§9.3 Inference For Matched Pairs

$H_0: \mu_d = 0$

$H_1: \mu_d >, <, \neq 0$

Two Lists of Data from One person

L_1	62	64	80	72	54	58	Pulse
L_2	68	72	90	102	76	70	After Jumping Jacks
$L_1 - L_2 = d$	−6	−8	−10	−30	−22	−12	$= L_3$

Claim: Jumping jacks increases pulse rate.

Claim: $\mu_d < 0$ ← ↑ Supported when differences are Negative

$H_0: \mu_d = 0$ Do a T Test on L_3

$H_1: \mu_d < 0$

TS: $t = -3.841$

p-value $= .00605 < \alpha$

$\mu_0: 0$

List: L_3

$H_1: \mu_0 < 0$

Reject H_0

Jumping jacks ~~a~~ increase the heart rate.

there is a significant difference in heart rate.

Confidence Interval T Interval 90%

$-22.36 < \mu_d < -6.97$ bpm

No Zero → there is a significant difference

Write More,

the jumping jacks caused the heart rate to increase between Average 7 bpm and 22 bpm.

§9.3 Inference for Matched Pairs

Inference $\equiv$ Make CI, Do HT

§9.2 Indepent large Samples

Test 1 Fall 2018

Test 1 Fall 2017

Point estimate

PE: $\bar{x}_1 - \bar{x}_2$

= the difference of the Means

2-SampT Test

2-SampT Int

$H_0: \mu_1 = \mu_2$

Summary stats

$\bar{x}_1, s_1, n_1$

$\bar{x}_2, s_2, n_2$

Not the same

§9.3 Dependent or Matche pairs

Test 1 & Test 2
Same class

- Paired by student
- Paired by Year of Award

PE: $\bar{d}$ = mean of List of Difference

$d_i = L_1 - L_2 = L_3$

$d_i = x_{1i} - x_{2i}$

T-Test on L_3

T Interval Data in L_3 on

$H_0: \mu_d = 0$

Summary Stats

$\bar{d} = \bar{x}$ for L_3

$S_d = S_x$ on L_3

n = # of pairs

Designing an Experiment

Matched pairs is more likely to show a significant difference.

when $|TS| > |cv|$

p-value $< \alpha$

0 is Not in Confidence Interval for $\bar{d}$

Matched Pairs uses $SE = \sigma_{\bar{d}} = \frac{S_d}{\sqrt{n}}$

is usually much smaller than

Independent Samples use $SE = \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$

$$TS = t = \frac{\bar{d} - 0}{(S_d/\sqrt{n})} > t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

Smaller denominator

Margin of Error

$$E = t_{\alpha/2} \cdot \frac{S_d}{\sqrt{n}} < E = t_{\alpha/2} \cdot \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

Procedure

① Requirements $n > 30$ or Normal parent Population

Sample histogram is Bell and QQ is a line

② Find List of differences $d = L_3 = L_1 - L_2$

③ Find Summary Statistics

1-VarStat L_3 or

TTest on L_3

$\bar{d}$ = mean L_3

$S_d = S_x$ for L_3

n = # of pairs

④ HT → do a TTest on L_3 with $H_1: \mu_d > 0$, < 0, $\neq 0$

$\mu_0 = 0$ for all

↑ claim

CI → TInterval on L_3

⑤ Conclusions

If $|TS| > |CV|$, p-value $< \alpha$, or 0 is Not in CI

Reject H_0 → Support that there is a significant difference. group A ~~has~~ a larger Mean than group B

fail to Reject H_0 → There is Not a significant Difference.

CI: (LB, UB) & 0 is Not in here

~~Group~~ Score on A is between LB & UB higher than ~~group~~ Score on B

Paired Hypothesis Test

Is really just a TTest on the list of Differences.

