§4.2 Probability Rules

Addition Rule Probability of A OR B is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Logic: Add probabilities but don't count Anything twice

Complement Rule $P(\overline{A}) = 1 - P(A)$

Multiplication Rule

Probability of A and then B, two different sequential Events

$$P(A \text{ and then } B) = P(A) \cdot P(B)$$ A & B are independent

→ with Replacement

$$= P(A) \cdot P(B|A)$$ If B depends on A

→ without Replacement

$P(B|A)$ → "the probability of B given that A already occured."

4.3, ~~4.4 and 4.5~~

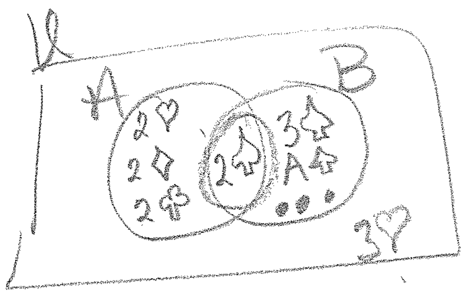
§4.2 Addition Rule

The Probability of event A or event B

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$\cup$ = union = $A \cup B$ = The set of elements in either A <u>or</u> B, but Don't List twice

$\cap$ = intersect, $A \cap B$ = Set of elements Common to both A <u>and</u> B



A = {2's}
= {2♥, 2♦, 2♠, 2♣}

B = {♠} ← the set of Spades

$\underbrace{\text{A 2 3 4 ... 10}}_{\text{Numbered}}$ $\underbrace{\text{J Q K}}_{\text{face}}$ Spades

K Clubs

Black {♠, ♣}

A

2

Red {♦, ♥}

2 Dimonds

2 Hearts

Suits = 4

Values = 13

$$P(2) = P(A) = \frac{4}{52} = \frac{1}{13}$$

$$P(B) = P(\spadesuit) = \frac{13}{52} = \frac{1}{4}$$

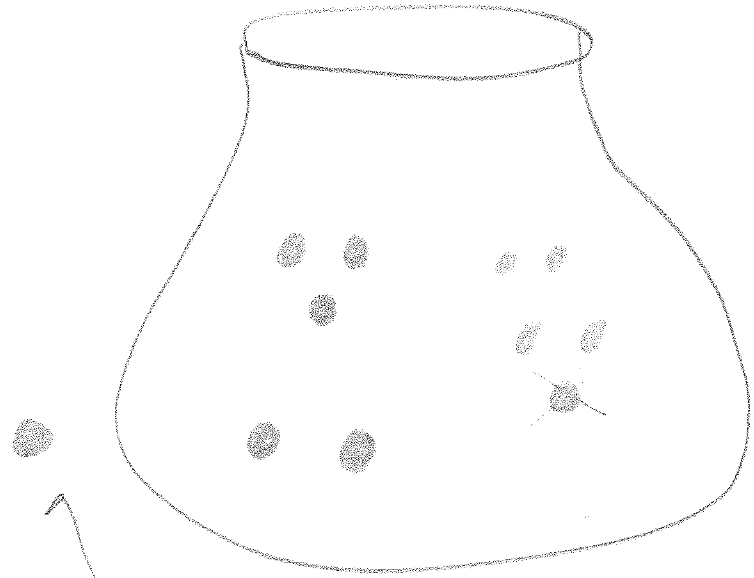
$$P(A \cap B) = P(2\spadesuit) = \frac{1}{52}$$

$$P(A \cup B) = P(2 \text{ or spade}) = \frac{16}{52}$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

