Repost Page 8, 4

Practice Test 3 Math 15 Spring 2011 Jones Name Key

Complete by May 2 for 5 points on your exam. You must show all work and explain conclusions .

Use the given degree of confidence and sample data to construct a confidence interval for the population proportion p.

1-PropZInt

§7.2

1) When 306 college students are randomly selected and surveyed, it is found that 115 own a car. Find the point estimate for the proportion of college students who own a car, and find a 99% confidence interval for the true proportion of all college students who own a car.

$\alpha = .01$

$\alpha/2 = .005$

$1-\alpha/2 = .995$

What is the point estimate of the population proportion? $\hat{p} = 115/306 = .376$

What is the critical value? $z_{\alpha/2}$ = invnorm $(1-.01/2) = 2.576$

What is the margin of error? E= $z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.576 \cdot \sqrt{\frac{.376 \cdot .624}{306}} = \boxed{.071 = E}$

Explain the meaning of the confidence interval.

What is the confidence interval?

$\hat{p} \pm E = .376 \pm .071$ $.305 < p < .447$ the True proportion of college students who own a car is between .305 and .447.

Use the given degree of confidence and sample data to construct a confidence interval for the population mean μ. Assume that the population has a normal distribution. T Interval Data

§7.4

2) The principal randomly selected six students to take an aptitude test. Their scores were:

76.5 85.2 77.9 83.6 71.9 88.6

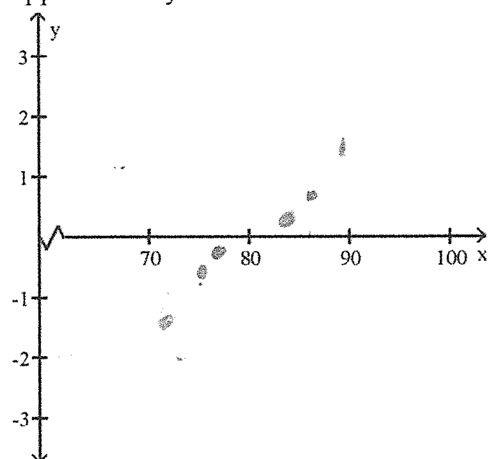
$\bar{x} = 80.6$

$s = 6.23$

Determine a 90% confidence interval for the mean score for all students.

a) Make a normal Probability plot for this data. Does it indicate that the data comes from a population that is approximately normal?

last graph



this is close to a straight line so the Data does appear to come from a Normally distributed population

df = 5 α = .10 $t_{\alpha/2}$ = tinv (1 − .10/2, 5) = 2.015

a) What point estimate of the population mean does this sample give? $80.6 = \bar{x}$

b) What is the margin of error? (Show work. Include critical value.)

$E = t_{\alpha/2} \cdot s/\sqrt{n} = 2.015 \cdot 6.23/\sqrt{6} = 5.12$

d) Find the confidence interval.

$\bar{x} \pm E = 80.6 \pm 5.1$ $75.5 < \mu < 85.7$

Check TI T Interval (75.493, 85.74) ✓

e) Interpret the meaning of this confidence interval. Is the principal reasonable confident that the average of his student's ~~are~~ scores is higher than the national average if the national average for the aptitude test is 70.

Yes, the principle is 90% confident that the mean score for all students at his school will be between 75.5 and 85.7

Use the given degree of confidence and sample data to find a confidence interval for the population standard deviation σ. Assume that the population has a normal distribution.

§7.5

3) The football coach randomly selected ten players and timed how long each player took to perform a certain drill. The times (in minutes) were:

7 10 14 15 15
5 12 15 11 11

What is the point estimate for the population standard deviation? $S = 3.47$

Find a 95 percent confidence interval for the population standard deviation σ.