TTest $L_3 = L_1 - L_2$ $\quad \bar{d}$ = mean of L_3

$H_0: \mu_d = 0$ $\qquad n$ = # of differences

$H_1: \mu_d >, <, \neq 0$ $\qquad \bar{x} = \bar{d}$ = mean of differences

$\qquad S_x = S_d$ = standard deviation of list of diff

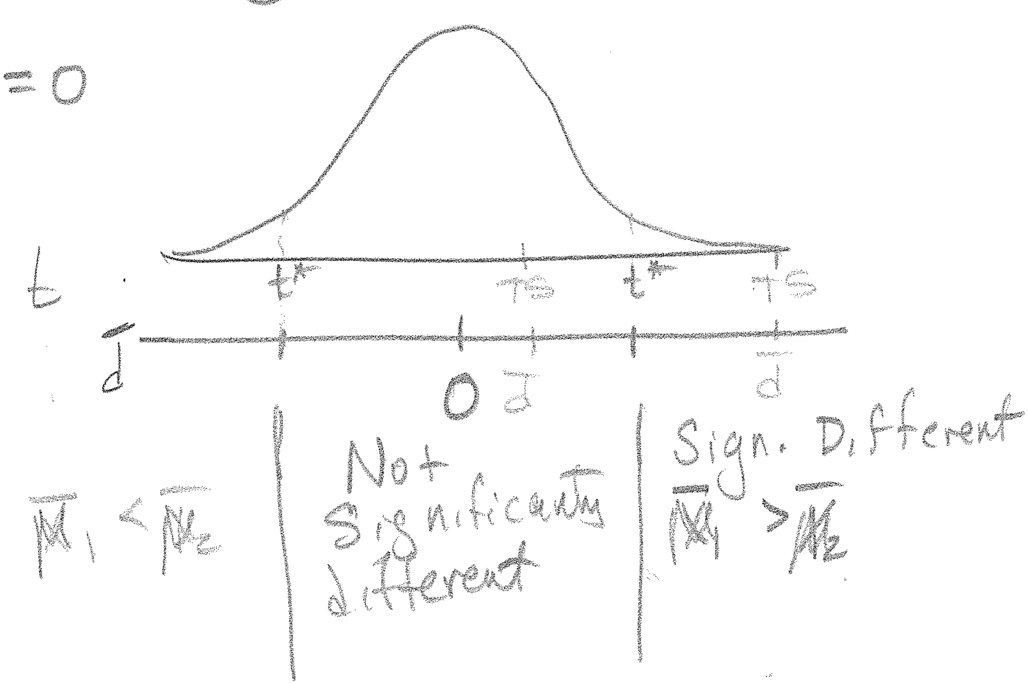
$$CV: \; t^* = invt(\underset{Lt}{\alpha}, \underset{Rt}{1-\alpha}, \underset{2\text{ tailed}}{1-\alpha/2}, df = n-1)$$

$$TS: \; t = \frac{\bar{d} - 0}{(S_d/\sqrt{n})}$$

← SE is smaller than the SE used for independent Samples.

this makes it easier to show that the two means are significantly different

No difference $\equiv \mu_d = 0$



Explain how you would set up the study so the samples are independent and then so the samples consist of matched pairs. Explain.

1) The accuracy of verbal responses is tested in an experiment in which individuals report their heights and then are measured. The data consist of the reported height and measured height for each individual.

Subject	Reported	Measured
1	75	74
2	64	63.5

Match pair

Determine whether the samples are independent or dependent.

2) The effectiveness of a headache medicine is tested by measuring the intensity of a headache in patients before and after drug treatment. The data consist of before and after intensities for each patient.

Dependent
Matched pair
Mean the Same thing

3) The effectiveness of a new headache medicine is tested by measuring the amount of time before the headache is cured for patients who use the medicine and another group of patients who use a placebo drug.

Independent

Explain how you would set up the study so the samples are independent and then so the samples consist of matched pairs. Explain.

4) The effect of caffeine as an ingredient is tested with a sample of regular soda and another sample with decaffeinated soda.

Independent → One group decaf & one regular ask how awake after 1 hour on Scale of 1 to 10.

dependent → double blind one day decaf one day caffee. Ask How Awake.

Ex Ten families are tested on # of gallons per day of water use before and after watching a water conservation video

L_1 = Before	33	33	38	33	35	35	40	40	40	31
L_2 = After	34	28	25	28	35	33	31	28	35	33
L_3 = diff	-1	5	13							

