Spring 2019

Practice Test 1 Math 15 Jones Name Key

Determine whether the given value is a statistic or a parameter.

1) After inspecting all of 55,000 kg of meat stored at the Wurst Sausage Company, it was found that 45,000 kg of the meat was spoiled. 1) Parameter

Identify the number as either continuous or discrete.

2) The number of stories in a Manhattan building is 22. 2) Discrete

Provide an appropriate response.

3) A researcher wants to obtain a sample of 100 school teachers from the 800 school teachers in a school district. Describe procedures for obtaining a sample of each type: random, systematic, convenience, stratified, cluster. 3) ________

Cluster - Order Schools, randomly select 5, Interview all teachers at those 5 Schools

Random - Order Teachers, randomly Select 100 Numbers from 1 to 800, Interview all teachers Selected

Systematic - Order teachers, randomly select number between 1 and 8, Select every 8 th teach after that

Stratified - Divide teachers by years of experiance, 1-5, 6-10, ... interview proportion corresponding to prop in each catagory

Determine which score corresponds to the higher relative position.

4) Which score has a better relative position, a score of 44 on a test for which $\bar{x} = 40$ and $s = 4$, or a score of 283.4 on a test for which $\bar{x} = 260$ and $s = 26$? 4) ________

$Z_{44} = \frac{44-40}{4} = 1$ $\quad Z_{283.4} = \frac{23.4}{26} = .9$

44 is a Better Score

Solve the problem.

5) The ages of the members of a gym have a mean of 40 years and a standard deviation of 14. Use the range rule of thumb to estimate the minimum and maximum "usual" ages. Is 72 an unusual age for a gym member? 5) 1- yes

Min = 12

Max = 68

$\text{Min usual} = \bar{x} - 2s = 40 - 2\cdot 14 = 12$

$\text{Max usual} = \bar{x} + 2s = 40 + 2\cdot 14 = 68$

Yes 72 is unusual large → Statistically Significantly large

$Z_{72} = \frac{72-40}{14} = 2.28 > 2$

b)

Men	120	77	89	97	124	68	72	96
Women	115	86	49	56	78	76	78	95

	$\bar{x}$	S_x	min	Q_1	med	Q_2	max	range	mode	Mid range	$Var = s^2$
Men	92.9	20.88	68	74.5	92.5	108.5	124	56	None	96	436.1
Women	79.125	20.84	49	66	78	90.5	115	66	78	82	434.41

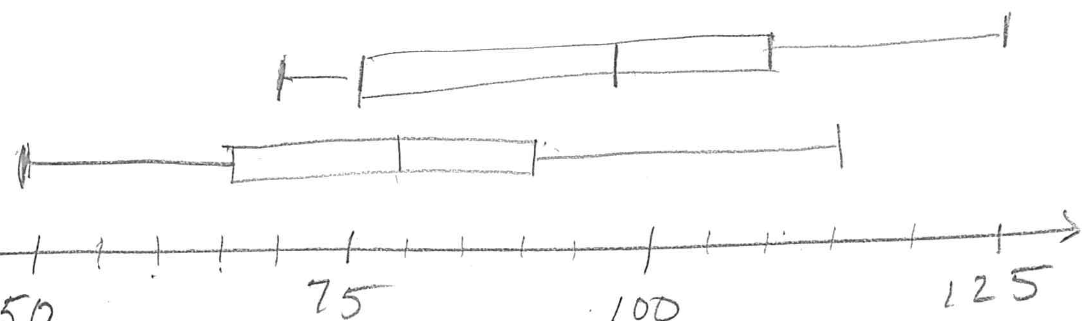
range = Max − Min = 56

$$\text{Midrange} = \frac{\text{Max} + \text{Min}}{2} = \frac{124+68}{2} = 96$$

$$S_x^2 = 436$$

b) Men

Women



c) Men's pulse center is <u>higher</u> than womens pulse

d) Spread of men's pulse rates <u>is about the same as</u> the Spread of womens pulse rates.

PT1 Page 2

Find the range, variance, and standard deviation for each of the two samples, then compare the two sets of results.