Jar of Marbles

3 purple
5 green
2 Yellow



a) $P(G) = \frac{5}{10} = \frac{1}{2}$

b) Pick Two Marbles
Without Replacement

$$P(2\,Green) = P(G_1 G_2) = \frac{5}{10} \cdot \frac{4}{9} = \frac{2}{9} = .2222$$

$$= P(G_1) \cdot P(G_2 \mid G_1)$$

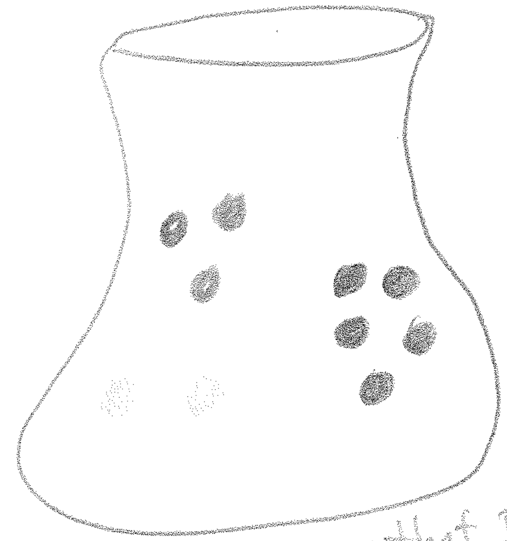
" the prob of green on first Trial • Prob of Green on 2nd Trial <u>Given That</u> We got green on first Trial

c) $P(G_1 P_2) = \frac{5}{10} \cdot \frac{3}{9} = \frac{1}{6} = .1667$

§4.3 The Multiplication Rule

Jar of Marbles

5 green, 2 yellow, 3 blue



$P(\text{Blue}) = P(B) = \frac{3}{10} = .3$

Two Tasks pick blue on first draw & Also pick a blue Marble on Second draw

given that B_1 Occured

Without Replacement

$$P(B_1 \text{ and then } B_2) = P(B_1) \cdot P(B_2 | B_1)$$

$$= \frac{3}{10} \cdot \frac{2}{9} = \frac{6}{90} = .0667$$

3 Significant Digits

Indepent of B_1

With Replacement

$$P(B_1 B_2) = P(B_1) \cdot P(B_2)$$

$$= \frac{3}{10} \cdot \frac{3}{10} = .09$$

$n=2 \qquad N=10$

$2 > 5\% \text{ of } 10 = .5$

Does Not Satisfy Cumbersom Calc Rule

d) Pick Two Marbles with Replacement

$$P(G_1 G_2) = \frac{5}{10} \cdot \frac{5}{10} = \frac{1}{4} = .25$$

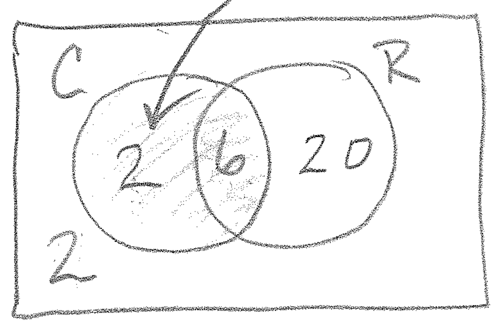
Back to a contingency Table

<u>Ex</u> Composting or Recycling

$8-6=$

$n(C) = 8$ $\quad n(R) = 26$ $\quad C \cap R' = 2$

$n(U) = 30$ $\quad n(C \cap R) = 6$



	C	$\overline{C}$	
R	$n(C\cap R) = 6$	$n(R \cap \overline{C}) = 20$	$26 = n(R)$
$\overline{R}$	$n(C \cap \overline{R}) = 2$	$n(\overline{R} \cap \overline{C}) = 2$	$4 = n(\overline{R})$
	$8 = n(C)$	22	$30 = n(U)$

a) $P(C) = \frac{8}{30} = \boxed{.2667}$

b) Prob someone composts <u>or</u> recycles?

$$P(C \cup R) = P(C) + P(R) - P(C \cap R)$$

$$= \frac{8}{30} + \frac{26}{30} - \frac{6}{30} = \frac{28}{30} = \boxed{.9333}$$

~~Contingency Table~~ Doesn't work

	Y	N	
Recycle	75	25	100
Compost	70	30	100

Can't be in both

Must be Disjoint groups

Set up like this

	C	$\bar{C}$	
R			
$\bar{R}$			

c) Select two people from class and Find the probability that they Both Compost.

