§4.3 Conditional Probabilities

Manufacturing Produces 70% good & 30% Bad

Goal: Understand "AT LEAST ONE"

Ex a) Box has 10 calculators, 7 good, 3 bad. We Select 3

Find $P(\text{At least 1 bad}) = 1 - P(\text{None Bad})$

$P(X \geq 1) = 1 - P(\text{All } 3 \text{ good})$

$= 1 - \frac{7}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} = \boxed{.708}$

B B B G G G G G G G

X = # of Bad Calculators

$P(G_1)\,P(G_2|G_1)$ Dependent

b) Ship 100 Calculators

$P(\text{At least one Bad}) = 1 - P(\text{None bad})$

$= 1 - \frac{70}{100} \cdot \frac{69}{99} \cdot \frac{68}{98} = \boxed{.661}$

c) On Assembly line 70% good

$P(\text{At least One Bad}) = 1 - P(\text{None Bad})$

$= 1 - .7 \cdot .7 \cdot .7$ Independent

$= 1 - (.7)^3 = \boxed{.657}$

b) & c) are close enough by cumbersome calc. Rule

but a) must use the Conditional Probabilities.

Air plane → Replace Valves

$P(\text{Valves good}) = .99$

$P(\text{At least one engine works}) = 1 - .99^4 = .9999$ ← only if independent!

4.3b Independence and Dependence

With Replacement → Independent

$P(R_2) = P(R_2 | B_1)$ ← Put it back

Without Replacement → Dependent

Draw 3 Marbles

$P(\underline{\text{At least One Red}})$

$= 1 - P(\text{None are Red})$

$= 1 - \frac{5}{9} \cdot \frac{4}{8} \cdot \frac{3}{7}$ ← Dependent

$= 1 - \left(\frac{5}{9}\right)^3$ ← Ind.

Without Replacement

With replacement

If $P(B) = P(B|A)$ then B is independent of A

Ex Alarm Clock

	W	W'
Watch	.90	.1
iPhone	.98	.02
Alarm Clock	.99	.01

What is probability that At least one will wake You?

$$P(\text{At least 1 Works}) = 1 - P(\text{None Work})$$

$$= 1 - P(\text{watch does Not work}) \cdot P(\text{iPhone } W') P(\text{AC } W')$$

$$= 1 - .1 \cdot .02 \cdot .01$$

$$= .99998$$

Redundancy

Class Try

Ex A multiple choice test has 5 Responces A, B, C, D, E for each of 15 Questions. What is the Probability of getting At Least One right?

$$P(\text{At least one Right}) = 1 - P(\text{None Right})$$

$$= 1 - \left(\frac{4}{5}\right)^{15} = \boxed{.9648}$$

$$= 1 - .0352$$

Ex

	C	$\bar{C}$	Total
R	6	26	32
$\bar{R}$	2	2	4
Total	8	28	36

If 3 class members are Randomly Selected what is the prob. that at least one recycles?

$$P(\text{At least one Recycles}) = 1 - P(\text{None of 3 Recycle})$$

$$= 1 - \frac{4}{36} \cdot \frac{3}{35} \cdot \frac{2}{34} = 1 - .000560$$

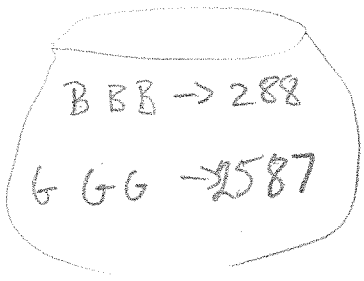
(.000560: Not Significant; 3 significant Digits)

$$= .999\underline{439}8$$

§4.2

#28 $n = 2875 = \#$ of containers.