6) When investigating times required for drive–through service, the following results (in seconds) were obtained. 6) ______

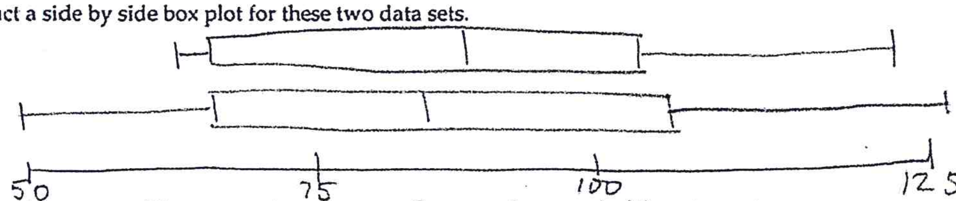
Use One Var Stat (L1) to find Summary Statistics

SKIP

Restaurant A	120	67	89	97	124	68	72	96
Restaurant B	115	126	49	56	98	76	78	95

	A	B		A	B
Mean	91.6	86.6	S_x = Standard Deviation	22.2	27.0
Q1	70	66	Minimum	67	49
Median	92.5	86.5	S_x^2 = Variation	493.98	728.0
Q3	108.5	106.5	Maximum	124	126
Mode	none	none	Range	57	77
$\frac{max+min}{2}$ = Midrange	95.5	87.5			

Construct a side by side box plot for these two data sets.



Compare the centers of these two sets.

A is lower than B
But Similar

Compare the spread of these two sets.

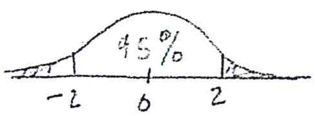
A has less Spread than B.
But Similar

7) Explain how two data sets could have equal means and modes but still differ greatly. Give an example with two data sets to illustrate. Different spreads 7) ______

65 69 70 70 71 75 $\bar{x}$ = 70 mode = 70 mode = 7 S = Small

40 60 70 70 80 100 Same middle S = Big

8) The textbook defines **unusual** values as those data points with z scores less than z = -2.00 or z scores greater than z = 2.00. Comment on this definition with respect to "the Empirical Rule"; refer specifically to the percent of scores which would be defined as **unusual** according to "the Empirical Rule". 8) ______



5% of Scores are unusual by the empirical Rule

9) Sometimes probabilities derived by the relative frequency method differ from the probabilities expected from classical probability methods. How does the law of large numbers apply in this situation? 9) ______

As an experiment is repeated many times the relative frequency of an event approaches the value given by the Classical Probability.

65.3

Find the indicated probability.

10) A class consists of 69 women and 68 men. If a student is randomly selected, what is the probability that the student is a woman? 10) ________

$$\frac{69}{69+68} = .504$$

11) If you pick a card at random from a well shuffled deck, what is the probability that you get a face card or a spade? 11) ________

$$P(☺ \text{ or } ♠) = P(☺) + P(♠) - P(☺ \text{ and } ♠)$$
$$= \frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{22}{52} = .423$$

12) A sample of 100 wood and 100 graphite tennis rackets are taken from the warehouse. If 5 wood and 10 graphite are defective and one racket is randomly selected from the sample, find the probability that the racket is wood or defective. 12) ________

$$P(W \text{ or } D) = P(W) + P(D) - P(W \text{ and } D)$$
$$= \frac{100}{200} + \frac{15}{200} - \frac{5}{200}$$
$$= \frac{110}{200} = .55$$

13) A bag contains 7 red marbles, 4 blue marbles, and 1 green marble. Find P(not blue). 13) .666

$$P(B) = \frac{4}{12} \qquad P(\bar{B}) = 1 - \frac{4}{12} = \frac{8}{12} = \frac{2}{3}$$

Find the indicated probability.

14) A restaurant offers 9 entrees and 11 desserts. In how many ways can a person order a two-course meal? 14) $9 \cdot 11 = 99$

Find the indicated probability.