a) With Replacement $P(C_1 C_2) = \frac{8}{30} \cdot \frac{8}{30} = .0711 = \left(\frac{8}{30}\right)^2$

b) With out Replacement $P(C_1 C_2) = \frac{8}{30} \cdot \frac{7}{29} = .0644$

Cumbersome Calculation Rule

If Sample Size n is less than 5% of the population size N, then Calculations, You can use with Replacement Ans or Probability

→ Calc Easier

→ Ans. Very close

$N = 30$

$n = 2$

5% of 30 = $.05 \cdot 30 = 1.5 < 2$

Does Not Satisfy Rule!

Conditional Probability

"Probability that someone Recycles
Given that they compost."

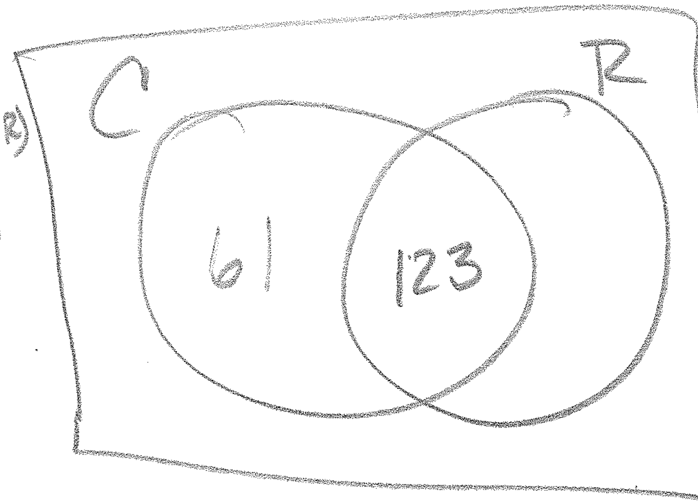
universe is those
who compost.

$$P(R|C) = \frac{6}{8} = .75$$

$$= \frac{P(R \cap C)}{P(C)} = \frac{n(R \cap C)}{n(C)}$$

Q4

$n(C \cap \overline{R}) = n(C) - n(C \cap R)$
$= 184 - 123$
$= 61$



$n(C \cup R) = n(C \cap \overline{R}) + n(C \cap R) + n(R \cap \overline{C})$
$61 \quad + 123 \quad + \underline{\qquad}$

$= n(C) + n(R) - n(C \cap R)$

$=$

§4.2 The Addition and Multiplication Rule

Apply Probability Rules to Contingency Tables

	Non Smokers N	Occational Smokers O	Regular Smokes R	
Men M	436	42	71	549
Women W	326	47	78	451
	762	89	149	1000

a) What is the probability of randomly Selecting a Nonsmoking Women?

$P(W \cap N)$ = % of Nonsmoking women = Proportion of Nonsmoking Women = Relative frequency

$$= \frac{n(W \cap N)}{\text{Grand Total}} = \frac{326}{1000} = .326$$

b) Prop. of Women "OR" Nonsmoker

$$P(W \cup N) = P(W) + P(N) - P(W \cap N)$$

$$= \frac{451}{1000} + \frac{762}{1000} - \frac{326}{1000} = .887$$

$$= \frac{436 + 326 + 47 + 78}{1000} = .887$$

c) $P(\overline{W \cup N}) = 1 - P(W \cup N)$

$= 1 - .887 = \boxed{.113}$

Complement

d) Multiplication Rule with Table

What is the probability of selecting two women in a row that are both Non Smokers?

