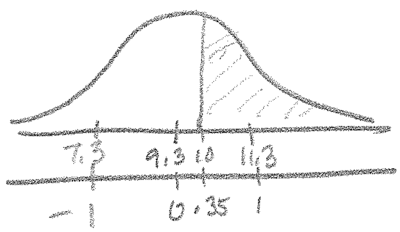
PRACTICE TEST 2 Math 15 STATISTICS Jones Spring 2019 Name Key

Show all work! Draw a normal distribution when needed.

Solve the problem.

1) Suppose that replacement times for washing machines are normally distributed with a mean of 9.3 years and a standard deviation of 2 years. $\mu = 9.3$ $\sigma = 2$

a) Draw this distribution showing an axes for the age of the machine and a z–axes.



$z = \frac{x - \bar{x}}{s} = \frac{10 - 9.3}{2} = \frac{.7}{2} = \boxed{.35 = z}$

$x = 10$

b) What proportion of washing machines last more than 10 years?

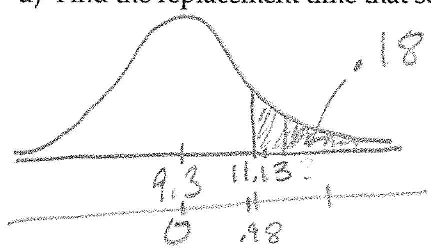
$P(x > 10) = \text{ncdf}(10, 9999, 9.3, 2) = \boxed{.3632}$

c) If a store sells 100 washers how many do they expect to last more than 5 years?

$P(x > 5) = \text{ncdf}(5, 9999, 9.3, 2) = .9842$

So 98.42% or $\boxed{98}$ out of 100 washers last more than 5 years

d) Find the replacement time that separates the top 18% from the bottom 82%.



$X = \text{invnorm}(1 - .18, 9.3, 2) = \boxed{11.13 \text{ years}}$

$Z = \text{invNorm}(.82, 0, 1) = \boxed{.91}$

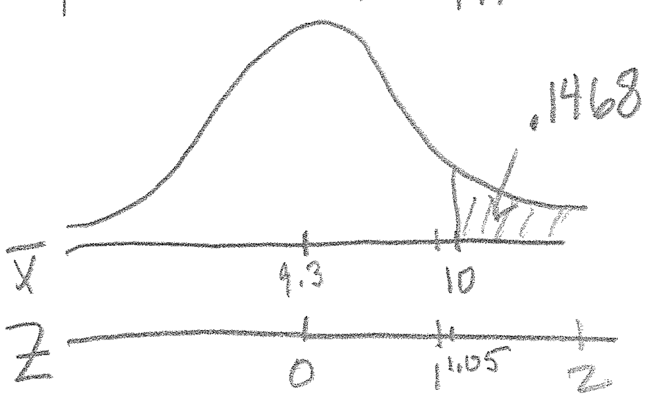
e) If you sell 9 washing machines and insure them for 10 years. Wht is the probability that the mean life ot the 9 machines is more than 10 years? Find Mean and standard deviation of sampling distribution. Draw Sampling distribution.

$P(\bar{x} > 10) = \text{ncdf}\left(10, 9999, 9.3, 2/\sqrt{9}\right) = .1468$

$Z_{\bar{x}=10} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma/\sqrt{n}} = \frac{(10 - 9.3)}{(2/\sqrt{9})} = 1.05$

$\mu_{\bar{x}} = 9.3$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{3}$

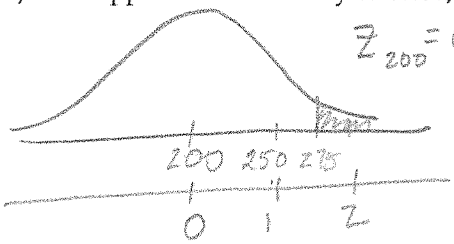
It is not unusual to have the mean life over 10 hrs for a group of 9 Washing Machines



Find the indicated probability. Show graphs with both x and z axis.

