§5.2 Binomial Distribution

Plan Thur. 2/21 Today Cover 5.2 & 5.3

Tues. 2/26 ~~Collect~~ $\boxed{\text{PT1}}$, H5.2, Q6

Thurs. 2/28 Test 1

Four Properties of a Binomial Distribution

① Fixed # of Trials $= n$

② Independent or Satisfy Cumbersome Calculation Condition $n < 5\% N$

③ Two Outcomes

④ Probability for Success is the Same on every trial.

$P(S) = p$

$P(F) = 1 - p = q$

Ex Not a Binomial Distribution

A shipment of 12 calculators with 5 Defective. We Select and Test 3 calculators.

Explain why this is Not Binomial

1. $n = 3$ Tested Calculators is fixed
2. Independent? No Probabilities change
4. $P(S_1) = \frac{5}{12}$ = prob. I get a defective on first trial

$P(S_2) = \frac{4}{11}$ or $\frac{5}{11}$ is Not the Same

This is Not Binomial

However if $N = 1200$ shipped with 500 Defective $\Rightarrow$

use Cumbersome calc Rule to

Use Binomial Even though

it is Not Quite right.

Determine if example is Binomial

Classic Binomial Multiple Choice Test with 8 Questions and 5 answer choices, a,b,c,d,e

this is Binomial because

1. $n = 8$ trials is fixed
2. If Guess are random, then answer on one question does Not effect the correctness on Next Question
3. Two Outcomes are Correct & incorrect
4. $P(S) = p = \frac{1}{5} = .20$
   $P(F) = q = \frac{4}{5} = .80$

Find the Binomial Probability Distribution for this problem

1. Formula
2. Table
3. Graph in Statcrunch
4. $\boxed{\text{Dist}}$ Binomial pdf

Multiple Choice Test (Classical Example)

there are 8 Questions
there are 5 choices a, b, c, d, e

a) Determine if this Describes a binomial Distribution

yes. ① $n = 8$ Trials

② Independent

③ Two Outcomes right or wrong

④ $P(S) = p = .2 = \frac{1}{5}$ $P(F) = q = .8 = \frac{4}{5}$

⇒ 5 choices $\frac{1}{5}$ is right & $\frac{4}{5}$ are wrong
a, b, c, d, e

b) Write Down the Probability Distribution

X	0	1	2	3	4	5	6	7	8	
P(X)	.1678	.3355	.2936	.1468	.0459	.0092	.0011	.0001	0+	= 1

$X \geq 5$

Small + Numb

STO▷ L_2

c) what is the probability of randomly Guessing and getting exactly 5 correct?

$P(X=5) = .0092 = \text{binpdf}(8, .2, 5)$

d) What is probability of At least 5 correct?

$P(X \geq 5) = .0092 + .0011 + .0001 + 0 = .0104$
$= 1 - P(X \leq 4) = 1 - \text{Binomial Cdf}(8, .2, 4) = .0104$

4 Methods to get a Binomial Probability Distribution

① Derive with a formula $X = \#$ correct

$P(X=r) = {}_nC_r\, p^r \cdot q^{n-r}$ $p = .2$ $q = .8$ $n = 8$

$P(X=0) = {}_8C_0\, .2^0\, .8^8 = 1(.8)^8 = .1678$

② Table in Book $p = .2$ $n = 8$

③ Calculator

$\boxed{\text{DISTR}}$

A: binompdf(n,p) = Distribution

Exactly r $P(X=r) = \text{binomialpdf}(n, p, r)$

= Probability of r Successes

At most r $P(X \le r) = \text{binomialcdf}(n, p, r)$

At least r $P(X \ge r) = 1 - P(X \le r-1)$

$= 1 - \text{binomial cdf}(n, p, r-1)$

Questions About Distribution

a) Find Probability of at most 2 Correct.

$$P(X \le 2) = .168 + .336 + .294 = .798$$
$$= \text{binomialcdf}(8, .2, 2) = .7969$$

Cummulative Relative frequency

Formula Page

At most

$$\boxed{P(X \le r) = \text{Binomial cdf}(n, p, r)}$$

b) Find the prob. of at least 3 correct

$$P(X \ge 3) = .147 + .046 + .009 + .001 + 0 + 0$$
$$= 1 - P(X \le 2) = 1 - \text{bincdf}(8, .2, 2)$$
$$= 1 - .7969 = .2031$$

$$\boxed{P(X \ge r) = 1 - \text{bincdf}(n, p, r-1)}$$

At least

Mult. choice Test $\boxed{\begin{array}{l} n=8 \\ p=.2 \end{array}}$ Questions = prob. of Success