15) Describe an event whose probability of occurring is 1 and explain what that probability means. Describe an event whose probability of occurring is 0 and explain what that probability means. 15) ________

$P(E) = 0$, E can't happen $E = \{$ pigs fly today $\}$

$P(E) = 1$, E is certain $E = \{$ Sun rose today $\}$

PT I

16) Consider the frequency table below which has single values as classes: 16) ____________

12 classes

Value	Frequency
10	1
11	3
12	7
13	18
14	10
15	4
16	2
17	7
18	16
19	10
20	6
21	2

4 classes

Value	Frequency
10-12	11
13-15	32
16-18	25
19-21	18

6 classes

Value	Frequency
10-11	4
12-13	25
14-15	14
16-17	9
18-19	26
20-21	8

Construct a new frequency table for this data with 4 classes.
Now construct a another frequency table for this data with 6 classes.

Suppose that you construct a histogram corresponding to the original data and histograms corresponding to each of the new frequency tables. Describe the shapes of the three histograms. Does the histogram with six classes capture the distribution of the data? Does the histogram with four classes capture the distribution of the data?

The Bimodal Nature of the data is seen in the Original data and with 6 classes, but is missed when there are only 4 classes.

Solve the problem.

Provide an appropriate response.

17) A computer company employs 100 software engineers and 100 hardware engineers. The personnel manager randomly selects 20 of the software engineers and 20 of the hardware engineers and questions them about career opportunities within the company. a) What sampling technique is being used? b) Does this sampling plan result in a random sample? c) Simple random sample? d) Explain.

a) Stratified

b) Yes

c) No, since Not every Sample of 40 employees is Possible it is Not a Simple random Sample.
for instance we could never get a sample with 19 Software engineers and 21 of the hardware engineers.

#19 b) prob. Dist

X = # of Defectives

X	$P(X)$
$X=0$	$.7744$
$X=1$	$.2112$
$X=2$	$.0144$

$.7744 = .88^2$ What is the probability of No Defectives

$.2112 = .12 \cdot .88 + .88 \cdot .12$

$.0144 = .12^2$

$X=1$ happen two ways

gb or bg

$P(X=1)$ $.88 \cdot .12 + .12 \cdot .88$

gg

$.88 \cdot .88$

bb

$.12 \cdot .12$

$1 - (.88^2 + .12^2)$

Find the indicated probability.

19 18) A batch consists of 12 defective coils and 88 good ones. 18) ________
Find the probability of getting two good coils when two coils are randomly selected if the first selection is replaced before the second is made.____________

$\frac{88}{100} \cdot \frac{88}{100} = \boxed{.7744}$

If X=the number of defective coils when 2 are selected. Make a probability distribution for the number of defective coils out of 2. You may assume that the selections are done with replacement.

X = # of defective

X	P(X)	
0	.7744	= P(Both good) = .88 · .88
1	.2112	= 1 − (.0144 + .7744) or = 2 · .88 · .12
2	.0144	= P(Both Bad) = .12 · .12

18) 19) Among the contestants in a competition are 42 women and 28 men. If 5 winners are randomly selected, what is the probability that they are all men? 19) ________
In how many ways can 5 people be selected from this group of 70?

${}_{70}C_5 = 12103014$

In how many ways can 5 men be selected from the 28 men?

${}_{28}C_5 = 98280$

$P(\text{All men}) = \frac{28}{70} \cdot \frac{27}{69} \cdot \frac{26}{68} \cdot \frac{25}{67} \cdot \frac{24}{66}$

Find the probability that the selected group that will consist of all men.

$P(\text{all Men}) = \frac{98280}{12103014} = .00812$

Solve the problem.

20) 8 basketball players are to be selected to play in a special game. The players will be selected from a list of 27 players. If the players are selected randomly, what is the probability that the 8 tallest players will be selected? 20) $\frac{{}_8P_8}{{}_{27}P_8}$

$P(8\ \text{tallest}) = \frac{{}_8C_8}{{}_{27}C_8} = \frac{1}{2220075} = .0000004504$

21) There are 9 members on a board of directors. If they must elect a chairperson, a secretary, and a treasurer, how many different slates of candidates are possible? 21) ________

${}_9P_3 = 504$

BCA
CBA

PTI

olve the problem involving probabilities with independent events.

22) A single die is rolled twice. Find the probability of getting a 2 the first time and a 2 the second time. 22) ______

$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

Find the indicated probability.

