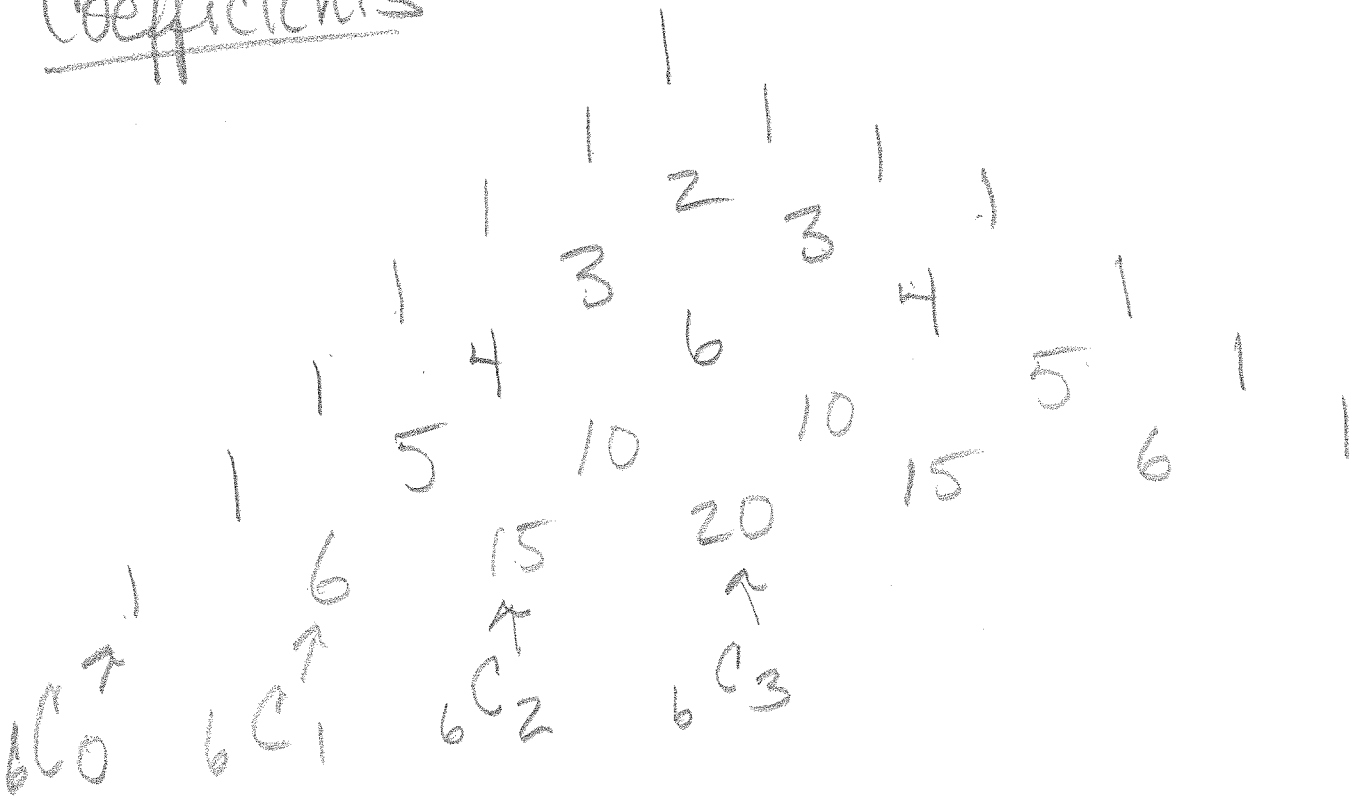
Each combination of 3 people can be written as $6 = 3!$ Permutations.

So $_5C_3 = \frac{60}{6} = 10$

= # combinations of 5 objects taken 3 at a time.

$$ {}_nC_R = \frac{{}_nP_r}{r!} = \frac{n!/(n-r)!}{r!} = \frac{n!}{(n-r)!\,r!} $$

Pascal's Triangle gives the Binomial Coefficients



Ex 27 players and 3 are Selected
What is the probability that I Selected the 3 Tallest?

3 Methods

(1) Method 1 Conditional

$P(\text{Tallest}) = \frac{3}{27} \cdot \frac{2}{26} \cdot \frac{1}{25} = \boxed{.0003419}$

(2) Method 2 $P(E) = \frac{n(E)}{n(S)} = \frac{{}_3C_3}{{}_{27}C_3} = \frac{1}{2925} = \boxed{.0003419}$

$E = \{$ ~~Combinations~~ of 3 tallest Players $\}$

$E = \{$ GED $\}$ $n(E) = {}_3C_3 = 1$

$S = \{$ All combinations of 3 from All 27 players $\}$ $= \{$ ALED ZAED $\}$

$n(S) = {}_{27}C_3 =$

(3) S = set of Permutations

$\frac{n(\text{Permutations of 3 tallest})}{n(\text{Permutations of 27})} = \frac{{}_3P_3 = 3! = 6}{{}_{27}P_3 = 27 \cdot 26 \cdot 25 =}$

$= \frac{6}{17550} = \boxed{.0003419}$

Ex At an Amusement park there are 5 rides and I have 3 tickets

a) If we can ride the same ride more than once and order Matters

Z R W G S

S	S	S
Z	R	S
S	R	W
↑	↑	↑
5	5	5 = **125**

b) Three different rides order Matters

A Permutation of 5 items selected 3 at a time. $_5P_3$ =

Z	R	S
Z	S	R
↑	↑	↑
5	4	3 = **60**

$$\text{factorial} = \frac{5!}{2!} = \frac{5\cdot4\cdot3\cdot2\cdot1}{2\cdot1} = 5\cdot4\cdot3$$

the Number of permutations of n items taken r at a time is

Arrangements

$$_nP_r = \frac{n!}{(n-r)!}$$

= **Math** ▷▷▷ **PRB** 2: nPr

Can't ride the same twice

c) What three rides did you go on? Order you ride them in does not matter.