Binomial

Table

X	0	1	2	3	4	5	6	7	8
P(x)	.168 *	.336 *	.294 *	.147	.046	.009	.001	0+	0+

$\underbrace{.168 \quad .336 \quad .294}_{P(x \le 2)}$ $\quad \underbrace{.147 \quad .046 \quad .009 \quad .001 \quad 0+ \quad 0+}_{P(x \ge 3)}$

Calc 2nd $\boxed{\text{DISTR}}$ △ A: binompdf

STO▷ L_2

Formula $P(X=2) = {}_8C_2\,(.2)^2\,(.8)^6$

$P(X=r) = {}_nC_r\,(p)^r\,q^{n-r}$

$E = \{$ CWCWWWWW $\quad 3 = 28$

prob of Each is $.2^2 \cdot .8^6$

$$\boxed{\begin{array}{l} P(X=r) = \text{binomial pdf}(n, p, r) \\ P(X=2) = \text{binomial pdf}(8, .2, 2) \\ \quad = \text{probability of exactly 2} \end{array}}$$

Is it Binomial?

Ex ① A shipment of 12 calculators and 5 are defective. 3 are selected to be tested.

Is this Binomial? Check 4 properties

① $n = 3$ Tested $3 > 5\%$ of $12 = .6$ (Cumbersome does Not Apply)

② Prob Not Replaced ⇒ Dependent

$P(\text{Def of 1st}) = \frac{5}{12}$ $P(\text{def on 2nd}) = \frac{4}{11}$ or $\frac{5}{11}$
Not the same

NO, Binomial Can not be used.

Ex 2 A shipment has 20,000 calculators and 20% are bad. We Test 3.

① $n = 3$ fixed

② Not independent but treat as independent because $n = 3 < 5\%(20{,}000) = 1000$

③ Defective or Not (good)

④ $P(D) = .2 = p$ doesn't change per trial

$\frac{4000}{20{,}000} \approx \frac{3999}{19999}$ close enough

The Mean and Standard Deviation of a Binomial Distribution

Standard
Prob Dist

$\mu = \Sigma\, x\, p(x) = E(x)$

$\theta = \sqrt{\Sigma\, x^2 p(x) - \mu^2}$

↑ ↑ Never

1-Varstat L_1, L_2

$L_1 = x$

$L_2 = P(x)$

$\bar{x} = 1.6$

$\theta = 1.1314$

Binomial

$\mu = np$

$\theta = \sqrt{npq}$

$n = 8$

$p = .2$

$q = 1-p = 1-.2 = .8$

$\mu = np = 8 \cdot (.2) = 1.6$

$\theta = \sqrt{npq} = \sqrt{8 \cdot .2 \cdot .8}$

$\theta = 1.13137$

Ex In Sonoma County 80% of Kindergardeners have been imunized against Measles. In a group of 17 Kindergardeners Is it Statistically Significant if we get at most 11 with imunity?

① Z-Score

② P(E) < .05

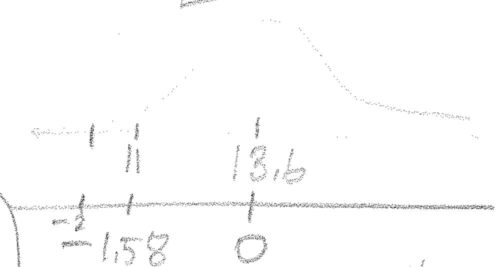
② $P(X \le 11) = \text{binomialcdf}(n, p, r)$

$= \text{binomialcdf}(17, .8, 11)$

$= .1057$ or 10.57% of the time

Not statistically significantly low!

① $Z = \dfrac{X - \mu}{\sigma} = \dfrac{11 - 13.6}{1.65} = \boxed{-1.58} \ge -2$



Not statistically low.

Below $\bar{X} - 2S = 13.6 - 2(1$
$= 10.3$ would be Considered statistically low

Binomial

$\mu = np = 17 \cdot .8 = 13.6$

$\sigma = \sqrt{npq} = \sqrt{17 \cdot .8 \cdot .2} = 1.65$

$q = 1 - p$

Ex 2 Kindergarteners have been immunized against Measles at a rate of .8.
In a group of 17 children is it Statistically Significant to get at most 11 with immunity?

2 Methods

① Range Rule is $11 \le \mu - 2\sigma$

Mean binomial $= \mu = np = 17 \cdot .8 = 13.6$

Standard deviation of Binomial $= \sigma = \sqrt{npq} = \sqrt{17 \cdot .8 \cdot .2} = 1.65$

Min. Usual $= \mu - 2\sigma = 13.6 - 2(1.65) = 10.3$

11 is Not below min usual so it is Not Statistically low.