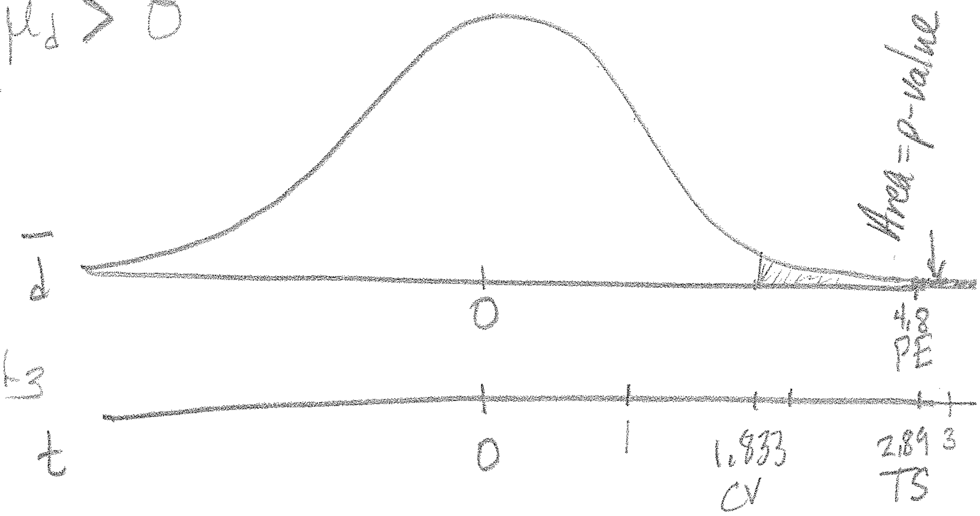
Test the claim that the usage decreased at $\alpha = .05$

Claim: $\mu_d > 0$

$H_0: \mu_d = 0$

$H_1: \mu_d > 0$

R t



PE: $\bar{d} = \bar{x}$ 1-VarStats L_3

$\boxed{\bar{d} = 4.8}$

$S_d = 5.25$

$n = 10$

TS: $t = \dfrac{\bar{d} - \mu_d}{S_d/\sqrt{n}} = \dfrac{4.8 - 0}{5.25/\sqrt{10}} =$

$\boxed{TS: t = 2.89}$

CV: $t^* = \text{invT}(\text{Area}, df)$

$t^* = \text{invT}(1 - .05, 9)$

$\boxed{CV: t^* = 1.833}$

P-value = ① Prob. of getting a ② Sample more extreme than sample given ③ H_0 is True

$= \text{tcdf}(2.89, 9999, 9) = .0089 < .05$

T Test on L_3 (Same Answers ✓) :)

c) Write Conclusion Reject $H_0: \mu_d = 0$

The Mean water usage difference is Not zero

We Support H_1

We Support that the usage before is greater than after watching conservation video

d) Write Meaning of p-value

① In a sample of size 10, there is less than a 1% chance

② Water usage would decrease by 4.8 gallons or more

③ If sample were selected from a population where water usage did Not decrease.

Use the traditional method of hypothesis testing to test the given claim about the means of two populations. Assume that two dependent samples have been randomly selected from normally distributed populations.

7) Ten different families are tested for the number of gallons of water a day they use before and after viewing a conservation video. At the 0.05 significance level, test the claim that the mean is the same before and after the viewing. $\alpha = .05$

L_1 = Before	33	33	38	33	35	35	40	40	40	31
L_2 = After	34	28	25	28	35	33	31	28	35	33
$L_2 - L_1 = d$	1	-5	-13							

Claim: $\mu_d = 0$

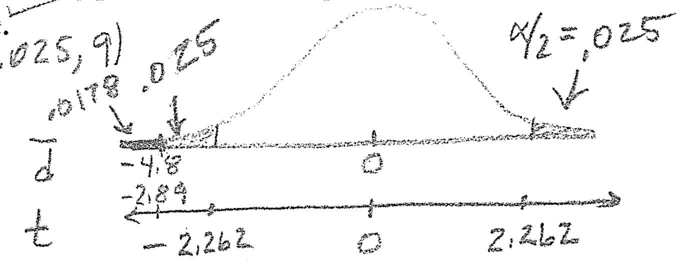
← Neg. # indicate conserved used less

a) (5 Points) State the null and alternate hypothosis. Graph and shade the critical region. Find the critical value.

$H_0: \mu_d = 0$ CV: $t^* = \text{invT}(.025, 9)$

$df = n - 1 = 9$

$H_1: \mu_d \neq 0$ $n = \#$ of pairs



b) (5 Points) Find point estimate for the mean of the differences and it's test statistic. Label these on your graph. State the initial conclusion.

PE: $\bar{d} = -4.8$ Line up TS & PE on Graph Vertically

$S_d = 5.25$

TTest L_3

TS: $t = -2.89$

d) (5 points) Find the p–value, draw a new graph and label this area. Explain the meaning of this p–value.

p-value = $.0178 < \alpha$

There is only a 1.78% chance of seeing a sample difference of -4.8 gal if the video did not effect usage.

e) (5 Points) Clearly state your final conclusion in language addressing the original claim.