$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

$\chi^2_L = \text{inv}\chi^2(.025, 9) = 2.700$

$\chi^2_R = \text{inv}\chi^2(.975, 9) = 19.023$

$$\sqrt{\frac{9 \cdot 3.47^2}{19.023}} < \sigma < \sqrt{\frac{9 \cdot 3.47^2}{2.70}}$$

$2.39 < \sigma < 6.34$

Identify the null hypothesis, alternative hypothesis. Find and graph the point estimate for the population Proportion and test statistic,. Find the P-value. State your conclusion about the null hypothesis, and final conclusion that addresses the original claim. 1-PropZ Test $x_1 = .45 \cdot 100 = 45$

§8.3

4) According to a recent poll 53% of Santa Rosans would vote for the incumbent president. However a random sample of 100 people results in 45% who would vote for the incumbent, test the claim that the actual percentage is 53%. Use a 0.10 significance level.

a) (3 Points) State claim and the null and alternate hypothesis. b) (3 Points) Graph and shade the critical region.

c) (3 Points) Find the critical value, point estimate of the population proportion and it's test statistic.

d) (3 points) Label these values on your graph.

e) (5 Points) Clearly state your initial conclusion and your final conclusion so that it is understandable without knowing ~~what the problem is.~~ Statistics

f) (5 Points) Find and emplain the meaning of the P-value. Shade a graph showing the area equal to the p-value.

a) Claim: $p = .53$

$H_0: p = .53$

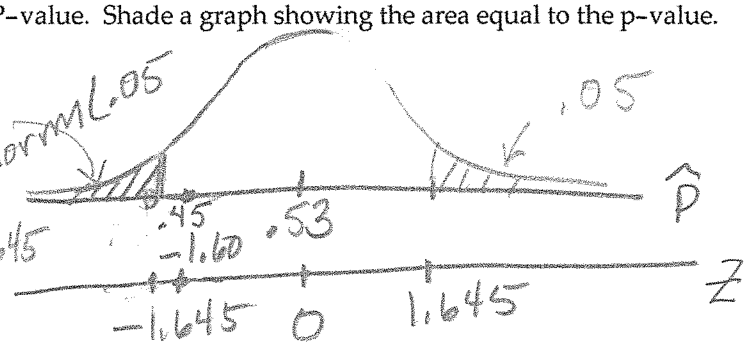
$H_1: p \neq .53$

b) $\alpha = .10$

$\alpha/2 = .05$

CV: $z^* = \text{invnorm}(.05)$

$z^* = \pm 1.645$



c) TS: $z = \dfrac{.45 - .53}{\sqrt{\dfrac{.53 \cdot .47}{100}}} = -1.60$ point estimate is .45

e) fail to Reject Ho. We don't have significant evidence to show that the proportion of Santa Rosans who would for the incumbent president is Not 53%. There is Not a statistical difference between 45% and 53%.

f) P-value $= 2 \cdot \text{Normalcdf}(-999, -1.60) = .1089$

There is a 11% chance of getting a sample proportion as low or lower than 45% when the 53% is correct.

§8.2 1-Prop Z Test

5) A poll of 1,068 adult Americans reveals that 513 of the voters surveyed prefer the Democratic candidate for the presidency. At the 0.05 level of significance, test the claim that at least half of all voters prefer the Democrat.
(3 Points) State claim and the null and alternate hypothesis. (3 Points) Graph and shade the critical region.
(3 Points) Find the critical value, test statistic. (3 points) Label these values on your graph.
(5 Points) Clearly state your initial conclusion and your final conclusion so that it is understandable without knowing statistics. 1-prop Z Test

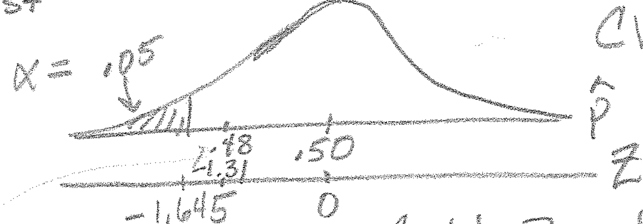
a) Claim $p \geq .50$

$H_0: p = .50$

$H_1: p < .50$

$\hat{p} = \frac{513}{1068} = .48$

b) $\alpha = .05$



CV: $Z^* =$ invnorm(.05) $= -1.645$

c) TS: $Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{.48 - .50}{\sqrt{\frac{.5 \cdot .5}{1068}}} = -1.31$

d) fail to Reject H_0
there is Not enough evidence to reject the claim that at least half the voters are Democrat.

Construct the indicated confidence interval for the difference between population proportions $p_1 - p_2$. Assume that the samples are independent and that they have been randomly selected.

We are 95% confident that the prop of 20–24 year olds who smoke is between 3 & 13% higher than the prop of 25 to 29.