$P(WN_1 \text{ and then } WN_2) = P(WN_1) \cdot P(WN_2 \mid WN_1)$

$= \frac{326}{1000} \cdot \frac{325}{999} = \boxed{.106}$

$= .106056$

Without Replacement

Dependent

$P\begin{pmatrix} \text{2 women Non Smokers} \\ \text{With Replacement} \end{pmatrix} = \frac{326}{1000} \cdot \frac{326}{1000} = (.326)^2$

Independent

$= .106276$

$n = 2 = \#$ picked

$N = 1000$ and 5% of $1000 = 50$

$2 < 50 \Rightarrow$ use cumbersome calc. Rule

The 5% Rule for Cumbersome Calculations

If $n < 5\%$ of N

n = # you pick = Sample Size
N = # you are picking from = Population

Then you can treat dependent event calculations as independent, so the <u>Math is easier.</u>
and the probabilities are very close to the same.

Dice Throw Two 6 Sided Dice

Sample space =

11	12	13	14	15	16
21	22	23	24	25	$\boxed{26}$
31	32	33	34	$\boxed{35}$	36
41	42	43	$\boxed{44}$	45	46
51	52	$\boxed{53}$	54	55	56
61	$\boxed{62}$	63	64	65	66

Probability the Sum of Numbers is 7

a) $P(7) = \frac{6}{36} = \frac{1}{6} = .1667$

b) $P(\text{Sum is } 5) = \frac{4}{36} = \frac{1}{9} = .1111$

c) $P(\text{Sum is 5 or 7}) = P(5) + P(7) = .2778$

Disjoint Sets so We can just Add

d) $P(\text{Doubles or a Sum of } 8) = \frac{10}{36} = .2778$

$= P(\text{Doubles}) + P(\text{Sum of } 8) - P(\text{Double \& Sum } 8)$

$\frac{6}{36} + \frac{5}{36} - \frac{1}{36} = \frac{10}{36}$

Dice Example

Probability of a 7 or a 5

Sum of ... Sum of

Disjoint – Cant happen at the same time.

a) $$P\left(\begin{matrix}\text{Sum is}\\ 7 \text{ or } 5\end{matrix}\right) = P\left(\begin{matrix}\text{Sum is}\\ 7\end{matrix}\right) + P\left(\begin{matrix}\text{Sum is}\\ 5\end{matrix}\right)$$

$$= \frac{6}{36} + \frac{4}{36} = \frac{10}{36} = \boxed{.278}$$

b) Find prob that one die $\{\text{shows a } 5\} = B$ or the $\underbrace{\{\text{Sum is } 7\}}$ ← A

Use Addition Rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

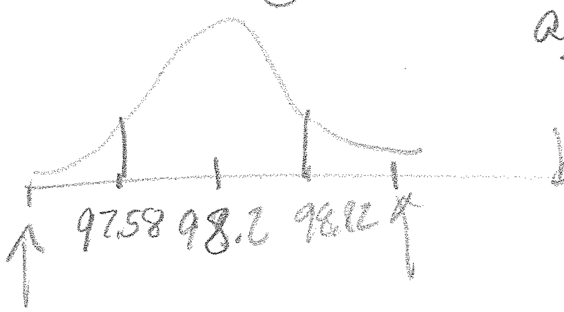
$$= \frac{6}{36} + \frac{11}{36} - \frac{2}{36}$$

$$= \frac{15}{36} = \boxed{.417}$$

37) ~~If~~ 57 out of 104 or 55% correctly predict the gendar 32.7% of the time when randomly guessing so it is NOT Statistica Significant. (To be stat. sig. probability of occurring by chance is less than 5%)
No practical significance either