Since p-value = $.0178 < \alpha = .05$ Reject H_0

TISE ~~we~~ reject that the mean difference in water useage is zero, or that useage is the same

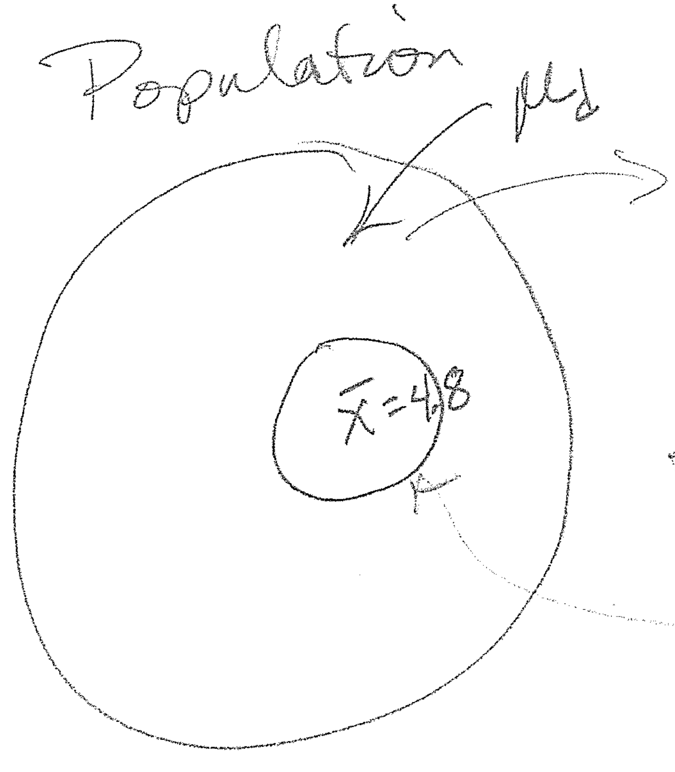
TISE to support that the useage is Not the same.

e) Find the confidence interval for the mean of difference & state its meaning.

CI (1.0479, 8.5521)

Meaning

1. Zero is Not in interval so there is a significant difference
2. Mean usage will decrease by between 1.0479 and 8.5512 gallo for families that watch video.



$1.05 < \mu_d < 8.55$

Inference About Population base on small sample

13) Below are the test scores for two tests in a class. Enter them into Excel. Then copy and paste them with Variable Names into StatCrunch.

1. Generate Summary Statistics and make Histograms, Normal Probability Plots and Side-by-Side Box Plots for the data sets. Paste them into your excel spread sheet.
2. Compare the displays. Comment on their similarities in center, spread, and shape.

Student	Test 1	Test 2
1	72	84
2	66	61
3	77	72
4	76	75
5	93	85
6	84	62
7	62	68
8	87	89
9	84	76
10	90	78
11	81	95
12	55	20
13	74	51
14	72	61
15	79	58
16	94	91
17	71	62
18	75	87
19	79	91
20	75	48

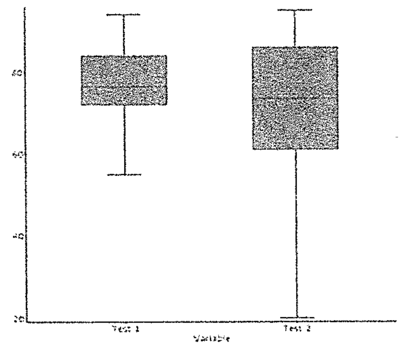
3. Create two 95% confidence interval for the difference between the means. First assume that the data sets are independent and then assume that they are matched pairs. Use **Calc->Estimate->2-sample t-interval for μ1-μ2** and **Calc->Estimate->Paired t-interval.** Clearly state the meaning of each confidence interval and determine which is the appropriate one to use and why.

4. The teacher claims that the students did better on the first test. Perform both hypothesis tests at the .05 level of significance. Use **Calc->Test->2-sample t test for μ1-μ2** and **Calc->Test->Paired t-test.** Write a statement about the conclusion of your t-test. Indicate which is the appropriate test to use and why.

Example Project – Assume test 1 & Test 2 are the Same Student.

