Section 3.1
Definition

- An experiment
- outcome
- A sample space
- An event
- The probability

Example Since Suzanne did not study, Suzanne needs to guess the answer to a multiple choice question where there are five possible answers to the question.

1. What is the probability that she will answer the question correctly?
2. What is the probability that she will answer the question incorrectly?

Example Consider flipping a fair coin.

1. Suppose that you flip the fair coin 4 times. Should you expect to get exactly 2 heads and 2 tails? Explain
2. Suppose that you flip the fair coin 400,000 times. Should you expect to get about 200,000 heads and 200,000 tails? Explain.

## Definition

- Equally likely
- empirical
- unfair, or biased.
- An outcome is the event A OR B
- An outcome is the event A AND B

Example Suppose that you pick one card out of a standard 52 card deck and record the suit.

- How many cards in the deck would satisfy the event: "Draw a three"?
- How many of those cards would satisfy the event: "Draw a red card"?
- How many cards in the deck would satisfy the event: "Draw a red three" (Which is the same as "Draw a red card AND draw a three"?
- How many cards in the deck would satisfy the event: "Draw a red card OR draw a three"?

Definition The complement of event $A$, denoted by $A^{C}$ or $A^{\prime}$ is

Example Suppose that you pick one card out of a standard 52 card deck and record the suit.

- How many cards in the deck would satisfy the event: "Draw a red card"?
- How many cards in the deck would satisfy the event: "Draw a card that is not red"?
- How many cards in the deck would satisfy the event: 'Draw a red card OR a card that is not red"?

Definition The event $A \mid B$, called a conditional event:

## Examples

- The event that a person wants to have a girl child vs. the event that a person wants to have a girl child given that they first had a boy child.
- The event that a person draws an Ace from a deck of cards vs. the event that a person draws an Ace from a deck of cards given that they are holding three Aces in their hand.
- The event that a fair coin toss results in "heads" vs. the event that a fair coin toss results in "heads" given that the three previous coin tosses each resulted in "heads".

Understanding Terminology and Symbols We will use the following symbols representing events to write the symbols for the probabilities of the events 1 through 5 listed below.

- Let $F$ be the event that a student is female.
- Let $L$ be the event that a student has long hair.

1. The probability that a student is female.
2. The probability that a student does not have long hair.
3. The probability that a student is a female and has long hair.
4. Of all the female students, the probability that a student has long hair.
5. The probability that a student is female or has long hair.

Section 3.2
Definition
Two events $A$ and $B$ are independent:
$A$ and $B$ are independent if the following are true:
-
-
-

To show two events are independent, you must
dependent events:

Example Suppose that you roll a pair of fair dice and record the sum. Then, you repeat the experiment. Are the two outcomes independent?

Example Suppose that you pick one card out of a standard 52 card deck and record the suit. Then, without replacing the card that was drawn you then draw another card and record the suit. Are the two outcomes independent?

Independent Event Example Let event $G=$ taking a math class. Let event $H=$ taking a science class. Then, $G$ AND $H=$ taking a math class and a science class. Suppose $P(G)=0.6, P(H)=0.5$, and $P(G$ AND $H$ ) $=0.3$. Are $G$ and $H$ independent?

Independent Event Example In a particular college class, $60 \%$ of the students are female. $50 \%$ of all students in the class have long hair. $45 \%$ of the students are female and have long hair. Of the female students, $75 \%$ have long hair. Let F be the event that the student is female. Let L be the event that the student has long hair. One student is picked randomly. Are the events of being female and having long hair independent?

Definition Two events are mutually exclusive if

Example Suppose that you have a bag containing 1000 M \& M candies that are either Red or Green. Are the events "Draw a Red" and "Draw a Green" mutually exclusive?

## Section 3.3



Example Consider rolling a pair of fair dice and recording the sum.

1. What is the probability that the sum is seven?
2. What is the probability that the sum is not seven?
3. What is the probability that the sum is four or six?
4. What is the probability that the sum is even or at most four?
5. What is the probability that the sum is at most ten?
6. Would it be considered a "rare event" to roll a sum of twelve?

Example Carlos plays college soccer. He makes a goal $65 \%$ of the time he shoots. Carlos is going to attempt two goals in a row in the next game. $A=$ the event Carlos is successful on his first attempt. $P(A)=0.65 . B=$ the event Carlos is successful on his second attempt. $P(B)=0.65$. Carlos tends to shoot in streaks. The probability that he makes the second goal GIVEN that he made the first goal is 0.90 .

1. What is the probability that he makes both goals?
2. What is the probability that Carlos makes either the first goal or the second goal?
3. Are A and B independent?

## Section 3.4

Example The contingency table below results from cross classifying U.S. hospitals by type and number of beds. Suppose a hospital is chosen at random. Use proper notation when answering the questions.

|  | 6-24 beds | 25-74 beds | 75 or more beds | Total |
| :---: | :---: | :---: | :---: | :---: |
| General | 299 | 1894 | 3945 | 6138 |
| Psychiatric | 17 | 121 | 378 | 516 |
| Chronic | 0 | 7 | 40 | 47 |
| Tuberculosis | 0 | 1 | 10 | 11 |
| Other | 22 | 131 | 162 | 315 |
| Total | 338 | 2154 | 4535 | 7027 |

1. What is the probability that the hospital has at least 25 beds?
2. What is the probability that the hospital is a psychiatric hospital or has 75 or more beds?
3. What is the probability that the hospital is a general hospital, given that it has 75 or more beds?
4. What is the probability that the hospital has 75 or more beds, given that is a general hospital?
5. Would it be a rare event for a hospital to be a tuberculosis hospital?
6. Are the events general hospital and 6-24 beds independent? Explain.

Example The Journal of National Cancer Institute published results of a study that investigated the association between cigar smoking and death from tobacco-related cancers for American males. Data are provided in the following table. Use proper notation when answering the questions.

|  | Died? | Died? |  |
| :---: | :---: | :---: | :---: |
| Cigars | Yes | No | Total |
| Never Smoked | 782 | 120747 | 121529 |
| Former Smoker | 91 | 7757 | 7848 |
| Current Smoker | 141 | 7725 | 7866 |
| Total | 1014 | 136229 | 137243 |

What is the probability that an American male chosen at random:

1. who died from tobacco-related cancer was a former smoker?
2. never smoked or died from tobacco-related cancer?
3. was a former smoker or a current smoker?
4. was a former smoker or current smoker, given that he died of tobacco-related cancer?
5. Are the events "former smoker" and "died from tobacco-related cancer" independent?