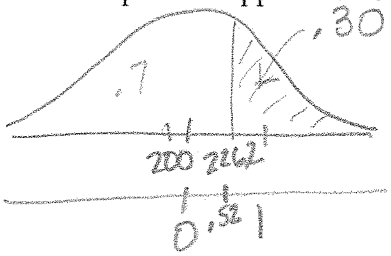
2) A bank's loan officer rates applicants for credit. The ratings are normally distributed with a mean of 200 and a standard deviation of 50. $\mu = 200 \quad \sigma = 50$

a) If an applicant is randomly selected, find the probability of a rating that is between 200 and 275.



$Z_{200} = 0 \quad Z_{250} = \frac{x - \bar{x}}{s} = \frac{275 - 200}{50} = 1.5$

b) In todays market the loan officer is only giving loans to the top 30% of applicates. What rating will separate the top 30% of applicants from the bottom 70%.



$x = \text{invN}(1 - .3, 200, 50) = 226.2$

$Z = \text{invNorm}(.7, 0, 1) = .52$

Solve the problem. Graph of the distribution of sample means is required.

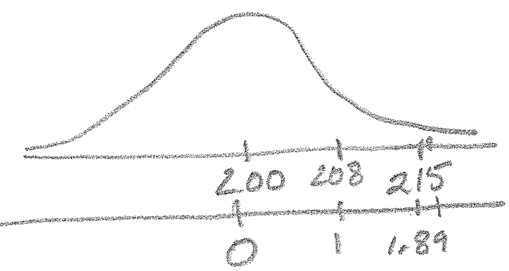
3) A bank's loan officer rates applicants for credit. The ratings are normally distributed with a mean of 200 and a standard deviation of 50. $\mu = 200 \quad \sigma = 50 \quad n = 40$ CLT & MEAN

If 40 different applicants are randomly selected, find the probability that their mean score is above 215.

$P(\bar{x} > 215) = \text{ncdf}(215, 999; 200, 50/\sqrt{40}) = .0289$

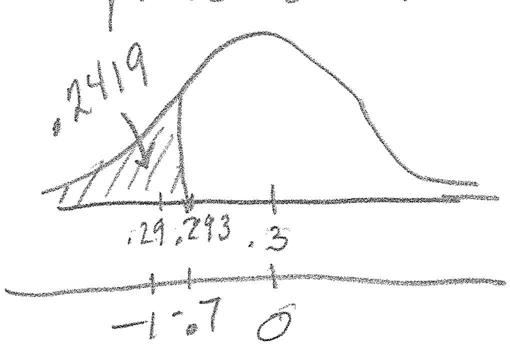
$\mu_{\bar{x}} = 215 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{50}{\sqrt{40}} = 7.9057$

$Z_{\bar{x} = 215} = \frac{(\bar{x} - \mu_{\bar{x}})}{(\sigma/\sqrt{n})} = \frac{(215 - 200)}{(50/\sqrt{n})} = 1.89$



4) The diameters of pencils produced by a certain machine are normally distributed with a mean of 0.30 inches and a standard deviation of 0.01 inches. In a random sample of 450 pencils, approximately HOW MANY would you expect to have a diameter less than 0.293 inches? (Hint: Find a proportion first.)

$\mu = .3 \quad \sigma = .01 \quad n = 450 \quad P(X < .293) = \text{ncdf}(-999, .293, .3, .01) = .2419$



$Z = \frac{x - \mu}{\sigma} = \frac{.293 - .3}{.01} = -.7$

24.19% of the pencils have diameters less than .293. In a batch of 450 pencils that gives $.2419 \cdot 450 = \boxed{109 \text{ pencils}}$

Provide an appropriate response.

5) Sampling without replacement involves dependent events, so this would not be considered a binomial experiment. Explain the circumstances under which sampling without replacement could be considered independent and, thus, binomial. To make calculations easier.

When $n < 5\%$ of N then the probabilities corresponding to Selection with replacement are so close to the probabilities when sampling is done without replacement that we can use the independent (with Replacement) Values instead.

6) Under what conditions can we apply the results of the central limit theorem?

For Means when $n > 30$ or dist. of the Parent population is Normal.

For proportions when $np > 5$ and $nq > 5$.

(Note: np & $nq > 9$ is Much better)

7) The typical computer random–number generator yields numbers in a uniform distribution between 0 and 1 with a mean of 0.500 and a standard deviation of 0.289. (a) Suppose a sample of size 50 is randomly generated. Find the probability that the mean is below 0.300. (b) Suppose a sample size of 15 is randomly generated. Find the probability that the mean is below 0.300. These two problems appear to be very similar. Only one can be solved by the central limit theorem. Which one and why?