Student	Test 1	Test 2	d
1	72	84	-12
2	66	61	5
3	77	72	5
4	76	75	1
5	93	85	8
6	84	62	22
7	62	68	-6
8	87	89	-2
9	84	76	8
10	90	78	12
11	81	95	-14
12	55	20	35
13	74	51	23
14	72	61	11
15	79	58	21
16	94	91	3
17	71	62	9
18	75	87	-12
19	79	91	-12
20	75	48	27

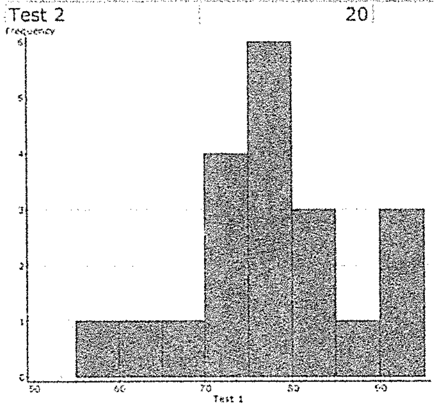
Box Plot

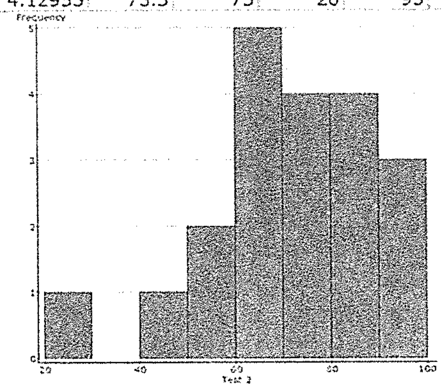


Summary statistics:

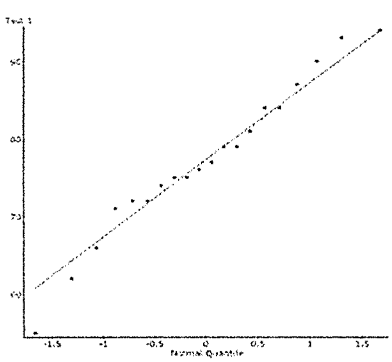
Column	n	Mean	Variance	Std. dev.	Std. err.	Median	Range	Min	Max	Q1	Q3
Test 1	20	77.3	98.32632	9.91596	2.21728	76.5	39	55	94	72	84
Test 2	20	70.7	341.0632	18.4679	4.12955	73.5	75	20	95	61	86

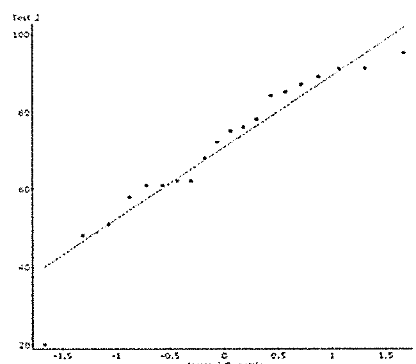
Histograms





QQ Plots





Both sets of data appear to come from populations that are skewed to the left slightly, but since the Normality requirements are loose for tests about means this data should give reliable Confidence itervals and hypothosis tests.

95% confidence interval results:
μ : Mean of variable

Variable	Sample Mean	Std. Err.	DF	L. Limit	U. Limit
Test 1	77.3	2.2172767	19	72.6592	81.9408
Test 2	70.7	4.1295469	19	62.0568	79.3432

(1) We are 95% confident that the mean of all stats students who take test 1 will be between 72.7 and 81.9.

(2) We are 95% confident that the mean of all stats students who take Test 2 will be between 62.1 and 79.3.

95% confidence interval results:
$\mu_D = \mu_1 - \mu_2$: Mean of the difference between Test 1 and Test 2

Difference	Sample Diff.	Std. Err.	DF	L. Limit	U. Limit
Test 1 - Test 2	6.6	3.1367096	19	0.03479	13.1652

Differences stored in column, Differences.

(3) We are 95% confident that the mean difference for all stats students will be between 0 and 13 pts higher on their first test than their Second Test.
Zero is Not in interval so there is a significant difference.