§9.2

6) In a random sample of 500 people aged 20–24, 22% were smokers. In a random sample of 450 people aged 25–29, 14% were smokers. Construct a 95% confidence interval for the difference between the population proportions $p_1 - p_2$. Find the Critical Value z*., the point estimate of p1–p2, and the margin of error,. State the meaning of this confidence interval. 2-Prop Z Interval (.03156, .12844)

$n_1 = 500$

$x_1 = .22 \cdot 500 = 110$

$x_2 = .14 \cdot 450 = 63$

$n_2 = 450$

$\hat{p}_1 - \hat{p}_2 = .22 - .14 = .08$ → there is a Significant difference

$E = Z_{\alpha/2}\sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} = 1.96 \cdot \sqrt{\frac{.22 \cdot .78}{500} + \frac{.14 \cdot .86}{450}} = .048$

Provide an appropriate response.

$E = .048 = (.12844 - .0316)/2$ ← Values from TI

§9.2

7) Suppose the proportion of sophomores at a particular college who purchased used textbooks in the past year is p_s and the proportion of freshmen at the college who purchased used textbooks in the past year is p_f. A study found a 95% confidence interval for $p_s - p_f$ is (0.235, 0.427). Does this interval suggest that sophomores are more likely than freshmen to buy used textbooks? Explain what this interval says. We are 95% confident that the proportion of Sophomores who buy used texts is between 23.5% and 42.7% higher than the proportion of freshmen who purchase used texts

Interpret the confidence interval.

8) A random sample of clients at a weight loss center were given a dietary supplement to see if it would promote weight loss. The center reported that the 100 clients lost an average of 43 pounds, and that a 95% confidence interval for the mean weight loss this supplement produced has a margin of error of ±9 pounds.

§8.4

We are 95% Confident that the mean weight lost by all clients is between 34 and 52 pounds

Use the traditional method to test the given hypothesis. Assume that the samples are independent and that they have been randomly selected 2-PropZTest $x_1 = 110$ $x_2 = 63$

9) In a random sample of 500 people aged 20–24, 22% were smokers. In a random sample of 450 people aged 25–29, 14% were smokers. Test the claim that the proportion of smokers in the two age groups is the same. Use a significance level of 0.01.
State the null and alternate hypothesis. claim
Graph and shade the critical region.
Find the critical value, point estimate of p1–p2, and it's test statistic.
Label these values on your graph above.
Find and Explain the meaning of the P–value. Shade a graph showing the area equal to the p–value.
Clearly state your initial and final conclusion.
Explain what a Type I Error and a Type II Error would be in this case.

§9.2

$\alpha = .01$

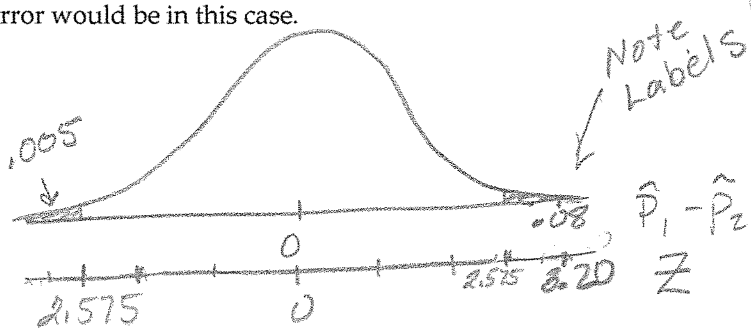
Type I: Showing a difference when there isn't one

Type II: Not showing a difference when there really is one.

Claim $p_1 = p_2$

$H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$



CV: $Z_{\alpha/2} = \text{invnorm}(1 - \alpha/2) = \pm \text{invnorm}(.995) = \pm 2.575$

Point estimate $= \hat{p}_1 - \hat{p}_2 = .22 - .14 = .08$ $\quad \bar{p} = \dfrac{\hat{p}_1 + \hat{p}_2}{2} = \dfrac{.22 + .14}{2} = .18$

$$TS: \; Z = \frac{.08 - 0}{\sqrt{\frac{.18 \cdot .82}{500} + \frac{.18 \cdot .82}{450}}} = 3.20$$

P-value $= 2 \cdot \text{normalcdf}(3.20, 999) = .00137$

There is only a 0.14% chance of seeing this large a differenc for this size random sample from populations with equal proportions of smokers.

Reject H_0. There is sufficient evidence to show the the proportion of smokers are different. It appears that the prop is higher for people aged 20-24.