23) The following table contains data from a study of two airlines which fly to Small Town, USA. 23) ______

	Number of flights which were on time	Number of flights which were late
Podunk Airlines	33	6
Upstate Airlines	43	5
	76	11

87

a) If one of the flights is randomly selected, find the probability that the flight selected arrived on time given that it was an Upstate Airlines flight.

$\frac{43}{48} = .8958 = P(O|u)$

b) If one of the flights is randomly selected, find the probability that the flight selected arrived on time and was an Upstate Airlines flight.

$\frac{43}{87} = .4943 = P(O \cap u)$

c) If one of the flights is randomly selected, find the probability that the flight selected arrived on time or was an Upstate Airlines flight.

$P(O \cup u) = \frac{76}{87} + \frac{48}{87} - \frac{43}{87} = \frac{81}{87} = .9310$

d) If two flights were randomly selected find the probability that both flights were on time. Calculate this probability with and without replacement.

With replacement

$P(\text{1st Ontime and 2nd Ontime}) = \frac{76}{87} \cdot \frac{76}{87} = .763$

without replacement

$P(O_1 \& O_2) = \frac{76}{87} \cdot \frac{75}{86} \stackrel{3}{=} .762$

Cumbersome calculation Rule

$2 < .05 \cdot 87$ yes

Course Tools | Document sharing

Find the indicated probability.

19) A batch consists of 12 defective coils and 88 good ones. 8) ______

a) Find the probability of getting three defective coils when three coils are randomly selected if each selection is replaced before the next is made. Show method used to get answer.

$$P(X=3) = \left(\frac{12}{100}\right)^3 = \boxed{.001728}$$

$N=100$
$n=3$
$p = 12/100 = .12$

b) If X=the number of defective coils when 3 are selected. Make a probability distribution for the number of defective coils out of 3 when the selections are done with replacement.

Dist $\boxed{\text{Binomialpdf}(3, .12)}$ - defective or not, with replacement so Independent with unchanging probability of Success = p = .12 on each trial. The Number of trial is 3.

STR → L_2

X	P(X) =
0	.6815
1	.2788
2	.0380
3	.001728

c) Find the probability of getting at least one defective coil. You get extra credit if you can find both methods for solving this problem.

$$P(X \geq 1) = .2788 + .0380 + .0017 = 1 - .6815 = 1 - (.88)^3 = \boxed{.3185}$$

18. Among the contestants in a competition are 42 women and 28 men. If 5 winners are randomly selected, what is the probability that they are all men? 9) ______

a) In how many ways can 5 people be selected from this group of 70? $_{70}C_5 = 12{,}103{,}014$

b) In how many ways can 5 men be selected from the 28 men?

c) Find the probability that the selected group that will consist of all men. $_{28}C_5 = 98280$

$$P(\text{All Men}) = \frac{_{28}C_5}{_{70}C_5} = \boxed{.00812} = \frac{28}{70} \cdot \frac{27}{69} \cdot \frac{26}{68} \cdot \frac{25}{67} \cdot \frac{24}{66}$$

Solve the problem.

20) 8 basketball players are to be selected to play in a special game. The players will be selected from a list of 27 players. If the players are selected randomly, what is the probability that the 8 tallest players will be selected? 10) ______

only one group of the 8 tallest player, order selected doesn't Matter

$$P(\text{Tallest } 8) = 1/{_{27}C_8} = 1/2220075 = \boxed{.0000004504}$$

21) There are 9 members on a board of directors. If they must elect a chairperson, a secretary, and a treasurer, how many different slates of candidates are possible? 11) ______

order Selected ⇒ which job → order Matters ⇒ Permutations

$$_9P_3 = \boxed{504}$$

Solve the problem involving probabilities with independent events.

22) A single die is rolled twice. Find the probability of getting a 2 the first time and a 2 the second time. 12) ______

$$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} = \boxed{.0278}$$

Then use the Binomial Theorem to find the probability exactly.