(4) **Hypothesis test results:**
$\mu_D = \mu_1 - \mu_2$: Mean of the difference between Test 1 and Test 2
$H_0 : \mu_D = 0$
$H_A : \mu_D \neq 0$

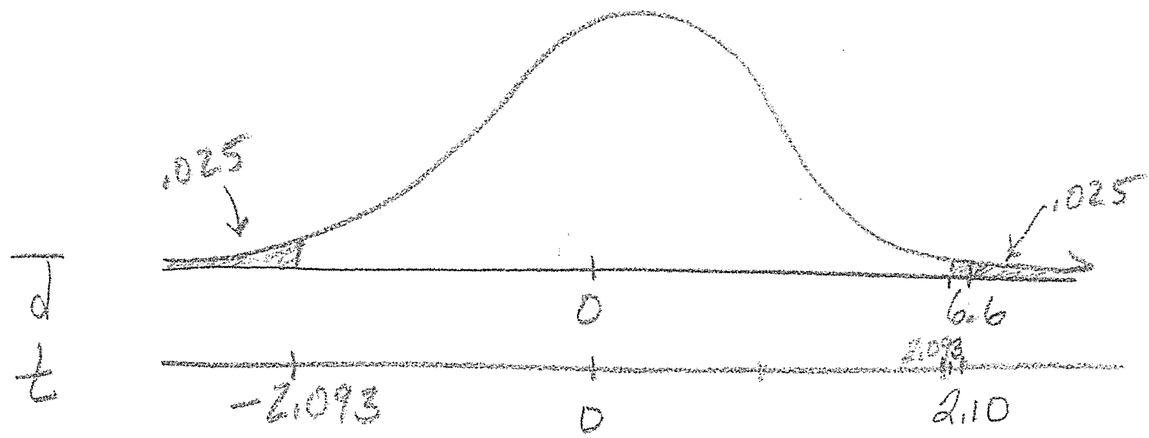
Claim: Students did the Same on Both Tests.

Difference	Sample Diff.	Std. Err.	DF	T-Stat	P-value
Test 1 - Test 2	6.6	3.1367096	19	2.10412	0.0489

Differences stored in column, Differences.

Reject H_0 p-value $= .0489 < \alpha = .05$
There is sufficient evidence to reject that the Students did the Same on both Test.
TISE to Support that they didn't do the same.
they did better on the first Test
p-value = .0489 says there is only a 4.89% chance of seeing this large a difference in test scores if the samples are from populations with equal Test scores. Hence, we conclude that the population mean test scores are Not the same.