② Prob method $P(X \le 11) \le .05$

$P(X \le 11) = \text{binomialcdf}(17, .8, 11) = \boxed{.1057}$

$P(X \le r) = \text{binomialcdf}(n, p, r)$

No, Not Statistically low because Prob of event is larger than .05.

Range Rule of thumb for Probability Distributions

Only use if dist is close to Normal

When Outcome $< \mu - 2\sigma$ we say that outcome is Statistically Sig. LOW

Outcome $> \mu + 2\sigma$ it's Statistically high

§5.1 28, 30

#28

	Win	Lose
X=$	30	-5
P(X)	5/38	33/38

$E(x) = \sum x \cdot p(x)$

$= 30 \cdot \frac{5}{38} - 5 \cdot \frac{33}{38} = -.39$

Single # = -.26 ← lose less here on a single bet.

Better to keep Money E(x) = 0 :)

#30

	Live	Die
X	-226	49,774
P(x)	.9968	.0032

a) Lives she pays $226
Dies her kids get
50,000 - 226 =

b) $E(x) = -226 \cdot .9968 + \underline{49774} \cdot .0032 = \$66$

c) On Average the company Makes $66
Per policy

$\bar{x} = \frac{226 + 226 + 226 - 49{,}774 + 226 \ldots}{10{,}000}$

= $66

Provide an appropriate response.

1) The random variable x represents the number of credit cards that adults have along with the corresponding probabilities. This distribution is not Binomial.

a) Find the mean and standard deviation.

x	P(x)
0	0.07
1	0.68
2	0.21
3	0.03
4	0.01

b) Find the probability of at most 2 credit cards.

c) Find the probability of at least 3 credit cards.

Assume that a researcher randomly selects 14 newborn babies and counts the number of girls selected, x. The probabilities corresponding to the 14 possible values of x are summarized in the given table. Answer the question using the table.

Probabilities of Girls

x(girls)	P(x)	x(girls)	P(x)	x(girls)	P(x)
0	0.000	5	0.122	10	0.061
1	0.001	6	0.183	11	0.022
2	0.006	7	0.209	12	0.006
3	0.022	8	0.183	13	0.001
4	0.061	9	0.122	14	0.000

$n = 14$

$p = .5$

2) For each of the following write the correct probability notation and the correct calculator entry to use to get the answer without the above table. Probability notation = Calculator input = Probability

a) Find the probability of exactly 10 girls. $P(X=10) =$ binomialpdf(14, .5, 10) $= .061$

b) Find the probability of at most 4 girls. $P(X \le 4) = .061 + .022 + .006 + .001 =$

= binomialCdf(14, .5, 4) =

c) Find the probability of at least 10 girls.

$P(X \ge 10) = 1 - P(X \le 9) =$

d) Find the probability of at least 12 girls.

e) What is the mean and standard deviation of this probability distribution?

f) Is it unusual to get at most 4 girls? Why?

Quiz 4 Math 15 Spring 2018 Jones Name________________

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c) Find the probability of at least 10 girls.

d) Find the probability of at least 12 girls.

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f) Is it unusual to get at most 4 girls? Why?

Provide an appropriate response.

3) Suppose you pay $2.00 to roll a fair die with the understanding that you will get back $4.00 for rolling a 2 or a 3, nothing otherwise. What is your expected value?

Use the binomial distribution to find the desired probability.

4) Merta reports that 74% of its trains are on time. A communittee group questions this parameter. In a random sample of 60 trains 38 of them arrived on time.

a) Use a binomial distribution to find the probability of getting a sample where among 60 trains, 38 or fewer arrive on time, if the overall ontime rate is 74%.

b) Based on the result, does it seem plausible that the "on-time" rate of 74% could be correct?

Determine if the outcome is unusual. Consider as unusual any result that differs from the mean by more than 2 standard deviations. That is, unusual values are either less than $\mu - 2\sigma$ or greater than $\mu + 2\sigma$.

5) A survey it is determined that 68% of consumers avoid products that have excesive packaging. A survey of 700 randomly selected consumers is to be conducted.

a) For such groups of 700, would it be statistically significant to get 521 consumers who avoid products with excesive packaging?

b) Find the probability randomly selecting of at least 521 consumers out of 800 who avoid products with excesive packaging?

Ex In the lunch room 68% of students use correct disposal container.

a) In an observation of 800 students what values would be considered statistically high or low? $\mu = np = 800 \cdot .68 = 544$ expected

$\sigma = \sqrt{800 \cdot .68 \cdot (1-.68)}$

$\sigma = 13.19$

$\sigma = \sqrt{npq}$

Stat high $\geq \mu + 2\sigma = 544 + 2(13.19)$

$\geq \underset{\text{Max expected}}{570.38}$ Students

Stat Low $\leq \underset{\text{Min expected}}{517.62} = \mu - 2\sigma$

b) If Guy observes 593 student use the correct bin is he happy?