Construct the indicated confidence interval for the difference between the two population means. Assume that the assumptions and conditions for inference have been met. 2SampT Interval DATA

10) A researcher was interested in comparing the number of hours of television watched each day by two-year-olds and three-year-olds. A random sample of 18 two-year-olds and 18 three-year-olds yielded the follow data. Not Pooled

§9.3

df = 17

$\alpha = .05$

Two Tail

$t_{\alpha/2} = 2.093$ Table

$\bar{X}_2 = 1.25$

$S_2 = .809$

$n_2 = 18$

2-year-olds		3-year-olds	
0.5	1.5	2.0	3.0
1.5	2.0	1.5	1.5
1.5	0	1.5	2.0
1.0	1.0	1.0	0
1.0	0	0	1.5
2.0	1.5	1.5	2.0
2.5	2.0	2.5	2.0
0.5	0	3.0	1.0
1.5	2.5	1.5	0.5

$\bar{X}_3 = 1.56$

$S_3 = .856$

$n_3 = 18$

TI INVT(.925,17) = 2.11

Does this data represent independent samples or matched pairs? Independent 2 Separate groups of Subjects

Find the point estimate $\mu_2 - \mu_3$. $= \bar{X}_2 - \bar{X}_3 = 1.25 - 1.56 = .031$

~~Find the critical value?~~

FInd a 95% confidence interval for the difference, $\mu_2 - \mu_3$, between the mean number of hours for two-year-olds and the mean number of hours for three-year-olds. $(-.8696, .25852)$

Explain the meaning of this confidence interval.

We are 95% confident that the difference between the means is between −.87 and .26. Since zero is in this interval we cannot show there is a difference in View Times for 2 & 3 year olds.

Use the traditional method of hypothesis testing to test the given claim about the means of two populations. Assume that two dependent samples have been randomly selected from normally distributed populations.

11) A coach uses a new technique to train gymnasts. 7 gymnasts were randomly selected and their competition scores were recorded before and after the training. The results are shown below.

§9.4

Subject	A	B	C	D	E	F	G
Before	9.4	9.5	9.7	9.4	9.5	9.7	9.6
After	9.5	9.7	9.7	9.3	9.6	10	9.4

L_3 = differences −.1, −.2, 0, .1, −.1, −.3, −.2

T-Test on differences on L_3

Using a 0.01 level of significance, test the claim that the training technique is effective in raising the gymnasts' scores. Note: effective ⇒ increased scores ⇒ Negitive differences

a) (3 points) State the claim, null and alternate hypothesis.

b) (7 Points) Graph and shade the critical region. Find the critical value, a point estimate for the mean difference, and it's test statistic. Label these values and areas on your graph above.

~~c) (5 points) Find and explain the meaning of the P-value. Shade a graph showing the area of the p-value.~~

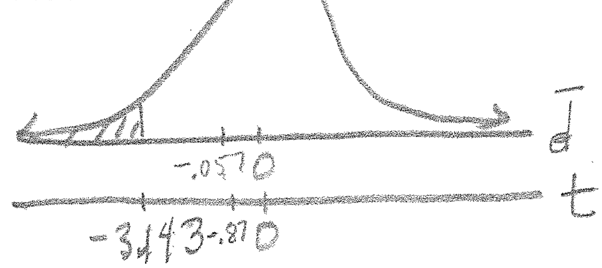
d) (5 points) Clearly state your initial and final conclusion.

Claim $\mu_d < 0$ PE: $\bar{d} = -.057$

$S_d = .172$

$H_0: \mu_d = 0$

$H_1: \mu_d < 0$ one Tail



CV: $t^* = invt(.01, 6) = -3.143$

TS: $t = \dfrac{\bar{d} - \mu_d}{S_d/\sqrt{n}} = \dfrac{-.057 - 0}{.172/\sqrt{7}} = -.87$ TI gives Z = −.88

P-Value = .2064 fail to Reject H_0. There is Not enough evidence to show the scores have improved.

Decide whether or not the conditions and assumptions for inference with a two-sample t-interval are satisfied. Explain your answer.

12) A study was conducted to determine which cab company gives quicker service. Companies A and B were each called at 50 randomly selected times. The response times were recorded. The results were as follows.

§9.3

	Company A	Company B
Mean response time $\bar{x}_1$	7.6 minutes	6.9 minutes
Standard deviation s_1 =	1.4 minutes	s_2 = 1.7 minutes

The conditions and assumptions for inference appear to be satisfied. The samples are independent, random, and $n > 50$ so the Central Limit Theorem applies.