CV: $t = \text{invT}(.025, 19) =$



Proportion Data

$\hat{p}_1 = \frac{x_1}{n_1}$, $\hat{p}_2 = \frac{x_2}{n_2}$

Need

1) 95% CI for $\hat{p}_1$ 1-prop Z Int

2) 95% CI for $\hat{p}_2$ 1-prop Z Int

3) 95% CI for $\hat{p}_1 - \hat{p}_2$ 2 prop Z Int

4) HT $H_0: p_1 = p_2$ 2 prop Z Test

$H_1: p_1 \quad p_2$

Choose $>, <, \neq$ Address claim in Conclusion

	Requirements	Sample size	Confidence Interval		Hypothesis Test	
	Formula page Requirements	Statistics	Estimate Population Parameter with Confidence Interval		Testing a claim with a Hypothesis Test	
	n=Size of population	Find Sample Size	Confidence Intervals One sample	Confidnece Interval Two samples	Test Statistic One Sample	Test Statistic Two sample
PROPORTION	**Proportion** requirements np>5, nq>5 Use Normal Distribution $\hat{p}=x/n$ $\hat{q}=1-\hat{p}$	approx. p known 7.1 $n=\frac{z_{\alpha/2}^2\,\hat{p}\cdot\hat{q}}{E^2}$ $\hat{p}$ = given approx. approx. p Unknown 7.2 $n=\frac{z_{\alpha/2}^2\cdot .25}{E^2}$	7.1 1-PropZInt PE: p = x/n $E=z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$ CV: $z_{\alpha/2}$ = invN$(1-\alpha/2,0,1)$ $\hat{p}-E<p<\hat{p}+E$ $(LB, UB)=(\hat{p}-E,\hat{p}+E)$ $E=\frac{UB-LB}{2}$, $\hat{p}=\frac{UB+LB}{2}$	9.1 2-PropZInt PE: $\hat{p}_1-\hat{p}_2$ $E=z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}$ $(\hat{p}_1-\hat{p}_2)-E<p_1-p_2<(\hat{p}_1-\hat{p}_2)+E$	8.2 1-PropZTest Ho: $p=p_0$ PE: $\hat{p}=x/n$ TS: $z=\frac{\hat{p}-p_0}{\sqrt{pq/n}}$ CV: z* = Invnorm(area) P-value= normalcdf(TS,999) RT normalcdf(-999,TS) LT mult. by 2 for 2tail	9.1 2PropZTest Ho: $p_1=p_2$, $p_1-p_2=0$ PE: $\hat{p}_1-\hat{p}_2=\frac{x_1}{n_1}-\frac{x_2}{n_2}$ TS: $z=\frac{\hat{p}_1-\hat{p}_2}{\sqrt{\frac{\bar{p}\bar{q}}{n_1}+\frac{\bar{p}\bar{q}}{n_2}}}$ CV: z* = Invnorm(area) P-value=ncdf(LB,UB) $\bar{p}=(x_1+x_2)/(n_1+n_2)$
MEANS	Mean = μ σ Known σ = Pop. SD n>30 or normal	7.2 $n=\left(\frac{z_{\alpha/2}\cdot\sigma}{E}\right)^2$	skip ZInterval $E=z_{\alpha/2}\cdot\sigma/\sqrt{n}$ $\bar{x}-E<\mu<\bar{x}+E$	Always Assume that the Population Standard deviation is Unknown for two sample means.	8.4 Skip ZTest Ho: $\mu=\mu_0$ PE: $\bar{x}$ TS: $z=(\bar{x}-\mu_0)/(\sigma/\sqrt{n})$ CV: z^* = InvNorm(area)	Always Assume that the Population Standard deviation is Unknown for two sample means.
MEANS	Mean = μ σ Unknown Use t-distribution σ = Pop. SD unknown n>30 or normal parent population If n < 30 Check normal Histogram QQPlot	7.2 $n=\left(\frac{t_{\alpha/2}\cdot S}{E}\right)^2$ df = n-1 and S from preliminary Sample	7.2 TInterval PE: $\bar{x}$ $\alpha=1-CL$ $E=t_{\alpha/2}\cdot S/\sqrt{n}$ For Data use 1-VarStat(L_1) to find $\bar{x}$ and S $\bar{x}-E<\mu<\bar{x}+E$ $\bar{x}=\frac{UB+LB}{2}$ CV: $t_{\alpha/2}$ = invT$(1-\alpha/2, df)$	9.2 TInterval on L3=L1-L2 Matched Pairs, df = n - 1, n = # pairs PE: $\bar{d}=\bar{x}$ 1-VarStat L_3 $E=t_{\alpha/2}\cdot\sigma/\sqrt{n}$ $\bar{d}-E<\mu_d<\bar{d}+E$ 9.2 2-SampTInterval Independent df = smaller(n1-1,n2-1) PE: $\bar{x}_1-\bar{x}_2$ $E=t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$ $(\bar{x}_1-\bar{x}_2)-E<\mu_1-\mu_2<(\bar{x}_1-\bar{x}_2)+E$	8.3 TTest Ho: $\mu=\mu_0$ PE: $\bar{x}$ TS: $t=\frac{\bar{x}-\mu_0}{(s/\sqrt{n})}$ CV: t* = InvT(area,df) RT area = $1-\alpha$ LT area = α 2T area = $1-\alpha/2$ df = n-1 P-value= RT tcdf(TS,9999, df) LT tcdf(-9999,TS,df) 2T multiply by 2	9.3 TTest on L3=L1-L2 Matched Pairs, df = n - 1, n = # pairs Ho: $\mu_d=0$ 1-VarStat L_3 PE: $\bar{d}$ $\bar{x}=\bar{d}$ $S_x=S_d$ TS: $t=\frac{\bar{d}-\mu_d}{(s_d/\sqrt{n})}$ CV: t* = InvT(Area, df) P-value=tcdf(LB,UB) df 9.3 2-SampTTest Independent df = smaller(n1-1,n2-1) Ho: $\mu_1=\mu_2$ or $\mu_1-\mu_2=0$ PE: $\bar{x}_1-\bar{x}_2$ TS: $t=(\bar{x}_1-\bar{x}_2)/\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$ CV: t^* = InvT(Area,df) P-value=tcdf(LB,UB,df)
Standard Deviation	Standard Deviation	Table	7.3 S2INT(S², df, CL) $\sqrt{\frac{(n-1)s^2}{\chi_R^2}}<\sigma<\sqrt{\frac{(n-1)s^2}{\chi_L^2}}$			

UB = upper bound of confidence interval
LB = Lower bound of confidence interval

RT = right Tail
LT = Left Tail