Section 4.1

Definition A discrete probability distribution function has two characteristics:

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Example A child psychologist is interested in the number of times a newborn babys crying wakes its mother after midnight. For a random sample of 50 mothers, the following information was obtained. Let X = the number of times a newborn wakes its mother after midnight. For this example, $x = 0, 1, 2, 3, 4, 5$.

$P(x) =$ probability that X takes on a value x. The following is the Probability Distribution Function or Probability Distribution Table:

x	0	1	2	3	4	5
$P(x)$	$\frac{2}{50}$	$\frac{11}{50}$	$\frac{23}{50}$	$\frac{9}{50}$	$\frac{4}{50}$	$\frac{1}{50}$

Example Suppose Nancy has classes 3 days a week. She attends classes 3 days a week 80% of the time, 2 days 15% of the time, 1 day 4% of the time, and no days 1% of the time. Suppose one week is randomly selected. Let $X =$ the number of days Nancy attends class in that week.

1. X takes on what values?

2. Suppose one week is randomly chosen. Construct a probability distribution table (called a PDF table) like the one in the previous example. The table should have two columns or rows labeled x and $P(x)$. What does the $P(x)$ column sum to?

Section 4.2

Definitions/Vocabulary A quantitative random variable x

A *discrete* random variable

A *continuous* random variable

The expected value

Example Which of the following are continuous variables, and which are discrete?

a. $x =$ the number of siblings a person has.

b. $x =$ a person's height.

c. $x =$ the amount of time it takes a statistics student to finish an exam.

d. $x =$ the number of units a student takes in a quarter.

Example Consider the following probability distribution of a discrete random variable, x. Complete the table so that it is a valid probability distribution.

x	0	1	2	3
$P(x)$	0.4	0.2	0.1	

Example A dust mite allergen level that exceeds 2 micrograms per gram (μ_g/g) of dust has been associated with the development of allergies. Consider a random sample of 4 homes. Let $x =$ number of homes with a dust mite allergen level that exceeds 2 μ_g/g. The probability distribution for x is given below.

x	0	1	2	3	4
$P(x)$	0.09	0.30	0.37	0.20	0.04

a. Find the probability that at least three of the homes have a dust mite level that exceeds 2 μ_g/g.

b. Find the probability that fewer that two of the homes have a dust mite level that exceeds 2 μ_g/g.

c. Compute and interpret μ.

Example Consider the game consisting of rolling a pair of fair dice and recording the sum. It will cost you $\$1.00$ to play this game. If the sum is at least 10, then you will win $\$3.00$. If the sum is 2, you will win $\$2.00$. Otherwise, you will lose the game.

Probability Distribution: Winnings

x	\$3.00	\$2.00	\$0.00
$P(x)$			

a. What are your expected earnings per dollar wagered? Show your work and interpret your result.

b. Are your expected earnings equal to the cost to play?

c. If you played this game one million times, how much money would you expect to earn/lose?

Example Suppose that Nigel wants to take out a $\$100,000$ term insurance policy. Nigel is 60 years old and according to the actuarial tables, there is a 1.191% chance Nigel will die this year. If the insurance company wants to make $\$700$ profit per policies of this type, how much should the insurance company charge Nigel? Show your work and give your answer in the form of a sentence.

a. Determine the probability distribution table. There are only two possibilities, you either pay out on the policy this year or you don't:

x	$\$100,000.00$	$\$0.00$
$P(x)$		

b. Now calculate the expected pay out per person for this year.

c. If they want to make $\$700$ per person (on average) how much should they charge?

Example To raise money, the ASFC is holding a raffle. The prize is a Walt Disney World vacation for two valued at $3000. The ASFC sold 1382 raffle tickets at $5 each.

a. Antonia bought 10 tickets. What is the probability that she will win?

b. What are Antonia's expected earnings?

c. Are Antonia's expected earnings equal to the cost to play?

Example Revisited Consider the game consisting of rolling a pair of fair dice and recording the sum. It will cost you $\$1.00$ to play this game. If the sum is at least 10, then you will win $\$3.00$. If the sum is 2, you will win $\$2.00$. Otherwise, you will lose the game.