2 4) An engineer thinks that she had improved the quality of the circuit boards that she is designing. The defect rate has been 14%. But in the last sample of 50 parts she found that only 4 were defective. Is this conclusive proof that she improved her design or is this sample usual to see when the defect rate is 14% and more data is needed to be sure that the defect rate really has decreased. Assume that many thousands of parts are being produced. Indepencence

4) ____________

$p = .14$

$n = 50$

$x = 4$

$q = 1 - p$

$q = 1 - .14$

$= .86$

$\hat{p} = \frac{4}{50} = .08$ defect rate in Sample

$\boxed{\sigma = \sqrt{npq}}$

a) What is the mean and standard deviation of the binomial ~~distribution used for this~~ problem. $\boxed{\mu = n \cdot p} = 50 \cdot .14 = \boxed{7}$ $\sigma = \sqrt{50 \cdot .14 \cdot .86} = \boxed{2.45}$

b) How many do we expect to be defective? $\boxed{7 = E(x)}$ X = # of defective circuit boards in a Sample of 50.

c) What is the proability that we see a sample with at most 4 when the defect rate is 14%? Use the binomial Distribution. Draw the disribution and shade the rectangles with area corresponding to the probability that we are finding.

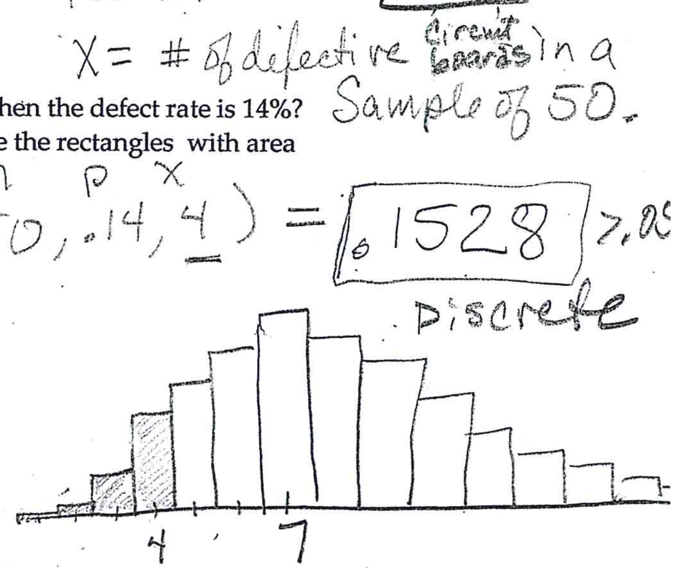
$P(X \le 4) = \text{binomialcdf}(\overset{n}{50}, \overset{p}{.14}, \overset{x}{4}) = \boxed{.1528} > .05$

$r \le 4$

$\boxed{P(x \le r) = \text{bincdf}(n, p, r)}$

Discrete

X
0
1
2
3
4
5
⋮
50



d) Does this sample verify her claim that the defect rate has been lowered?

No, it would No be unusual to See a Sample of 50 with only 4 defective pens. However her results are promising. A larger Sample with this rate would be conclusive.

Key

Show all work! Draw a normal distribution when needed.

Answer the question.

25) 1) Suppose that computer literacy among people ages 40 and older is being studied and that the accompanying tables describes the probability distribution for four randomly selected people, where x is the number that are computer literate. 1) ____________

a) is this a probability distribution_yes_

x	P(x)
0	0.16
1	0.25
2	0.36
3	0.15
4	0.08

$\sum p(x) = 1$

$0 \le p(x) \le 1$

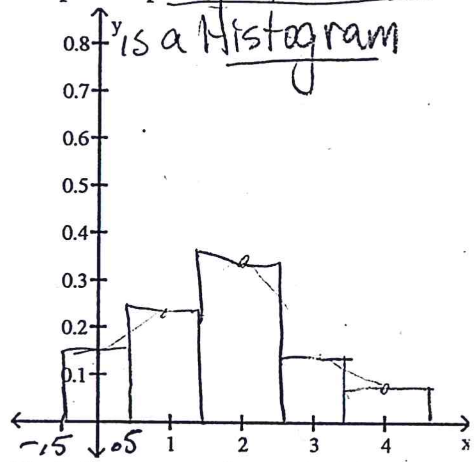
Is it unusual to find four computer literates among four randomly selected people? (WHY?) No, $P(X=4) = .08 > .05$ so Not Unusual

What is the probability of getting 2 or fewer people out of the 4 who are computer literate?__________

$P(X \le 2) = .36 + .25 + .16 = .77$

Graph this probabillity distribution.

is a Histogram



Find the mean of this probability distribution.