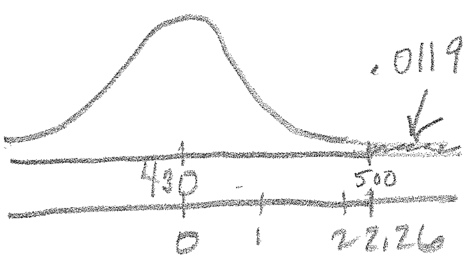
a) $n = 50 > 30$ so dist of Sample Means is Normal.

b) $n = 15 < 30$ & pop = Uniform $\neq$ Normal so distribution of Sample means Will Not be Normal (Yet) Bootstrap!

8) SAT verbal scores are normally distributed with a mean of 430 and a standard deviation of 120 (based on the data from the College Board ATP). If a sample of 15 students is selected randomly, find the probability that the sample mean is above 500. Does the central limit theorem apply for this problem?

$\mu = 430$ $\sigma = 120$ $n = 15$, Normal yes CLT Applys $\bar{x} = 500$

$$Z_{\bar{x}=500} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma/\sqrt{n}} = \frac{(500-430)}{(120/\sqrt{15})} = 2.26$$



$.0119 = P(\bar{x} > 500) = \text{ncdf}(500, 9999, 430, 120/\sqrt{15})$

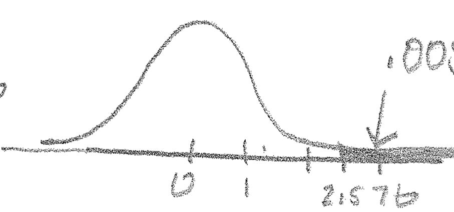
$= \boxed{.0119}$

9) Which of the following notations represents the standard deviation of the population consisting of all sample means?

A) $\sigma_{\bar{x}}$ B) s C) $\sqrt{npq}$ D) μ

Find the indicated value.

10) $z_{0.005}$ = invnorm$(1-.005, 0, 1) = 2.576$



Provide an appropriate response.

11) A poll of 1100 randomly selected students in grades 6 through 8 was conducted and found that 54% enjoy playing sports.
Is the 54% result a statistic or a parameter? Explain. Statistic, $.54 \cdot 1100 = 594 = X = \#$ in Sample who enjoy sports.
Does the 54% refer to sample mean or a sample proportion?

$$54\% = .54 = \frac{594}{1100} = \frac{x}{n} = \hat{p} = \text{a Sample Proportion.}$$

12) Tell whether the following statistic is a biased or unbiased estimator of a population parameter:
Sample proportion used to estimate a population proportion. unbiased
Sample mean used to estimate a population mean. unbiased
Sample standard deviation used to estimate a population standard deviation. biased (But used)
Sample varience used to estimate a population varience. unbiased
Median - Biased

13) Apply the Central Limit Theorem. Samples of size n = 800 are randomly selected from the population of numbers (0 through 9) produced by a random-number generator.
a) If the proportion of odd numbers is found for each sample what type of distribution is the distribution of the sample proportions? What is it's mean and what is it's standard deviation?

a) $\boxed{\text{Normal}}$

b) $\boxed{\mu_{\hat{p}} = .5} = p = \frac{5}{10} = \frac{\# \text{ odd}}{\# \text{ Values}}$

c) $\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.5 \cdot .5}{800}} = \boxed{.05 = \sigma_{\hat{p}}}$

b) If the mean of the 800 values is found for each of the samples what type of distribution is the distribution of sample mean? What is the mean and what is the standard deviation of the distribution of sample means? (please use correct notation.)

Population $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} = L_1$
1-Var stats $\mu = 4.5$ $\sigma = 2.87$

a) Normal

b) $\mu_{\bar{x}} = 4.5$

c) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.87}{\sqrt{10}} = .1015$

$n = 367 \quad x = 30$

Use the given degree of confidence and sample data to construct a confidence interval for the population proportion p.