Assume that the assumptions and conditions for inference with a two-sample t-test are met. Test the indicated claim about the means of the two populations.

13) Researchers wanted to compare the effectiveness of a water softener used with a filtering process with a water softener used without filtering. Ninety locations were randomly divided into two groups of equal size. Group A locations used a water softener and the filtering process, while group B used only the water softener. At the end of three months, a water sample was tested at each location for its level of softness. (Water softness was measured on a scale of 1 to 5, with 5 being the softest water.) The results were as follows. TI

§9.3

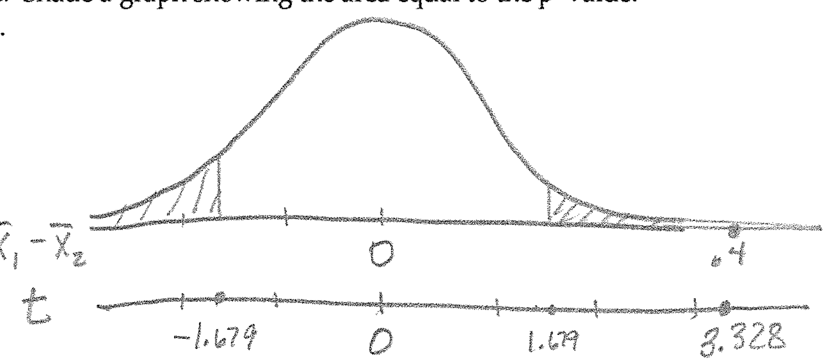
Group A (water softener and filtering)
$\bar{x}_1 = 2.1$
$s_1 = 0.7$ $n_1 = 45$
Group B (water softener only)
$\bar{x}_2 = 1.7$
$s_2 = 0.4$ $n_2 = 45$

Claim $\mu_1 \neq \mu_2$
$H_0: \mu_1 = \mu_2$
$H_1: \mu_1 \neq \mu_2$

2 SampTTest
with Stats
Not Pooled

Determine, at the 90% confidence level, whether there is a difference between the two types of treatments.
State the null and alternate hypothesis.
Graph and shade the critical region. Find the critical value, the point estimate for the difference in population means given by these samples, and it's test statistic. Label these values and areas on your graph above.
Find and explain the meaning of the P-value. Shade a graph showing the area equal to the p-value.
Clearly state your inital and final conclusion.

TS: $t = 3.328$
P-Value = .0014
CV: $t^* = \pm 1.679$
df = 44 two tail $\alpha = .10$
INVT Area = .05
D = 44
$t^* = \pm 1.680$



there is only a .14% chance of seeing a difference this large or larger when there is no difference in the true population means.

Point estimate $= \bar{X}_1 - \bar{X}_2 = 2.1 - 1.7 = .4$

Reject H_0 there is significant evidence to show the mean water softness is higher with filtering.

Provide an appropriate response.

14) (4 Points) A survey investigation whether the proportion of employees who commute by car to work is higher than it was five years ago finds a P-value of 0.011. Is it reasonable to conclude that more employees are commuting by car? Explain the meaning of this P-value.

§8.2

Yes, the P-value of .011 indicates that a random sample would only show this much increase or higher 1.1% of the time if in reality there was no increase. Since this is an unusually low probability we reject the Null hypothesis that mean is the same and accept the alternate hypothesis that more now commute by car.

15) A researcher wishes to determine whether listening to music affects students' performance on memory test. He randomly selects 50 students and has each student perform a memory test once while listening to music and once without listening to music. He obtains the mean and standard deviation of the 50 "with music" scores and obtains the mean and standard deviation of the 50 "without music scores". He then performs a hypothesis test for two means assuming large and independent samples. Is this approach appropriate? If not, how would you proceed?

§9.4

This data should be treated as matched pairs. Look at the mean of the differences = with music score − without music score of all participants. If this mean is significantly above zero we can show the with music scores average is higher.

16) (4 Points) Hannah selected a simple random sample of all adults in her town and, based on this sample, constructed a confidence interval for the mean salary of all adults in the town. However, the distribution of salaries in the town is not exactly normal. Will the confidence interval still give a good estimate of the mean salary?

§7.4

As long as her sample size is larger than 30 the CLT will apply and the confidence interval will be accurate.

Use the computer display to answer the question.