Probability Distribution: Winnings

x	\$3.00	\$2.00	\$0.00
$P(x)$	$\frac{1}{6}$	$\frac{1}{36}$	$\frac{29}{36}$

Find the Standard deviation of this Probability Distribution Table.

Section 4.3
Features of a binomial experiment

a.

b.

c.

d.

e.

Example Suppose I flip a fair coin three times and determine the number of heads in the three flips. What is the probability that I will get exactly two heads in the three flips?

Example An unfair coin is flipped ten times. For this coin, the probability of heads is $p = 0.6$. Use proper notation in answering the questions.

a. What is the probability of getting exactly two heads?

b. What is the probability of getting at most two heads?

c. What is the probability of getting at least six heads?

d. What is the probability of getting no heads?

Example Nigel has just been given a twenty question multiple choice quiz where there are four possible answers to each question. Unfortunately for Nigel, he has not studied and consequently must guess the answer to each question.

a. What is the probability that he will answer at least 70% of the questions correctly?

b. If this actually happened, might this cause you to believe that he is not guessing?

Example An experiment is to be conducted to see whether an acclaimed psychic has extra sensory perception (ESP). Four different cards are to be shuffled, and one chosen at random. The psychic will then try to identify which card was drawn without seeing the card. The experiment is to be repeated five times. Assume the five trials are independent. If the psychic is guessing, what is the probability that the psychic will correctly identify the card on at least two of the five times in which the experiment is performed.

Example A sales representative for a tire manufacturer claims that the company's steel-belted radials last at least 40,000 miles. A tire dealer decides to check that claim by testing twenty tires. If more than 80% of the twenty tires he tests last at least 40,000 miles, he will purchase tires from the representative. If, in fact, 90% of the steel-belted radials produced by the manufacturer last at least 40,000 miles, what is the probability that the tire dealer will purchase tires from the sales representative?

Example Alzheimer's disease is a progressive, irreversible disorder, and at this time the cause and cure are unknown. However, researchers at the University of Minnesota developed a "brief, efficient and accurate means of predicting Alzheimer's" in persons from 65 to 85 years of age. The test takes about 10 minutes and requires the patient to try to recall 10 words after using each word twice in sentences. According to the researchers, the test results are accurate 95% of the time. If the test is administered to a random sample of 25 persons from 65 to 85 years of age, what is the probability that the test results are **inaccurate** for more than 12% of those persons sampled?

Example Consider the binomial distribution where the number of trials is 5 and the probability of success on each trial is 0.8.

1. Graph the distribution.

2. Describe the shape of the distribution.

3. Find the mean of the distribution.

4. Find the standard deviation of the distribution.

Example A manufacturer of CDs claims that 99.4% of its CDs are defect-free. A large software company that buys and uses large numbers of the CDs wants to verify this claim, so it randomly selects 1600 CDs to be tested. The tests reveal 19 CDs to be defective.

1. Assuming that the manufacturer's claim is correct, would it be unusual to have at least 19 defective CDs?

2. Based on your answer to part (1), should finding 19 defective CDs out of 1600 randomly selected CDs cause you to doubt the manufacturer's claim? Explain.

Example Consider a binomial distribution with twenty trials.

1. For what value of p will the distribution be perfectly mound-shaped and symmetric?

2. Determine a value of p for which the distribution is skewed right.

3. Determine a value of p for which the distribution is skewed left.

Example USA Today reported that 20% of all people in the US are illiterate.

1. Suppose that you randomly select 10 people. If the claim made by USA Today is correct, what is the probability that at least 8 of the 10 can read and write?

2. Suppose you randomly select 500 people. If the claim made by USA Today is correct, would it be unusual to find at least 115 illiterate people out of the 500 randomly selected?

3. Suppose you randomly select 500 people. If the claim made by USA Today is correct, would it be unusual to find at least 130 illiterate people out of the 500 randomly selected?

Example The Denver Post reported that a recent audit of Los Angeles 911 calls showed that 85% were not emergencies. How many calls would the 911 operators need to answer to be 96% (or more) sure that at least one call is, in fact, an emergency?