14) Of 367 randomly selected medical students, 30 said that they planned to work in a rural community. Find a 95% confidence interval for the true proportion of all medical students who plan to work in a rural community. $\alpha = .05$

d) (2 Points) What is the critical value needed to calculate a 90% confidnece interval?____________

95% $Z_{\alpha/2} = Z_{.025} = \text{invnorm}(1-.025, 0, 1) = 1.96$

90% $Z_{\alpha/2} = Z_{.05} = \text{invnorm}(1-.05, 0, 1) = 1.645$

e) (2 Points) What is the point estimate for the population proportion?____________ $\hat{p} = \frac{x}{n} = \frac{30}{367} = .0817$

f) (2 Points) Show the formula and the values used to calculate the margin of error

95% $E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96 \cdot \sqrt{\frac{.0817 \cdot .9183}{367}} =$

E=____________

90% $E = 1.645 \cdot \sqrt{.0817 \cdot .9183 / 367} = .0235$

g) (2 Points) Find a 90 percent confidence interval for the proportion of doctors who plan to work in rural communities..____________________

90% $\boxed{.0582 < p < .1052}$

h) (4 Points) State the meaning of this confidence interval.

We are 90% confident that the proportion of All Docs who plan to work in a rural community is between .0582 and .1052 or 5.8% to 10.5%.

Use the given data to find the minimum sample size required to estimate the population proportion.

15) Margin of error: 0.044; confidence level: 95%; $\hat{p}$ and $\hat{q}$ unknown

$n = \frac{Z_{\alpha/2}^2 \cdot .25}{.044^2} = \frac{1.96^2 \cdot .25}{.044^2} = \boxed{497}$

16) Margin of error: 0.005; confidence level: 99%; from a prior study, p is estimated by 0.166.

$n = Z_{\alpha/2}^2 \cdot \hat{p} \cdot \hat{q} / E^2 = 2.576^2 \cdot .166 \cdot .834 / (.005)^2 = \boxed{36,748}$

Solve the problem.

17) A newspaper article about the results of a poll states: "In theory, the results of such a poll, in 99 cases out of 100 should differ by no more than 5 percentage points in either direction from what would have been obtained by interviewing all voters in the United States." Find the sample size suggested by this statement.

CL = "99 cases out of 100 contain true proportion"

CL = .99 $\quad \alpha = .01 \quad E = .05$ = difference between poll = $\hat{p}$ and True proportion = p

$Z_{.005} = 2.575$ or 2.576 or 2.58

$n = \frac{Z_{\alpha/2}^2 \cdot .25}{E^2} = 664$ or 664 or 666 ← I will take all but must correspond to your work.

↙ Find n

$E = .008$
$\alpha = .01$
$\hat{p} = .139$

Use the given data to find the minimum sample size required to estimate the population proportion.

18) (5 points) Margin of error: 0.008; confidence level: 99%; from a prior study, $\hat{p}$ is estimated by 0.139.

$$n = \frac{z_{\alpha/2}^2 \cdot \hat{p}\hat{q}}{E^2} = (2.575)^2 \cdot .139 \cdot .861 / .008 = 12{,}407.15$$

$\boxed{n = 12408}$

b) Does the size of the population effect the size of the sample needed to make this confidence interval?

NO! the population size N is Not part of calculation as long as $N > n$ above

19) a) (2 Points) Define confidence interval. A confidence interval is an interval of values that is likely to a CL to contain the population parameter p or μ.

b) (2 Points) Define margin of error. The maximum likely difference between the population parameter and the sample sta: μ or p ; $\bar{x}$, $\hat{p}$.

b) (2 Points) Suppose a confidence interval is 0.12< p < 0.20. Find the sample proportion $\hat{p}$ and the error estimate E.

$$\hat{p} = \frac{.2 + .12}{2} = \boxed{.16 = \hat{p}} \quad E = \frac{.2 - .12}{2} = \frac{.08}{2} = \boxed{.04 = E}$$

Solve the problem.

20) The sample data below consists of the heights of 30 randomly selected adults.
You wish to use the data to obtain a confidence interval estimate of the population mean.
a) Does the data set include any outliers?

b) How could you handle the outlier in this case? Explain your answer.

d) Calculate the confidence interval with and without the outlier.

e) Are confidence interval limits sensitive to outliers?