$3 \cdot 2 \cdot 1 = 6 = 3!$

ZRS
ZSR
SRZ
SZR
RZS
RSZ

} All the same group of rides

Each combination of 3 letters can be written as $3! = 6$ permutations

$${}_5C_3 = \frac{60}{6} = 10$$

the Number of Combinations of n objects taken r at a time is

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

Ex Sustainability Committee 20 members
Bike Bridge over 101 committee with
5 people. a) How many committees?

$$_{20}C_5 = 15{,}504$$

b) There are 6 faculty, 6 Adm, 8 classified
What is the prob of all faculty

Two Methods

Counting 4.4

$$P(\text{All faculty}) = \frac{n(\text{All faculty comm})}{n(\text{committees})} = \frac{{}_6C_5}{{}_{20}C_5}$$

$$= \frac{6}{15504} = 3.8699 \times 10^{-4}$$

$$= .000387$$

Conditional 4.3

$$P(\text{All faculty}) = \frac{6}{20} \cdot \frac{5}{19} \cdot \frac{4}{18} \cdot \frac{3}{17} \cdot \frac{2}{16}$$

$$=$$

§4.2 # 28, 26, 29

§4.3 #

#21-24

	Positive Test	Neg Test	
Uses	119 (True pos.)	3 (False Neg)	122
Doesn't Use	24 (False Pos.	154 (True Neg)	178
	143	157	300

#21 n(Study) = 300

n(True Neg) = 154

$$P(\text{True Neg}) = \frac{154}{300} = 0.5133$$

#24 $P(\text{Did Not Use}) = \frac{178}{300} = .593$

#26 Hospital Generators

$P(1 \text{ fails}) = .22$

a) $P(\text{Both fail}) = P(\text{1st fail}) \cdot P(\text{2nd fails})$ ← "And"

$= .22 \cdot .22$

$= \boxed{.0484}$

b) $P(\text{At least one Works}) = 1 - P(\text{Both fail})$

$= 1 - .0484$

$= \boxed{.9516}$

c) People will Die 5 out 100 times the power goes out

#28 $P(1 \text{ Bad}) = \frac{288}{2875} = .1002$ Fish $P(1 \text{ good}) = .8998$

a) $P(3 \text{ good}) = .8998^3 = \boxed{.7286}$

Apply Lumbersom Calc. Rule

3 < 5% of 2875

73% of the time the fish is accepted

Not catching the Bad fish P

10% of consumers getting sick is too high

Use Counting to find Probabilities

Theoretical Prob. Def

$$P(E) = \frac{n(E)}{n(S)}$$

Ex XSort – Suppose to increase likelyhood of a girl.

In a clinical trial with 20 births

a) How many birth orders with one boy?

$E = \{ bg\cdots g,\ gbg\cdots g, \cdots \}$ $\Rightarrow n(E) = 20$

↑ 20 spots & we choose one in which to put boy

b) How many Possible birth orders

$S = \{ gbbgbbggg^{b}\cdots, \cdots \}$ $n(S) = 2^{20}$

c) What is the probability of one boy?

$$P(\text{one boy}) = \frac{n(E)}{n(S)} = \frac{20}{2^{20}} = .00001907$$

Ex 27 players on the high school basketball team. What is the prob. that 3 randomly selected players are the 3 tallest.

Use Combinations

$E = \{$ Groups of 3 that contain the 3 tallest $\}$

$E = \{$ EBD $\}$ ← $n(E) = 1$

$S = \{$ All possible groups of 3 players $\}$

$n(S) = {}_{27}C_3 = 2925$

$P(E) = \frac{1}{2925} = .000342$

Permutations

$E = \{$ Arrangements of 3 tallest $\}$ = $n(E) = 6 = {}_3P_3$

$S = \{$ Permutations of 27 @ 3 at a time $\}$ $n(S) = {}_{27}P_3$

$P(E) = \frac{n(E)}{n(S)} = \frac{{}_3P_3}{{}_{27}P_3} = .000342$

Use

Class Try

A committee has 15 members
8 faculty and 7 administrators.

a) How many Subcommittees of 3 contain only faculty?

$E = \{F_1F_2F_3, \ldots\}$ $\quad n(E) = {}_8C_3 = 56$

b) How many 3 person Subcommittees if we choose from all 15 members?

$S = \{F_2A_1A_2, \ldots\}$ $\quad n(S) = {}_{15}C_3 = 455$

c) Find Probability that a randomly selected Subcommittee will only have faculty?

Both ways

$$P(\text{Only faculty}) = \frac{n(E)}{n(S)} = \frac{56}{455} = \frac{{}_8C_3}{{}_{15}C_3} = .123$$

+3 EC Conditional Probabilities also

$$P(3 \text{ faculty}) = \frac{8}{15} \cdot \frac{7}{14} \cdot \frac{6}{13} = .123$$

Ex A committee has 15 members
8 faculty and 7 administrators

a) How many sub committees of 3 have only faculty? ${}_8C_3 = 56$

b) How many sub committees of 3 people?

${}_{15}C_3 = 455$

c) Find the probability that you select all faculty out of 3.

$$P(\text{All faculty}) = \frac{56}{455} = .123 = \frac{{}_8C_3}{{}_{15}C_3}$$

$$P(\text{3 faculty}) = \frac{8}{15} \cdot \frac{7}{14} \cdot \frac{6}{13} = .123$$

$$= \frac{{}_8P_3}{{}_{15}P_3} =$$