17) When testing for a difference between the means of a treatment group and a placebo group, the computer display below is obtained. Using a 0.05 significance level, is there sufficient evidence to support the claim that the treatment group (variable 1) comes from a population with a mean that <u>is less than</u> the mean for the placebo population? Explain.

§8.2 and §9.3

	t-Test: Two Sample for Means		
1		Variable 1	Variable 2
2	Mean	65.10738	66.18251
3	Known Variance	8.102938	10.27387
4	Observations	50	50
5	Hypothesized Mean Difference	0	
6	t TS	-1.773417	
7	P(T<=t) one-tail	0.0384	
8	T Critical one-tail CV	1.644853	
9	P(T<=t) two-tail	0.0768	
10	t Critical two-tail	1.959961	

Claim: $\mu_T < \mu_P$ $H_1: \mu_1 < \mu_2$ is a one Tail test indicating that the TS: $t = -1.77$ is below the CV because its p-value is .0384. So we <u>Reject Ho</u> and accept H_1

There is sufficient evidence to show that the treatment group has a lower mean

Provide the appropriate answer.

18) (4 Points) An entomologist writes an article in a scientific journal which claims that fewer than 19% of male fireflies are unable to produce light due to a genetic mutation. Identify the Type I error in this context.

§8.2 If the entomologist made a Type I error he showed that fewer than 19% of male fruit flies are unable to produce light when in fact the proportion is at least 19%.

Do one of the following, as appropriate: (a) Find the critical value $z_{\alpha/2}$, (b) find the critical value $t_{\alpha/2}$, (c) state that neither the normal nor the t distribution applies.

19) 90%; n = 9; σ = 4.2; population appears to be very skewed.

§8.2 $n < 30$ and population is Not Normal So Neither applies

20) 93%; n = 40; σ is known; population appears to be very skewed.

σ is known so use $Z_{\alpha/2} = \text{invnorm}(1-\alpha/2) = \text{invnorm}(.965) = 1.81$

21) 90%; n = 27; σ is unknown; population appears to be normally distributed.

σ is unknown Use $t_{\alpha/2} = \text{INVT}(.95, 26) = 1.706$

$1 - \alpha/2 = .95$

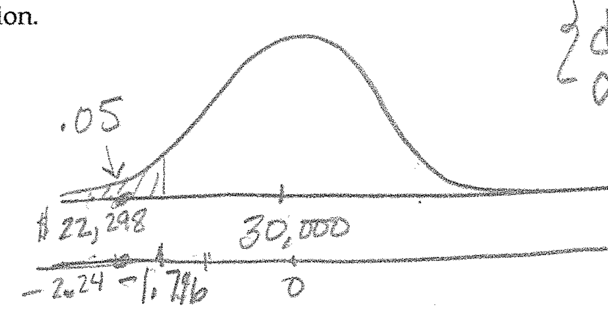
Test the given claim by using the P-value method of testing hypothesis. Assume that the sample is a simple random sample selected from a normally distributed population. Include the hypothesis, the test statistic, the p-value, and your conclusion. TTest Stats $\mu_0 = 30,000$

22) Test the claim that for the adult population of one town, the mean annual salary is ~~given by μ = $30,000.~~ less than $30,000
Sample data are summarized as n = 17, $\bar{x}$ = $22,298, and s = $14,200. Use a significance level of $\alpha = 0.05$.
a) State the claim, null and alternate hypothesis.
b) Graph and shade the critical region. Find the critical value, point estimate of the population mean, and test statistic. Label these values and areas on your graph above.
c) Explain the meaning of the P-value. Shade a graph showing the area equal to the p-value.
d) Clearly state your initial and final conclusion.

§8.5

Lt Tail
df = 16
$\alpha = .05$

Claim: $\mu < 30,000$
$H_0: \mu = 30,000$ One Tail $\alpha = .05$ df = 16
$H_1: \mu < 30,000$
CV: $t = -1.746 = \text{invt}(.05, 16)$



TS: $t = \frac{(\bar{x} - \mu)}{s/\sqrt{n}} = \frac{(22298 - 30,000)}{(14200/\sqrt{17})} = -2.24$

c) P-value = .01996 = tcdf(−999, −2.24, 17)

Meaning

There is only a 2% chance of seeing a sample mean of $22,298 or less if the true mean is at least 30,000.

d) Reject H_0 there is sufficient evidence to show the mean salary is below 30,000.