60.1	66.9	70.4	73.2	65.2	64.1
68.5	69.2	64.0	62.4	66.9	71.2
682	61.4	65.7	72.5	74.0	70.0
65.8	69.3	60.4	72.4	58.1	68.3
60.5	66.4	60.5	71.3	67.8	73.2

f) Find the confidence interval for the standard deviation of the heights of men.

Look up defenitions in Book

19) 26) a) (2 Points) Define confidence interval. An interval of values that is likely to contain the population parameter. 26) ______________

b) (2 Points) Define margin of error. the maximum likely difference E, between the Sample statistic, $\bar{x}$ or $\hat{p}$ and the population parameter, μ or p

b) (2 Points) Suppose a confidence interval is 0.12< p < 0.20. Find the sample proportion $\hat{p}$ and the error estimate E.

$\hat{p} = \frac{Max + Min}{2} = \frac{.2 + .12}{2} = \boxed{.16 = \hat{p}}$ $E = \frac{Max - Min}{2} = \frac{.2 - .12}{2} = \boxed{.04 = E}$

Solve the problem.

20) 27) The sample data below consists of the heights of 30 randomly selected adults. You wish to use the data to obtain a confidence interval estimate of the population mean. 27) ______________

a) Does the data set include any outliers? Yes, x = 682 Appears to be a Typo.

b) How could you handle the outlier in this case? Explain your answer.
Leave it out or correct it to 68.2

95% d) Calculate the confidence interval with and without the outlier. Use TInterval

With (45.421, 129.36) Without (65.143, 68.629)

e) Are confidence interval limits sensitive to outliers? Yes, the interval without the outlier is Much smaller

60.1	66.9	70.4	73.2	65.2	64.1
68.5	69.2	64.0	62.4	66.9	71.2
682	61.4	65.7	72.5	74.0	70.0
65.8	69.3	60.4	72.4	58.1	68.3
60.5	66.4	60.5	71.3	67.8	73.2

f) See Next Page

Use the given degree of confidence and sample data to construct a confidence interval for the population mean μ.

87.4 21) 28) A laboratory tested 80 chicken eggs and found that the mean amount of cholesterol was 213 milligrams with s = 12.8 milligrams. Construct a 95 percent confidence interval for the true mean cholesterol content, μ, of all such eggs. 28) ______________

$n = 80$
$\bar{x} = 213$
$s = 12.8$
$CL = .95$
$\alpha = .05$

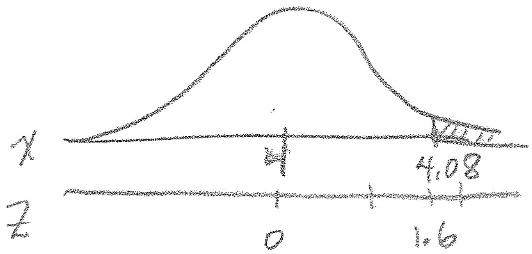
1. $\bar{x} = 213$
2. $z_{\alpha/2} = \text{invnorm}(1 - .05/2) = 1.96$
3. $E = z_{\alpha/2} \cdot s/\sqrt{n} = 1.96 \cdot 12.8/\sqrt{80} = 2.80$
4. $\bar{x} \pm E = 213 \pm 2.8$

$210.2 \le \mu \le 215.8$

5. We are 95% that The mean weight of all the eggs will be between 210.2 mg and 215.8 mg

23) 8) (21 Points) A machine in a saw mill cuts pieces of lumber to an average width of $\mu = 4$ inches with a standard deviation $\sigma = .05$ inches

a) (3 Points) Graph the distribution with both an x–axes and a z–axes. Show mean and standard deviation. Calculate the z–score of a 4.08 width for a piece of lumber and label on your graph.



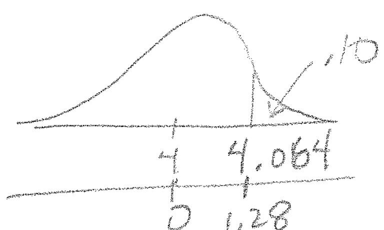
$z = \frac{x-\mu}{\sigma} = \frac{4.08-4}{.05} = 1.6$

3

b) (3 Points) What is the probability that width is at least 4.08 for a piece of lumber? Show all work. Use proper probability notation, calculator inputs and shade region with equal area on the graph above.