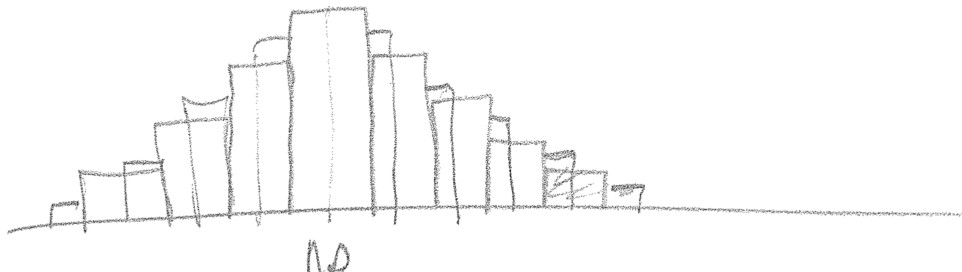
Yes! He's happy → This indicates that the compliance rate has increased above 68%.

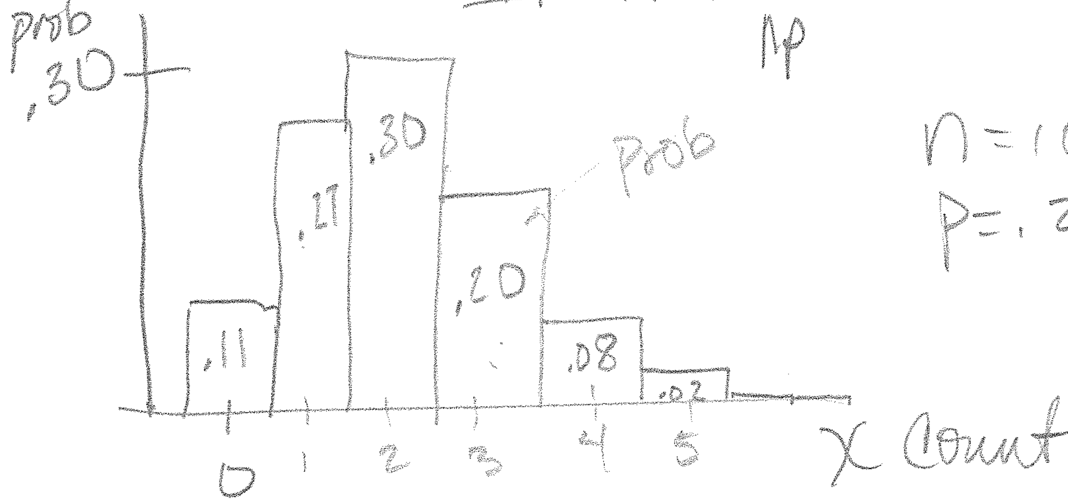
593 is Above Max expected When 68% use correct bin.

Note: Guy wants 100% Correct!
(Just Like Me!)

Approximate Binomial with Normal

Discrete Binomial have Prob. Histograms





$n = 10$
$p = .2$

If $np > 5$ and $nq > 5$ then Binomials Can be Approximated by Normal Distribution

$\mu = np \qquad \sigma = \sqrt{npq}$

Ex The probability that a customer will show up for a reserved seat is .97. The Airline sells 153 tickets for 150 seats. What is the prob. that there will Not be enough seats?

Airplaine Probability that a passenger arrives for their flight is .97. The plane has 150 seats but 153 tickets are sold. What is the probability that there will Not be enough seats?

Not enough

enough seats

$$P(\text{Not enough}) = P(X \geq 151)$$

$$= 1 - P(X \leq 150)$$

$$= 1 - \text{binomialCdf}(\underline{153}, .97, 150)$$

$$= .159$$

So they are over booking 16% of the time

Formula Page

Exactly r $= P(X=r) =$ binomialpdf(n, p, r)

At most r $= P(X \le r) =$ binomialcdf(n, p, r)

Cumulative Dist
Sum up to $X = r$

At Least r $= P(X \ge r) = 1 - P(X \le r-1)$

$= 1 -$ binomialcdf(n, p, r−1)

Mean and Standard Deviation Discrete

5.1 Discrete Not Binomial

$\mu = \sum X \cdot p(x) = E(x)$

$\sigma = \sqrt{\sum x^2 p(x) - \mu^2}$ (Don't use)

use

1-VarStat(L_1, L_2)

List freq

5.2 Binomial

$\mu = np$

$\sigma = \sqrt{npq}$