1-Var stat (L_1, L_2)

$\mu = \bar{x} = \boxed{1.74 = \mu}$

Find the Standard deviation of this distribution.

$\sigma = \sigma_x = \boxed{1.14}$

SKIP

~~What kind of probability distribution is this? (circle one)~~

~~Normal Distribution~~	~~Other Continuuous Distribution~~	~~Binomial Distribution~~	~~Discrete~~

$np = 4 \cdot p = 1.74$
$\Rightarrow p = .435$ but binomialpdf(4, .435) is Not the one given

Solve the problem.

26) 2) Suppose you buy 1 ticket for $1 out of a lottery of 1,000 tickets where the prize for the one winning ticket is to be $500. What is your expected value? X = Net winnings 2) ____________

	X	P(X)
Win	500	1/1000
Lose	−1	999/1000

$E(x) = 499 \cdot \frac{1}{1000} + (-1) \cdot 999/1000$

$= -.50$

one outcome should be Negitive

(4) Probability of a Sum of at least 10, $E = \{55, 56, 65, 66, 64, 46\}$ $P(E) = \frac{6}{36} = .1667$

(5) What is the probability that the Sum is even? $F = \{11, 13, 22, 31 \ldots$

$$P(F) = \frac{18}{36}$$

Deck of cards = 52 ← # cards

$\underbrace{\text{A } 2 \ 3 \ \ldots \ 10 \ \ \text{J Q K}}_{\text{Values} = 13}$ (face: J Q K)

Suits 4

♡

♢

♧

♤

$$P(\text{Face}) = \frac{12}{52}$$

Delt 2 face cards

$S = \{\text{Two Cards}\} = {}_{52}C_2$

$E = \{\text{Two face Cards}\} = {}_{12}C_2$

$$P(2\text{ face}) = \frac{{}_{12}C_2}{{}_{52}C_2}$$

#18 Probability Dist Pick Two
X = # of Defective

X	0	1	2
$P(X)$	.7744	.2112	.0144

a) $P(2 \text{ good}) = .88 \cdot .88 = .7744$

b) $P(2 \text{ Defective}) = .12 \cdot .12 = .0144$

$P(1 \text{ Defective}) = P(X=1) = P(GD) + P(DG)$

$= .88 \cdot .12 + .12 \cdot .88$

$= .1056 + .1056$

$= .2112$

c) $P(\text{At least one Defective}) = 1 - P(\text{No Defective})$

$= 1 - .7744$

$= \boxed{.2256}$

7) 8 9 10 10 11 12 $\bar{x} = 10$ Mode $= 10$

1 2 10 10 18 19 ← Spread is Much larger

4.2 & 4.3

23)

	(O)	(L)	
(P)	33	6	39
(U)	43	5	48
	76	11	87

a) $P(OT \mid US) = \cancel{\dfrac{P(O \cap US)}{P(U)}} = \dfrac{43}{48} = .8958$

Given That

only consider upstate flights

b) $P(O \text{ and } US) = \dfrac{43}{87} = P(O \cap US) = .4943$

c) $P(O \text{ or } US) = P(O \cup US) =$ Write Addition Rule

$= P(O) + P(US) - P(O \cap US)$

$= \dfrac{76}{87} + \dfrac{48}{87} - \dfrac{43}{87} = .9310$

d) $P(O_1 \text{ and then } O_2) = \dfrac{76}{87} \cdot \dfrac{75}{86}$ ← without Replacement $\left(\dfrac{76}{87}\right)^2$ ← with Replacement

e) Is Ontime <u>Independent</u> of Airline

$P(O) = P(O \mid U)$ then independent

$P(O) = \frac{76}{87} = .8735 \neq P(O \mid U) = \frac{43}{48} = .8958$

Not Independent

P(on time and Tham air
Upstate)