$P(x \geq 4.08) = \text{ncdf}(4.08, 9999, 4, .05) = .0548$ 3

c) (3 Points) What width separates the widest 10% of cuts? Show on a new graph.



$z = \text{invNorm}(.9, 0, 1) = 1.28$

$x = \text{invNorm}(.9, 4, .05) = \boxed{4.064 \text{ inches}}$

3

d) (3 Points) On a given day the inspector samples 16 boards, and finds the sample mean. Find the mean $\mu_{\bar{x}}$ and standard deviation $\sigma_{\bar{x}}$ of the population of sample means for samples of size n = 16.

$\mu_{\bar{x}} = 4 \qquad \sigma_{\bar{x}} = \frac{.05}{\sqrt{16}} = .0125$

3

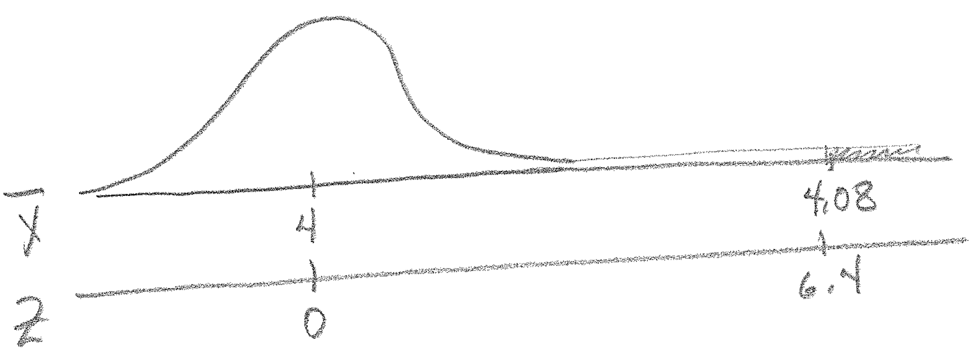
e) (3 Points) Find the z–score of a sample mean that is at least $\bar{x}$ =4.08 inches in the distribution of sample means.

$z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma/\sqrt{n}} = \frac{4.08-4}{.0125} = 6.4$

3

f) (6 Points) For a sample of size 16, what is the probability that the **mean** is at least $\bar{x}$ =4.08 inches in the distribution of sample means? Graph the distribution of sample means when the sample size is 16 with both an $\bar{x}$–axes and a z–axes. Does the data indicate that the machine is working properly? No,

6



the $P(\bar{x} > 4.08) \approx 0$

it would be very unusual to see this mean width if the machine is adjusted properly

21

Use the given degree of confidence and sample data to construct a confidence interval for the population mean [].

21) A laboratory tested 80 chicken eggs and found that the mean amount of cholesterol was 213 milligrams with $s = 12.8$ milligrams. Construct a 95 percent confidence interval for the true mean cholesterol content, μ, of all such eggs.

Provide an appropriate response.

22) What assumption about the parent population is needed to use the t distribution to compute the margin of error when $n < 30$?

We Need the parent population to be Normal so the CLT guarentees that the distribution of sample means follows a t-distribution.

23) (21 Points) A machine in a saw mill cuts pieces of lumber to an average width of $\mu = 4$ inches with a standard deviation $\sigma = .05$ inches

a) (3 Points) Graph the distribution with both an x–axes and a z–axes. Show mean and standard deviation. Calculate the z–score of a 4.08 width for a piece of lumber and label on your graph.

b) (3 Points) What is the probability that width is at least 4.08 for a piece of lumber? Show all work. Use proper probability notation, calculator inputs and shade region with equal area on the graph above.

c) (3 Points) What width separates the widest 10% of cuts? Show on a new graph.

d) (3 Points) On a given day the inspector samples 16 boards, and finds the sample mean. Find the mean $\mu_{\bar{x}}$ and standard deviation $\sigma_{\bar{x}}$ of the population of sample means for samples of size $n = 16$.

e) (3 Points) Find the z–score of a sample mean that is at least $\bar{x} = 4.08$ inches in the distribution of sample means.

f) (6 Points) For a sample of size 16, what is the probability that the **mean** at least $\bar{x} = 4.08$ inches in the distribution of sample means? Graph the distribution of sample means when the sample size is 16 with both an $\bar{x}$–axes and a z–axes. Does the data indicate that the machine is working properly.