3.2 # 42
3.3 # 14

42 Body Temp $\bar{x} = 98.2$ $s = .62$



a) $\boxed{68\%}$ of pop. will be within one S.d. of Mean

b) 99.7%

§3.3 # 14 Male $\bar{x} = 4.719$ $s = .490$ $x = 5.58$

female $\bar{x} = 4.349$ $s = .402$ $x = 5.23$

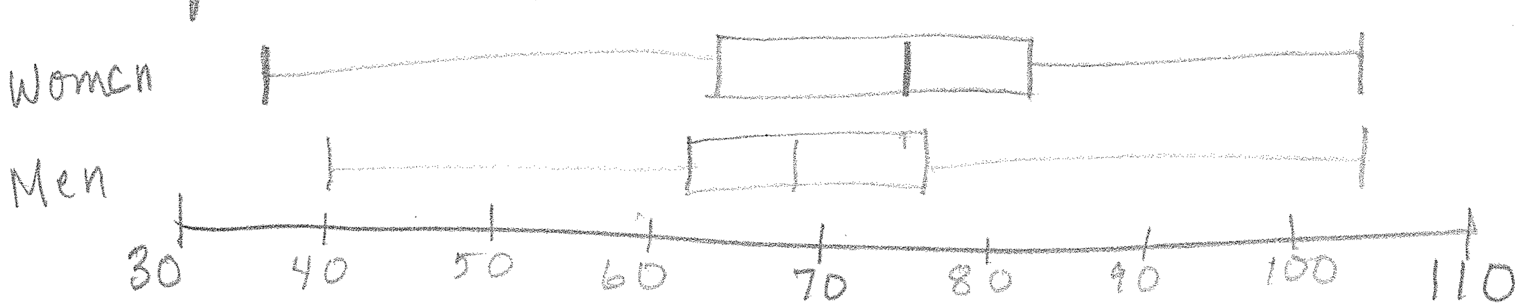
$$Z_{male} = \frac{x - \bar{x}}{s} = \frac{(5.58 - 4.719)}{.490} = \boxed{1.76}$$

$$Z_{female} = \frac{x - \bar{x}}{s} = \frac{(5.23 - 4.349)}{.402} = \boxed{2.19}$$

the female has a higher relative Red Blood Count.

33 Summary stats Group by Gendar

	min	Q1	med	Q3	max
Women	36	64	74	82	104
Men	40	62	68	76	104



#33 Men's pulse is lower than women
their spread in pulse rates is about
the same.

§3.4 #14 Sandra $Z = \frac{X - \bar{X}}{S} = \frac{45 - 35.9}{11.1} = \boxed{0.82 = Z_S}$

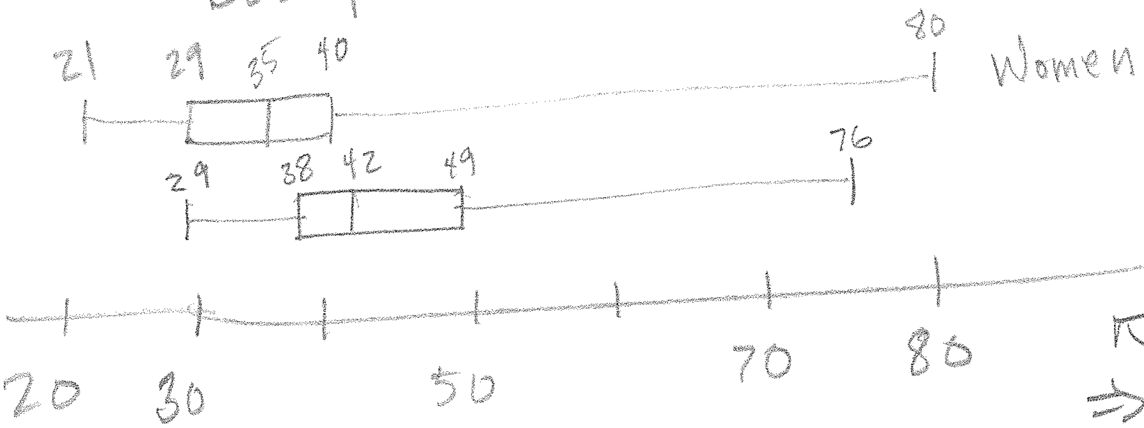
$X = 45 \quad \bar{X} = 35.9 \quad S = 11.1$

Jeff $= \boxed{Z_J = 1.77}$

$X = 60 \quad \bar{X} = 44.1 \quad S = 9.0$

Sandra is Relatively younger.

#34 Use Stat Crunch
Select Both Data Sets then Make
Box plot



Side-by Side
⇒ Same Number line