$X = 288 = \#$ Bad



Find prob of selecting 3 good.

$$P(3\text{good}) = \frac{2587}{2875} \cdot \frac{2586}{2875}$$

Cumbersome Coda. Rule or $\left(\frac{2587}{2875}\right)^3 = \boxed{.7286}$

$3 <$ 5% of 2875 so OK

#26 $p(f) = .22$

$P(2\text{ fails}) = .22 \cdot .22 = \boxed{.0484} < .05$ ✓

While unlikely that both will fail we need higher likelyhood that at least one will work. ✓

$$P(\text{At least one works}) = 1 - P(\text{All fail}) = 1 - .0484 = \boxed{.9516}$$ ✓

Contingence Table Questions

Smoking	Non (N)	Occ (O)	Regular (R)	
Men M	436	42	71	549
Women W	326	47	78	451
Totals	762	89	149	1000

Select 2

a) Prob. Both Women

$P(\text{2 Women}) = \left(\frac{451}{1000}\right)^2 = .2034$ ← With Replacement

$\left(\frac{451}{1000}\right)\left(\frac{450}{999}\right) = \boxed{.2032}$ ← Without Replacement

b) Find Prob of At least one male.

$P(\text{At least one Male}) = 1 - P(\text{No Males in 2 draws})$

$= 1 - P(\text{All Women})$

$= 1 - .2034$

$= \boxed{.7966}$

c) $P(R|M)$ = Prob of Regular Smoker given that the person is male.

$= \frac{71}{549} = .1293$ = Prop. of Males who Smoke

$P(R|F) = \frac{78}{451} = .1729$ = Prop. of Women that Smoke.

Since $P(R|M) \neq P(R|F)$

$.1293 \neq .1729$

Prop of Males who smoke is 13% is less than Prop of women who smoke 17%.

We say Smoking Rates are <u>dependent</u> on gendar

$P(R) \neq P(R|F)$

A is <u>independent</u> of B if

$P(A|B) = P(A)$

<u>Alarms</u> Watch & iphone are independent
Wall clock & Oven timer are dependent

5% Rule for Cumbersom Calculations

If Sample Size n < 5% of population Size N. then the probability Can can be calculated using the with Replacement formula even if we don't Replace.

- This Makes Math calculations easier
- the probabilities Don't change enough to matter.

Ex $n = 2$ selected

$N = 1000$ – Group you are selecting from

5% of 1000 is 50 & $2 < 50$

4.3-b Probability Rules with a Contingency Table

	Non Smokers N	Occational O	Regular R	
Men	436	42	71	549
Women	326	47	78	451
	762	89	149	1000

a) What is the probability of randomly selecting a Non Smoking women?

Prob. Notation

$$P(W \cap N) = \frac{326}{1000} = .326$$

fraction → decimal

b) A Women or a Nonsmoker?

$$P(W \cup N) = P(W) + P(N) - P(W \cap N)$$

$$= \frac{451}{1000} + \frac{762}{1000} - \frac{326}{1000} = .887$$

c) Two Women that are Non Smokers?

think "Pick one WN_1, and then Another WN_2"

this is when we multiply → Two Tasks

Without Replacement

$$P(WN_1 \text{ and then } WN_2) = \frac{326}{1000} \cdot \frac{325}{999} = .106$$

.106056056

With Replacement

$$P(\text{2 women}) = \frac{326}{1000} \cdot \frac{326}{1000} = .106276$$

Conditional Probabilities

$P(A|B)$ Reads the Probability of A given that B has already occured.

With Marble

$P(B_2|B_1)$ = Prob of Blue on Second Given that first is a blue

$= \frac{2}{9}$

With Smokers

$P(M_2|M_1) = \frac{548}{999}$

$P(N|W)$ = The probability of a Non Smoker given that the person selected is a women.

$= \frac{326}{451} = \frac{n(N \cap W)}{n(W)}$

Independence

Compare = $P(R|M)$ = the prob of Selecting a regular Smoker given that they are male

Prop. of Male Smoker Regularly = $\frac{71}{549}$ =

Prop. of female Smokers Regularly = $\frac{78}{451} = P(R|F)$

Independent of gender if

$$P(R|M) = P(R|F) = P(R)$$

$$.129 \neq .1729$$

→ Since these are Not the same we say smoking Reg. Depends on gendar.